Multi-Supplier and Single Retailer Contracts: Profit Splits under Equilibrium

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Abstract: We study contracts between suppliers with fixed capacities and a single retailer with access to customers' demand and the power to influence the purchases of customers. This results in horizontal competition between suppliers and vertical competition between the retailer and his suppliers. The suppliers present the retailer with contracts and the retailer decides how much capacity to buy from the suppliers and sell to the market to optimize his net profits from retail selling. We are interested in the effect of equilibrium contracts on supply chain profit and how profits are split between the suppliers and the retailer.

We assume that the products are substitutable, and there is an inverse demand function so the retailer can control sales through prices. We assume that the retailer can influence the decisions of customers. As products become closer substitutes, customers become more indifferent to the supplier, making the retailer's role more important in deciding which product to offer to customers. We use game theory to analyze the players' interactions and the resulting equilibria on the basis of the players' sets of strategies. We present the results in the Nash framework, where suppliers make their decisions simultaneously, without knowledge of other suppliers' decisions. The game is between suppliers, but must take into account how the retailer will react in response to the contracts. We assume that suppliers' capacities are exogenously determined and fixed. The fixed capacities may be a result from competition at a higher strategic level, and are considered fixed during the contract designing. We also consider a deterministic demand model to focus on the profit split among the players in equilibrium.

Under mild assumptions on the supply chain's profit as a function of aggregate capacity, we show that all equilibria result in a coordinated chain with a unique profit split. The profit of each supplier is equal to the marginal contribution of her capacity to supply chain profits.

Keywords: *supply chain management, game theory, Nash equilibrium, negotiation*

1 INTRODUCTION

We study contracts between suppliers with fixed capacities and a single retailer with access to customers' demand and the power to influence the purchases of customers. This results in horizontal competition between suppliers and vertical competition between the retailer and his suppliers. The suppliers present the retailer with contracts and the retailer decides how much capacity to buy from the suppliers to optimize his net profits from retail selling. We are interested in the effect that contracts have on supply chain profit and its split between the suppliers and the retailer.

We assume that the products are substitutable and that the retailer can influence the decisions of customers. In a supply chain, as products become closer substitutes, customers become increasingly indifferent toward the suppliers' brands. This ascribes greater importance to the roles of retailers in terms of determining which products to offer and those that should be allocated greater sales efforts. In such settings, suppliers have to depend more heavily on the retailer to sell their products, possibly by offering more attractive contracts than those offered by other suppliers. The demand is assumed to be common knowledge to all players.

We discuss three significant aspects of our study. First, the inefficiency or competition penalty (the percent loss in supply chain performance due to decentralized decision making) can be enormous in some industries. Based on Cachon (2003), in some cases it can be as high as 40%, which shows a considerable need for coordinating contracts. Second, we contribute to the understanding of coordination mechanism under competition. *Supply Chain Management* (SCM) is a relatively new concept in management, which dates back to approximately two decades ago. Coordination is the at heart of successful SCM. This importance has been demonstrated in the following definition of SCM, which is provided by the Council of Supply Chain Management Professionals: "Supply chain management encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third-party service providers, and customers. In essence, supply chain management integrates supply and demand management within and across companies." Finally, our study has diverse applications in different industries and settings. For example, a personal computer (PC) and laptop retailer that sells different brands, or car brokers which supply cars from different manufacturers. Another application of our study is in the service industry, and more specifically airlines, where travel agents play an intermediary role between airlines and customers. A supply chain in not limited to supplying only goods; it can also supply services, where the supply chain's upstream is a service provider and the chain's downstream is a broker agent.

2 LITERATURE REVIEW

Since our study merges the two aspects of competition and coordination, we will be reviewing both these aspects. From the competition perspective, early mathematical models of competition date back to the 19th century. Bertrand (1883) described a competitive setting wherein two firms offer similar products and compete on the basis of price. In this classic model, firms select their price simultaneously and the firm that sets the lowest price for its product attracts all the demand. In the original model, firms have infinite capacities and a duopoly is sufficient to push down the price levels to the levels of perfect competition. The Bertrand model has been extended in numerous directions during the last century. One of the most important extensions of this model is the study of the case where capacities are limited. In this case, it may be beneficial for one supplier to charge a higher price and take the residual demand. Another extension of the Bertrand model is the case wherein products are just partially, and not perfectly, substitutable. In this case, the products are not identical and some customers may purchase the more expensive products. Gallego and Hu (2011) extended the competitive literature by considering limited capacities and by permitting the firms to dynamically change their prices over time. Our research may be viewed as an extension of the classical Bertrand model of oligopoly competition. Our model extends the Bertrand model in the following two innovative directions: First, it investigates a supply chain, where the suppliers, instead of being in direct contact with the market, are in indirect contact with the market through a retailer. Second, the type of contract is selected endogenously and the wholesale price is not fixed.

With regard to the coordination aspect, much evidence exists in both theory and practice to indicate that supply chains are not necessarily coordinated. This implies that the profit of the entire supply chain in a decentralized system is less than that of an optimal chain in a centralized system. There are two streams of literature which focus on this subject: contract theory in economics and supply chain coordination in operations management. Tirole (1988) and Spengler (1950) provided an initial identification and a review of supply chain coordination

failure in economics and operations management literatures, respectively. The literature on supply chain coordination using sophisticated contracts has been surveyed by, among others, Cachon (2003). Table 1 presents a list of sophisticated contracts that are extensively used in the business world in order to increase the total revenue of a supply chain.

Table 1. Widely used contracts to coordinate a supply chain

Contract	Description
Revenue sharing	The retailer pays a wholesale price and a share of his revenue to the supplier.
Full return (Buy-back)	The supplier charges a wholesale price per unit, but the retailer returns the unsold units to the supplier at the end of the sales period for a predetermined amount per unit.
Sale-rebate (Target rebate)	The supplier charges the retailer a per unit wholesale price, but pays the retailer a rebate on the per unit sold over and above a fixed target and the retailer continues to salvage the leftover units.
Two-part tariff	The supplier charges a wholesale price per unit, as well as a fixed fee or side payment.
Franchise	A combination of two-part tariff and revenue sharing contracts, where the supplier charges a wholesale price per unit, a fixed fee, and a share of the revenue.
Quantity discount	The supplier charges a per unit price which is a decreasing function of the order size.

Despite the extensive literature on supply chain coordination, most of the current literature focuses on the monopoly case, or non-competing multiple suppliers and/or retailers. Very few studies consider these contracts together with suppliers' competition. Boyaci and Gallego (2004) studied two competing supply chains that compete for a higher market share by providing superior customer service. They indicated that although coordination is a dominant strategy for both supply chains, it does not necessarily increase their profits. However, Boyaci and Gallego (2004) did not investigate the type of contracts that can coordinate supply chains and did not permit the condition wherein a retailer can purchase from more than one supplier.

Our research is innovative in the setting, methodology, and outcomes. Our study explores a new setting wherein both coordination and competition exist. To our best knowledge this is the first time this setting is modeled and analyzed. Its innovation is due to the fact that we merge two aspects of supply chains by studying how coordination can be achieved among competing suppliers. In order to model competition, we consider the existence of indirect competition among suppliers to influence the retailer to sell additional quantities of their products. This research can be considered a natural extension of Talebian (2010) and Gallego and Talebian (2013b), where we analyzed a model that comprised two competing suppliers with fixed capacities and a single retailer, thereby allowing for both horizontal and vertical competition. We were able to demonstrate that competition between suppliers can lead to wholesale price instability in equilibrium. Therefore, agents select contractual agreements that are more complicated, i.e., all-units quantity discounts. This is a rather interesting result, since we demonstrate that it is competition, and not channel coordination, that determines the need for sophisticated contracts.

3 MODEL AND METHODOLOGY

Figure 1 represents a schematic picture of our setting, where the suppliers choose their contracts. A wholesale price contract corresponds to a linear function; while more complicated contracts correspond to non-linear functions. Based on selected contracts by the suppliers, the retailer decides how much to sell from each supplier to maximize his profit.

We use *game theory* to analyze the players' interactions and the resulting equilibria on the basis of the players' sets of strategies. Suppliers face a dilemma in selecting the type of contract and its parameters. They must offer the most suitable contract or else, they face the possibility of losing sales since the retailer's sales efforts may be directed toward products offered by other suppliers. Thus, although the game is among the suppliers, they must consider the retailer's reaction in response to the contracts offered by them.

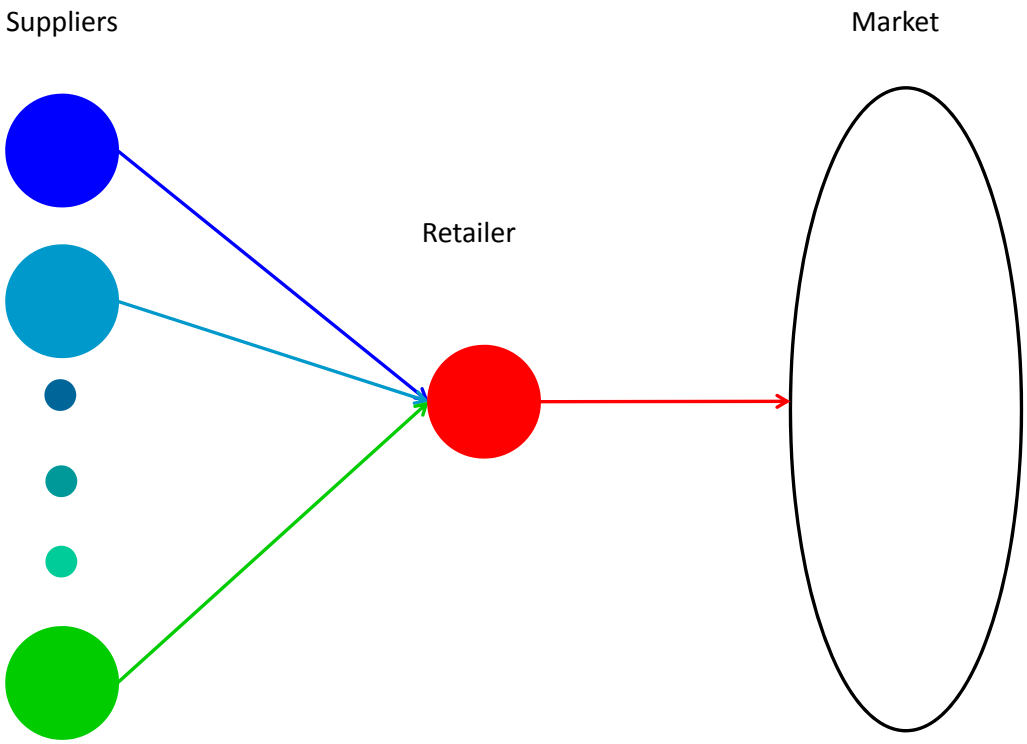

Figure 1. Game Scheme

Game theory was mainly developed in 1940s and 1950s by mathematicians. Its application in economics and managements can be tracked to 1980s and 1990s, as stated by Shubik (2002): "In the 50s ... game theory was looked upon as a curiosum not to be taken seriously by any behavioral scientist. By the late 1980s, game theory in the new industrial organization has taken over ... game theory has proved its success in many disciplines." Regarding the supply chain management, the use of game theory started in the late 20th century and is considered an active and young methodology in the community.

Game theory has two types of applications: descriptive and prescriptive. In descriptive applications, the main objective is to explain what is observed in practice, for example, why the prices are set higher than the optimum level or why certain suppliers avoid direct sale and prefer to sell their products through retailers. In prescriptive applications, the main objective is to identify how firms should establish their strategies by determining the best response to other firms' strategies.

A solution in game theory consists of defining a strategy for each player. A player's strategy is a plan of action for each possible situation in the game. We present the results in the *Nash* framework, where suppliers make their decisions simultaneously, without knowledge of other suppliers' decisions. A set of actions is Nash equilibrium (N.E.) if each player's action is the best response to the other players' actions. Nash equilibrium is a solution concept for games, wherein none of the players have incentive to deviate from the equilibrium. It concludes that in equilibrium, when all players observe other players' strategies, none of them would benefit from selecting another strategy. If players act simultaneously, the game is known as the strategic game, which can be represented in a normal form. We refer to Osborne and Rubinstein (1994) for the main concepts. A more elementary introduction to the subject can be found in Gibbons (1992).

Theorem 1 states our main result about the profit split in equilibrium. The result holds for a general set of models. More specifically, our model is general in the following directions: First, the contracts are unrestricted. Second, our model allows for a price setting or price taking retailer. Third, the retailer's sales effort have a general impact on sales and costs. Fourth, customers' behavior in choosing between different products is general and is not limited to a specific model.

Theorem 1. *In a competitive setting, all Nash equilibria result in a coordinated chain with the unique profit*

split. The split is such that the retailer pays each supplier the marginal profit she adds to the system as the last supplier.

Theorem 1 implies that the profit the retailer can make from suppliers does not change if one of suppliers withdraws, provided that the contracts do not change. To save space, we focus on our main results, and refer to Gallego and Talebian (2013a) for a detailed discussion of assumptions, results and their proofs. The proof includes establishing three main statements: the chain is coordinated in any N.E., profits are identical in any N.E., and the retailer is in equilibrium. The critical assumption for this result regards the the the maximum profit that the retailer can obtain from a fixed capacity, not including the procurement costs. We require that this function be submodular, i.e., have decreasing differences.

Theorem 1 shows that competition in addition to channel coordination drives the optimality of sophisticated contracts. Additionally, we show that all Nash equilibria result in a coordinated chain with a unique profit split such that each supplier gains her marginal contribution to the chain and results in a profit for the retailer when there are two or more suppliers.

Theorem 1 reduces to the single supplier setting profit split if all competitors, except one, have zero capacities. In this setting, the monopolistic supplier maximizes her profit and leaves the retailer with zero profit. It is consistent with the idea that the retailer's profit arises from the competition among suppliers, and therefore a unique supplier can collect all the profit using sophisticated contracts and leave the retailer with a minimum profit equal to his reservation value.

3.1 Example

Consider a setting of two suppliers each selling one product through the retailer. The two products are fully substitutable. The market size is 10, and prices are fixed at 20. To sell the vector (x_1, x_2), the retailer has a general corresponding sales cost of $(x_1 + x_2)^2$ and specific costs to sell suppliers' products as $.01x_1^2$ and x_2. It results in the profit function $\pi(x) = 20(x_1 + x_2) - (x_1 + x_2)^2 - .01x_1^2 - x_2$. For capacities 5 and 10, in equilibrium the profits of supplier 1, supplier 2, and the retailer, are given respectively, by 4.75, 20.25, and 70.

Graphes 2 and 3 show how profits change as the capacity of one of the suppliers changes. Graph 4 shows the minimum of suppliers' contracts.

Now, consider that each supplier has influenced on a part of the market such that the market size of 10 has been divided into 2 units loyal to Supplier 1, 3 units loyal to Supplier 2, and 5 units that are not loyal. As the result of this change, while total profit of the chain does not change, the first supplier gains at the expense of the retailer. The new profit split between suppliers and the retailer is 7, 20.25, and 67.75.

Finally, if both suppliers have non-limiting capacities, then in equilibrium, $x_1 = 9.90$ and $x_2 = 0$ which results in profit split as $\pi_1 = 8.75$, $\pi_2 = 0$, and $\pi_R = 90.25$. The result of this case is interesting as the presence of the second supplier prevents the first supplier from taking all the profits, although the second supplier does not gain any profit.

4 DISCUSSIONS

Our study elucidate the effects of business contracts in a competitive setting. We particularly believe that competition questions a few of the core insights of traditional models, especially, the emphasis on *negotiation* at the time of allocating coordination gains. Negotiation plays an important role in most monopolistic contracts since there exist multiple equilibria, and therefore no unique method to distribute the additional profits earned as a result of the coordination among the supply chain members. As Brandenburger and Stuart (2007) observe, different equilibria are usually an indicator that the competition is not having a powerful role in the model and players face a residual bargaining problem. However, in a competitive setting, the additional profits are distributed in a particular manner due to a more limited number of equilibria, which lowers the effect of negotiation on the outcome of the game. More broadly, we believe that this research makes an original and important contribution to our understanding of contracts in supply chains and their effect on the chain's efficiency and the total social welfare.

REFERENCES

Bertrand, J. (1883). Book review of theorie mathematique de la richesse sociale and of recherches sur les principles mathematiques de la theorie des richesses. *Journal des savants 67*, 499–508.

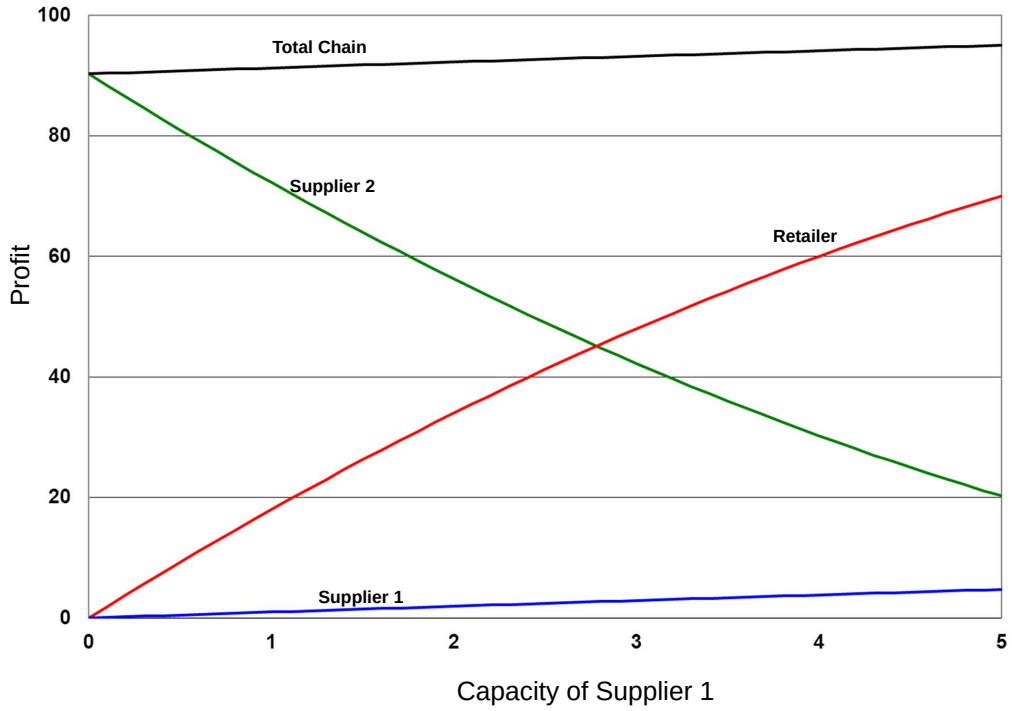

Figure 2.

Boyaci, T. and G. Gallego (2004). Supply chain coordination in a market with customer service competition. *Production Operations Management 13*(1), 3–22.

Brandenburger, A. and H. Stuart (2007). Biform games. *Management Science 53*(4).

Cachon, G. P. (2003). Supply chain coordination with contracts. In S. C. Graves and T. de Kok (Eds.), *Handbook in Operations Research and Management Science: Supply Chain Management: Design, Coordination and Operation*. North-Holland.

Gallego, G. and M. Hu (2011). Dynamic pricing of perishable assets under competition. *Working Paper, IEOR Department, Columbia University, NY*.

Gallego, G. and M. Talebian (2013a). Channel coordination under competition. *Working Paper, Columbia University*.

Gallego, G. and M. Talebian (2013b). Commissions and sales targets under competition. *Management Science*. Accepted conditionally.

Gibbons, R. (1992). *Game Theory for Applied Economists* (1 ed.). Princeton University Press.

Osborne, M. J. and A. Rubinstein (1994). *A Course in Game Theory*. Massachusetts: The MIT Press.

Shubik, M. (2002). Game theory and operations research: some musings 50 years later. *Operations Research 50*(1), 192–196.

Spengler, J. J. (1950). Vertical integration and anti-trust policy. *The Journal of Political Economy 58*(4), 347–352.

Talebian, M. (2010). *Ph.D. Thesis: Essays on Pricing and Contract Theory*. Columbia University, New York, NY.

Tirole, J. (1988). *The Theory of Industrial Organization*. Boston, MA: MIT Press.

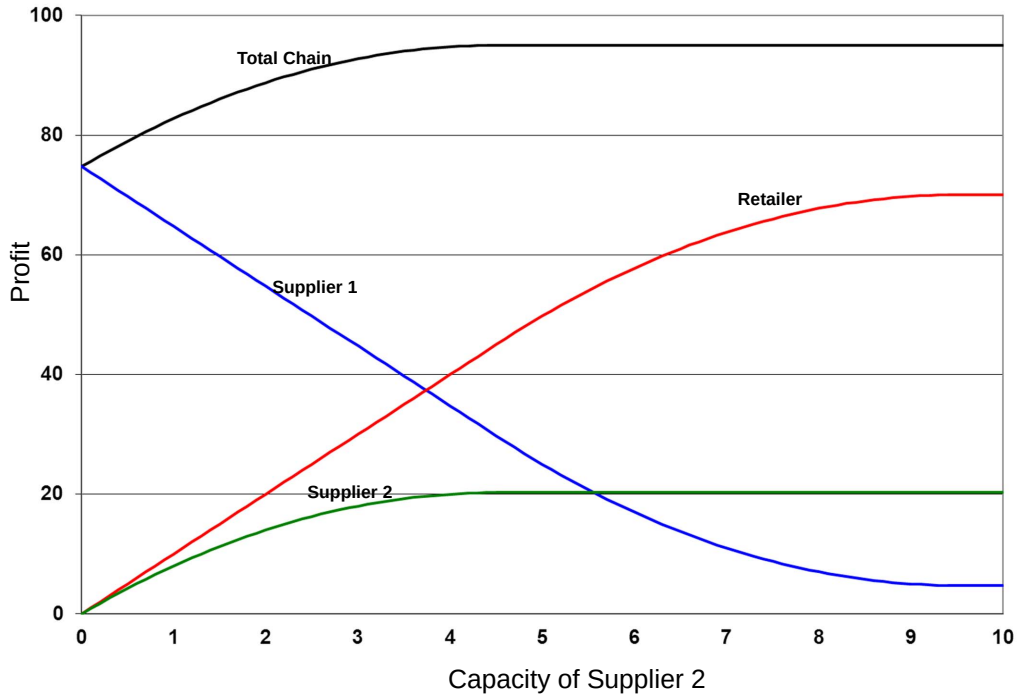

Figure 3.

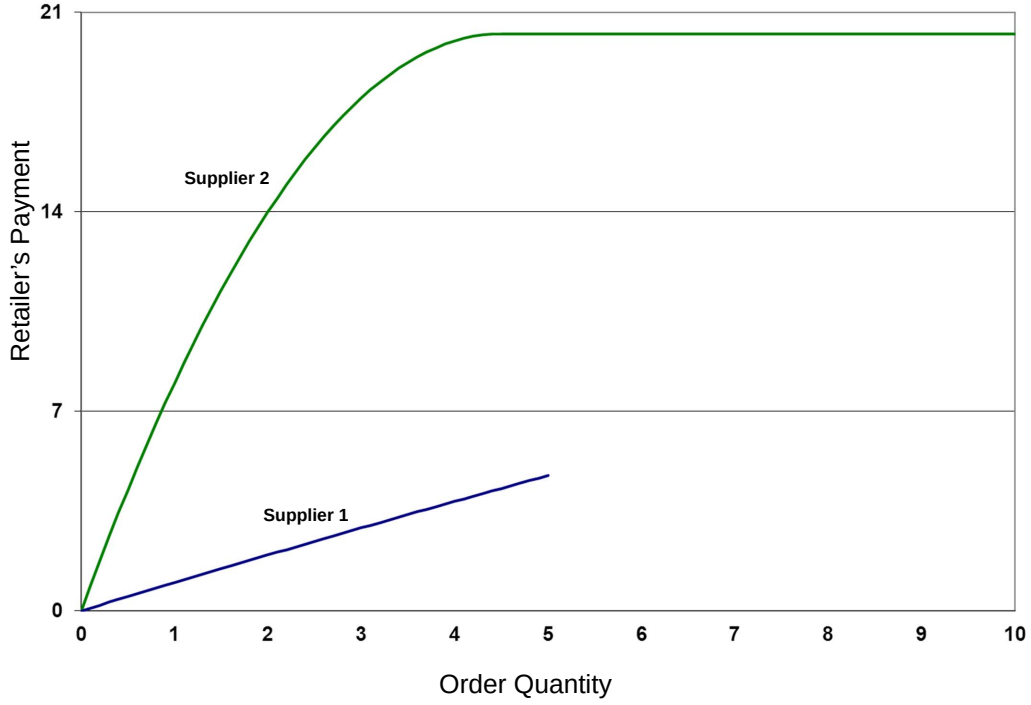

Figure 4.