Simulation model of crossing pedestrian movements for infrastructure planning

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Abstract: Infrastructure planning requires an extensive knowledge of potential pedestrian behavior, in particular at high crowd densities. The modelling and simulation of pedestrian movements is an important tool in the planning and operation of airports, railway stations, sports stadiums, shopping malls, and other public places. For example, in shopping malls, optimal models are needed to guide pedestrians on predefined itineraries. Evacuation scenarios, where all individuals move toward the same escape point, can be interpreted as single destination problems. These have been studied quite intensively. Multidestination problems, where distinct streams of pedestrians move from one or more starting points to multiple destinations, need more investigation. In particular, the crossing of pedestrian streams has not yet been thoroughly investigated.

In the last decade, various authors have modelled pedestrian flows by a macroscopic approach. Pedestrians in dense crowds behave much like gas particles. Consequentially, pedestrian flows are often modelled by partial differential equations that are similar to those used in models for gas or fluid dynamics.

A more realistic modelling of crossing situations can be obtained by an adequate description of pedestrian behaviour in the presence of crowded situations.

An adequate description of pedestrian behaviour in crowded situations leads to a more realistic model. We assume that pedestrians try to evade crowded spaces. This effect can be modelled as a function of local density. Thus, our model is based on the assumption that pedestrians avoid densely populated areas by moving in the direction of the negative gradient of the total local density $\varrho = \varrho_1 + \cdots + \varrho_n$, where $\varrho_i,\ i = 1,\dots,n$ is the density of a particular pedestrian group. This local orientation can be interpreted as the behaviour of blind persons with canes, who generally stick to their planned direction, but modify it by moving away from congestion, i.e. they detect the gradient. The assumption of sighted people would lead to a nonlocal model, which is beyond the scope of this contribution.

Our model developed has the general form

$$\frac{\partial \varrho_i}{\partial t} + \nabla \cdot \boldsymbol{f}_i(\varrho_1,\dots,\varrho_n;x,y) = \sum_{j=1}^{n} \nabla \cdot (b_{ij}(\varrho_1,\dots,\varrho_n)\nabla \varrho_j), \qquad i = 1,\dots,n, \tag{1}$$

where $\boldsymbol{f}_i$ and $b_{ij} \equiv b_{ij}(\varrho_1,\dots,\varrho_n)$ with $1 \le i,j \le n$ denote the flux vector and the components of a diffusion matrix $\boldsymbol{B}$, respectively. Different populations moving in different directions are represented by different phases. In the framework of modelling by balance laws with mass, momentum, and energy equations, Equation (1) corresponds to the set of mass equations. The constitutive functions make the momentum equations unnecessary. In this system of convection-diffusion equations, the convective term corresponds to a movement towards a strategic direction and the diffusion corresponds to a tactical movement that avoids jams. The convective and diffusion terms are deduced from a general mass balance. Thereby, we derive a nonlinear diffusion matrix that is superior to the linear diffusion matrix

$$\boldsymbol{B}(\varrho_1,\varrho_2) = \begin{pmatrix} \varepsilon & \delta \\ \delta & \varepsilon \end{pmatrix}. \tag{2}$$

Keywords: *Passenger simulation, Infrastructure planning, Multiphase continuum model, Convection-diffusion equation, Finite volume scheme*

1 INTRODUCTION

The two basic models for pedestrian behaviour are the microscopic and the macroscopic (Daamen et al., 2002). In the former, pedestrians are considered as individual objects interacting with each other; in the latter, their behaviour is analysed in terms of more global properties of a continuous stream. Macroscopic models interpret pedestrians as particles (with averaged flow intensity and speed) and focus on the balancing relationships of particle density. A third class, mesoscopic models, combine the main properties of the other two. For a detailed, comprehensive overview of both vehicular and pedestrian traffic and the main modelling and simulation approaches (in particular for macroscopic models), we refer to Helbing et al. (2001).

In the last decade, various authors Berres et al. (2012); Bruno et al. (2011); Hoogendoorn and Daamen (2005); Jiang et al. (2010); Nakayama et al. (2007); Xia et al. (2008) have modelled pedestrian flows by a macroscopic approach. Pedestrians in dense crowds behave much like gas particles, so models from gas or fluid dynamics are appropriate. Thus, most of the research on macroscopic models is focussed on the discussion and development of general partial differential equations, one- or two-dimensional in space, and based on physical principles such as mass, momentum, and energy balances. This contribution is a further development of our convection-diffusion model(Berres et al., 2011, 2012) with only linear diffusion. For $n = 2$, Berres et al. (2012) considered an equation of the form (1) with a diffusion matrix with constant coefficients. In Section 2, we develop a nonlinear diffusion matrix that models crowd-avoiding behavior. In Section 3, we simulate the equations for pedestrian streams moving in opposite directions.

2 MODELLING

The basic approach of our modelling assumes pedestrian flow to be a transport problem which is principally governed by a mass balance equation. Assume $n \in \mathbb{N}$ distinct pedestrian species. Let Ω be an open sufficiently smooth bounded domain in $\mathbb{R}^2$ and $(0, T)$ an open interval. For $(x, y) \in \Omega$, $t \in (0, T)$ the mass equation

$$\frac{\partial \varrho_i}{\partial t} + \nabla \cdot (\varrho_i \boldsymbol{v}_i) = 0, \qquad i = 1, \dots, n, \tag{3}$$

describes the mass flow where t denotes time and $\varrho_i \in [0, 1]$ the local densities of the i-th pedestrian species. The equations are coupled by the corresponding $\boldsymbol{v}_i = \boldsymbol{v}_i(\varrho_1, \dots, \varrho_n), 1 \leq i \leq n$, since each speed depends on all pedestrian densities.

When modelling the given transport problem, our solution has to reflect two aspects of pedestrian behaviour that correspond to strategic and tactical decision-making, respectively. On the one hand, a pedestrian has a target, which he tries to reach, and on the other hand he might to be forced to deal with local problems like high densities. To take account of these two aspects, we restart from the mass equation (3) and decompose the velocity as

$$\boldsymbol{v}_i(\varrho_1, \dots, \varrho_n) = \boldsymbol{v}_i^{\mathrm{s}}(\varrho_1, \dots, \varrho_n) + \boldsymbol{v}_i^{\mathrm{t}}(\varrho_1, \dots, \varrho_n), \qquad i = 1, \dots, n, \tag{4}$$

consisting of the following two components:

- a strategic component $\boldsymbol{v}_i^{\mathrm{s}}$, which reflects the disposition to follow the strategic goal of reaching a certain destination on a desired path
- a tactical component $\boldsymbol{v}_i^{\mathrm{t}}$, which locally avoids densely populated areas

These two components are not necessarily orthogonal, since the strategic component is prescribed “a priori”, whereas the tactical component adapts to the local arrangement of the pedestrians. Both velocity components are modelled as a product of velocity and direction,

$$v_i^{\mathrm{s}} = a_i V \boldsymbol{d}_i^{\mathrm{s}}, \qquad v_i^{\mathrm{t}} = b_i \varrho_i W \boldsymbol{d}_i^{\mathrm{t}}, \qquad i = 1, \dots, n, \tag{5}$$

where $\boldsymbol{d}_i^{\mathrm{s}}$ and $\boldsymbol{d}_i^{\mathrm{t}}$ denote the direction field giving a desired strategic and an adapted tactical walking direction, respectively. The functions $V = V(\varrho)$ and $W = W(\varrho)$ are velocity modules (scaling factors) in $[0, 1]$, where $\varrho = \sum_{i=1}^{n} \varrho_i$ is the pedestrian concentration in a representative elementary volume. The constants a_i and b_i are maximal direction velocities (maximum speeds).

Standard strategic directions are (for example) opposite or perpendicular. For a two-species model (where $n = 2$) the most simple examples for the strategic directions are given by $\boldsymbol{d}_1^{\mathrm{s}} = (1, 0)^{\mathrm{T}}$, $\boldsymbol{d}_2^{\mathrm{s}} = (-1, 0)^{\mathrm{T}}$

for a flow in opposite directions, or $\boldsymbol{d}_1^{\mathrm{s}} = (1,0)^{\mathrm{T}},\ \boldsymbol{d}_2^{\mathrm{s}} = (0,1)^{\mathrm{T}}$ for a flow in perpendicular directions. More sophisticated strategic directions are aligned to a potential field $\boldsymbol{P}$ with $\boldsymbol{d}_1^{\mathrm{s}} = \boldsymbol{P}_x,\ \boldsymbol{d}_2^{\mathrm{s}} = \boldsymbol{P}_y$. In this contribution, the strategic direction fields are designed such that the pedestrians try to reach their respective exit on the shortest path. This leads to the side effect that the corners of the exits are especially congested. Figure 1 illustrates the direction fields $\boldsymbol{d}_i^{\mathrm{s}}(x),\ i = 1,2$ of the two streams.

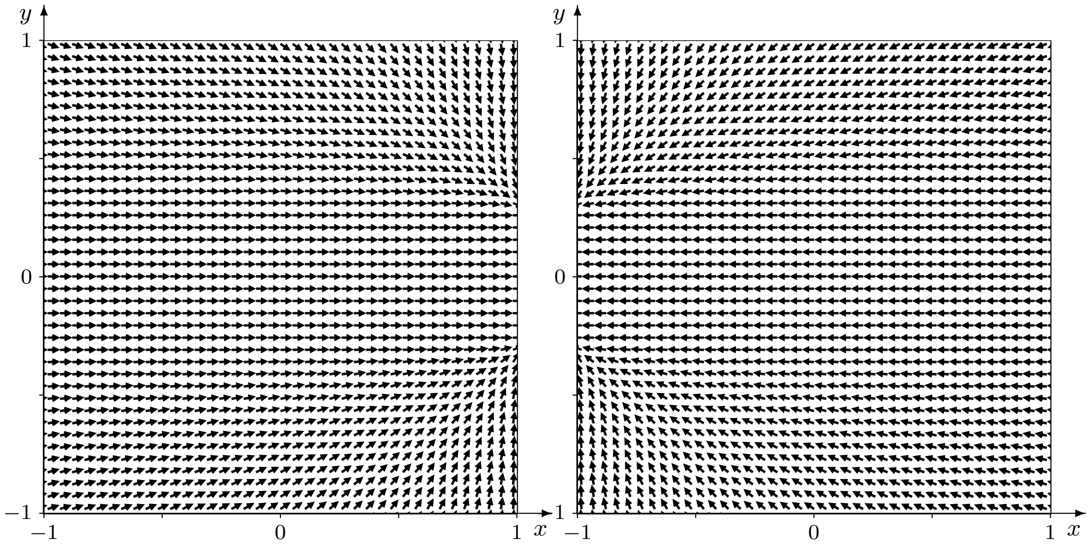

Figure 1. Direction fields of stream 1 (left) and stream 2 (right).

With these specifications equation (3) can be written as

$$\frac{\partial \varrho_i}{\partial t} + \nabla \cdot \left\{ \varrho_i \left[a_i V \boldsymbol{d}_i^s + b_i W \boldsymbol{d}_i^t \right] \right\} = 0. \tag{6}$$

The tactical direction is modelled using a partial normalization

$$\boldsymbol{d}_i^{\mathrm{t}} = \begin{cases} -\nabla\varrho/|\nabla\varrho| & \text{for} \quad |\nabla\varrho| > 1, \\ -\nabla\varrho & \text{for} \quad |\nabla\varrho| \leq 1, \end{cases} \tag{7}$$

by which unrealistic "escape" velocities can be avoided in the model. Defining

$$\chi(\varrho) = \begin{cases} 1/|\nabla\varrho| & \text{if} \quad |\nabla\varrho| > 1, \\ 1 & \text{if} \quad |\nabla\varrho| \leq 1, \end{cases} \tag{8}$$

we can interpret (6) as an example of model type (1) with

$$b_{ij}(\varrho_1, \ldots, \varrho_n) = b_i\, \varrho_i\, W\, \chi(\varrho), \quad i,j = 1, \ldots, n, \tag{9}$$

which describes a diffusive flux opposite to the gradient of the total density and proportional to ϱ_i. The variable V weights the strategic part of the pedestrian movement, and W the tactical. A generic assumption for V is that it is decreasing. This describes a throttling effect at higher concentrations: the more persons are in a given region, the more they get stuck on their way. The tactical velocity W is assumed to increase, which reflects the model assumption that the tendency to evade increases at higher concentrations.

The more persons are blocking the way, the stronger is the tendency to move along an alternate trajectory. Combining the qualitative behaviour of V and W, and modelling the concept that partitioning the flux into strategically and tactically caused parts results in a partitioning of total velocity (normalized to 1), one can impose the constraint

$$V + W = 1. \tag{10}$$

A pedestrian has an individual level of moving activity which he partitions between the two alternatives of moving to the desired target or evading jams.

In this contribution, we choose $V = 1 - \varrho$ and opt for assumption (10), which yields $W = \varrho$. Hence (9) becomes

$$b_{ij}(\varrho_1, \dots, \varrho_n) = b_i \, \varrho_i \, \varrho \, \chi(\varrho), \quad i, j = 1, \dots, n. \tag{11}$$

This leads to a diffusion matrix of the form

$$\hat{\boldsymbol{B}}(\boldsymbol{\varrho}) = \varrho \begin{pmatrix} b_1 \varrho_1 & \dots & b_1 \varrho_1 \\ \vdots & & \vdots \\ b_n \varrho_n & \dots & b_n \varrho_n \end{pmatrix} \chi(\varrho). \tag{12}$$

The convection-diffusion system (1) can be written in vector form as

$$\frac{\partial \boldsymbol{\varrho}}{\partial t} + \nabla \cdot \boldsymbol{f}(\boldsymbol{\varrho}) = \nabla \cdot \Big(\hat{\boldsymbol{B}}(\boldsymbol{\varrho}) \, \nabla \boldsymbol{\varrho} \Big), \tag{13}$$

where $\boldsymbol{\varrho} = (\varrho_1, \dots, \varrho_n)$ contains all concentration components.

3 NUMERICAL EXAMPLES

The first model variant uses the linear diffusion matrix $\mathbf{B}$, as specified in (2); the second uses the nonlinear matrix $\hat{\mathbf{B}}$ as specified in (12). We refer to the first as the "divergence" or Δ-simulation model, and the second as the "gradient" or ∇-simulation model. . The following examples illustrate the difference between the two models.

The crowd dynamics are considered to take place in the domain $\Omega = [-1, 1]^2$. The initial condition consists of an empty domain, i.e. $\varrho_i(0, x, y) = 0$ for all $(x, y) \in \Omega, \ i = 1, 2$. The subboundaries

$$\Gamma_w = \Big\{ (x, y) : x = -1, y \in [-0.3, 0.3] \Big\}, \qquad \Gamma_e = \Big\{ (x, y) : x = 1, y \in [-0.3, 0.3] \Big\},$$

represent combined entries and exits, while $\Gamma_c = \partial\Omega \setminus (\Gamma_w \cup \Gamma_e)$ denotes the walls. Streams 1 and 2 enter the domain by the doors Γ_w and Γ_e, respectively, and leave it by exits Γ_e and Γ_w, respectively.

We consider a two-dimensional situation of two pedestrian streams (where $n = 2$) meeting each other at an angle of $180°$, i.e. the streams move in opposite directions. Since the model is symmetric, the evolution of only one species (ϱ_1) is shown in the figures. The other species (ϱ_2) moves in the same way, but in the opposite direction.

The specification of the convective fluxes is completed by setting $a_1 = a_2 = 1$, $V = 1 - \varrho$. The diffusion term is $W \equiv 1$ in the linear model and $W = 1 - V = \varrho$ in the nonlinear. With these specifications, the model variants become

$$\frac{\partial \varrho_i}{\partial t} + \nabla \cdot \Big(\varrho_i (1 - \varrho) \boldsymbol{d}_i^{\mathrm{s}} \Big) = \varepsilon \, \Delta \varrho_i, \quad i = 1, 2,$$
$$\frac{\partial \varrho_i}{\partial t} + \nabla \cdot \Big(\varrho_i (1 - \varrho) \boldsymbol{d}_i^{\mathrm{s}} \Big) = \nabla \cdot \Big(\varrho_i \, \varrho \, \chi(\varrho) \nabla \varrho \Big), \quad i = 1, 2,$$

for the linear "Δ-model" and for the nonlinear "∇-model", respectively.

In order to illustrate the different qualitative behaviour of the two approaches, we use "normalized" parameters $\varepsilon = 0.01, \delta = 0$ in the linear diffusion matrix (2). This choice is motivated by the experimental observation that a larger ε reflects a larger diffusion that is spreading pedestrians rather quickly over the available space. This effect works against the assumption of pedestrian streams oriented towards a specific target. The coefficients of the non-linear diffusion matrix (12) are set to $b_1 = b_2 = 1$.

In Figure 2, the densitiy distribution for phase 1, simulated for the Δ-model, are shown at times 30, 40 and 480 (steady state). For better visualisation, the velocity field is magnified by the factors 0.55, 0.5 and 2 for the respective time steps. In Figure 3, the densitiy distribution of ϱ_1 and the corresponding velocity field of phase 1, simulated for the ∇-model, are shown at times 50, 75, and 100. The velocity field is magnified by the factors 0.5, 0.65, and 12 for the respective time steps. For both models, the accumulation of the streams

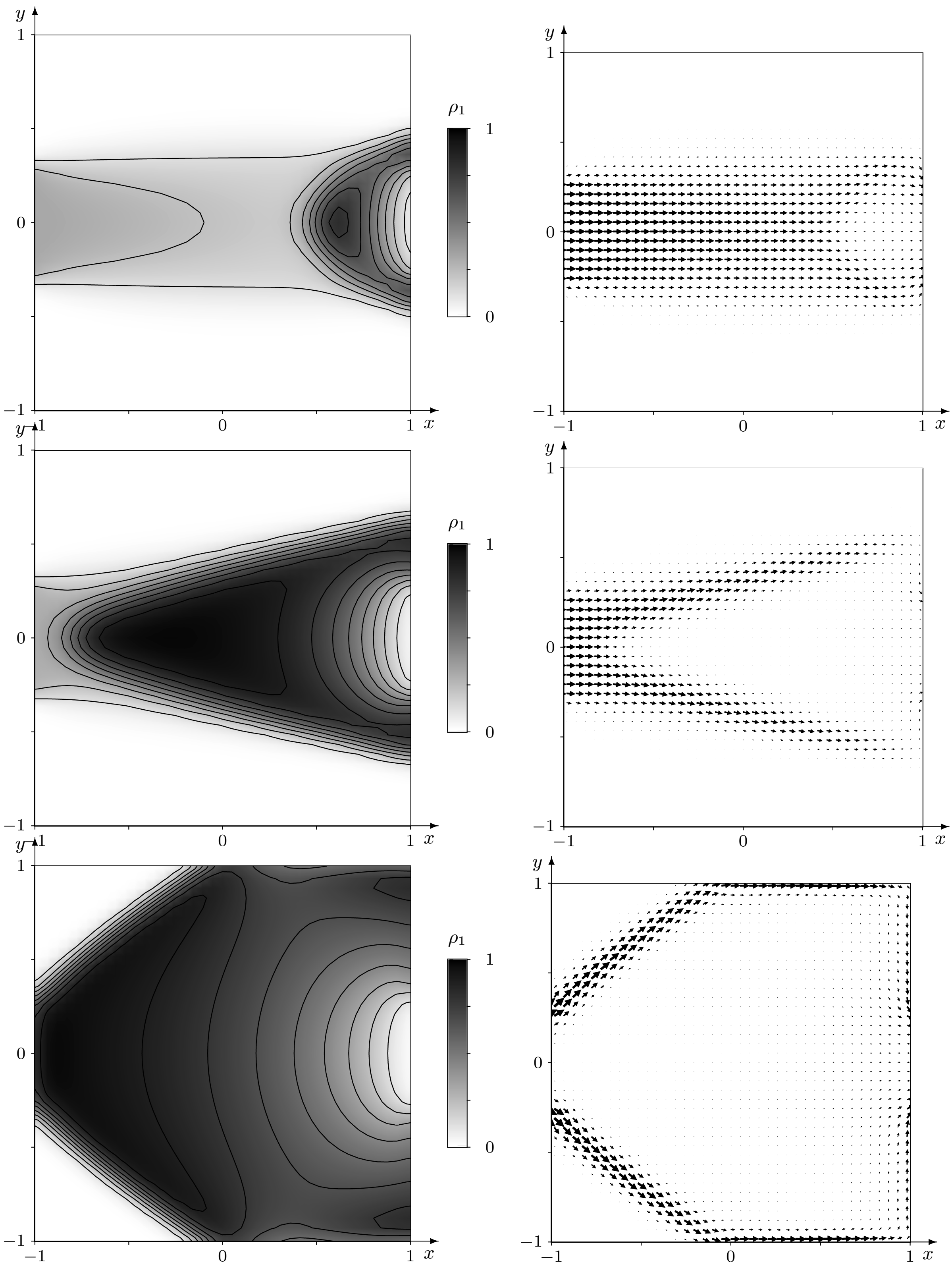

Figure 2. Density of ϱ_1 (left) and corresponding velocity field (right) for the Δ-model at times 30, 40 and 480 (steady state). The pedestrians diffuse back towards their entrance, blocking the opposing flow of pedestrians.

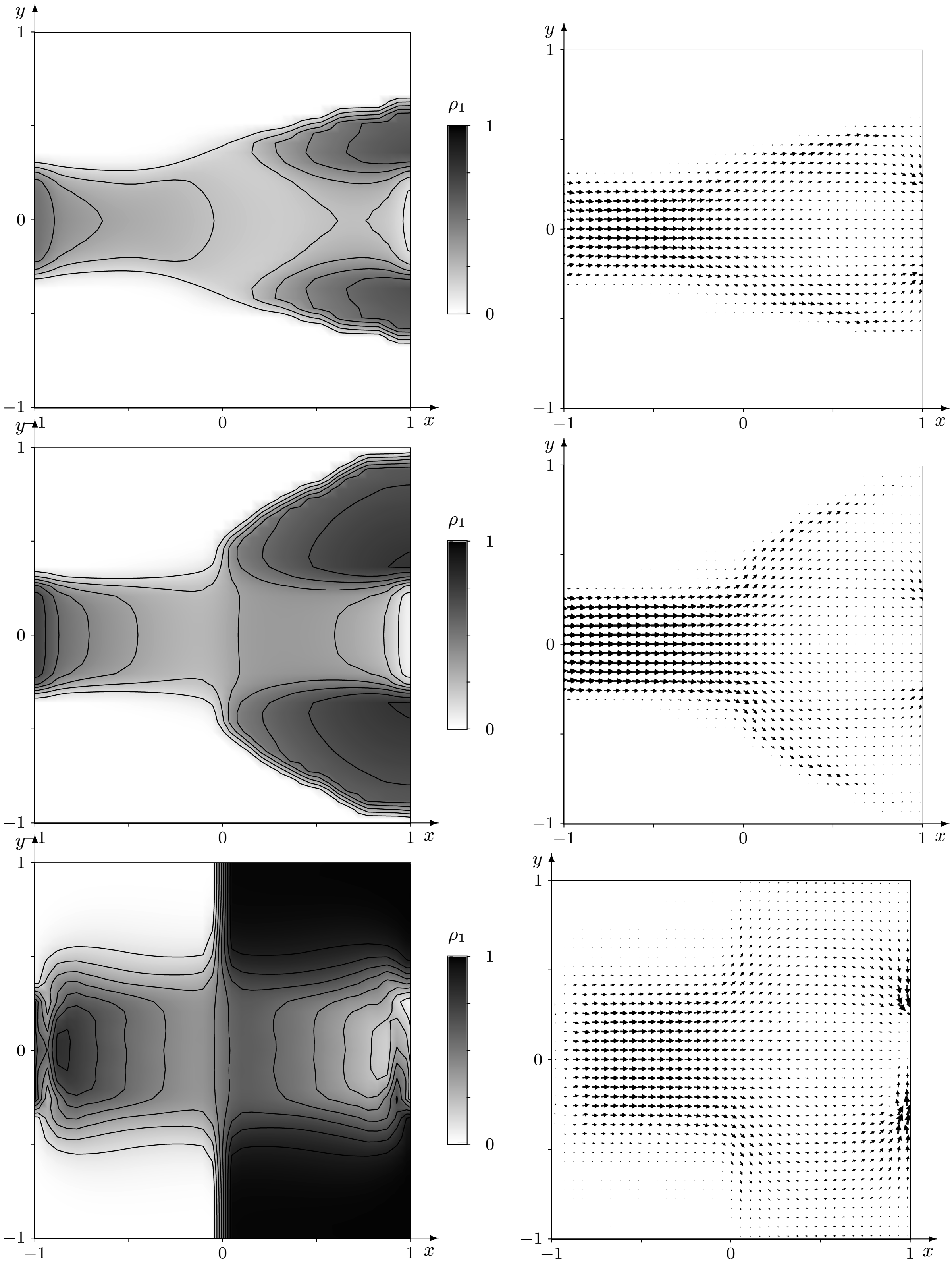

Figure 3. Density ϱ_1 (left) and corresponding velocity field (right) for the ∇-model at times 50, 75, 100. The pedestrians move to either side of their exit, allowing the opposing flow of pedestrians to move past.

starts at the side opposite to the entrance. This accumulation is built around the exit-door corners, leaving only a small entrance channel for the other species. In the ∇-model, this accumulation is concentrated at the sides of the exit door, leaving a broad entrance channel for the other species, whereas in the Δ-model, this accumulation is instead concentrated in the vertical center. Therefore, the accumulation covers the direct connection between the entrances and exists, leaving a only little space for the other species. This finally leads to a locking situation, in which the populations mutually block each other.

4 CONCLUSIONS

The Δ-model describes congestion situations by introducing diffusion in order to create evasion movements. The result, however, is a filling of empty spaces and does not align to our goal of a development of a simulation model capable to describe the crossing of pedestrian streams, where distinct particle streams try to reach their particular target without merging with the other streams. This goal prefers a mechanism for a better separation of the phases in particular at crossings. Such a separation is generated by an avoidance behaviour.

The linearity of the Δ-model does not provide a separation mechanism. The ∇-model assumes that pedestrians trying to reach a specific target are flexible enough to change direction in case of congestion or traffic jam. The movement towards the target is temporarily restricted by withdrawing from regions with higher densities, i.e., in the direction opposite to the gradient of total density. This model respects phase separation and circumvents congestion.

ACKNOWLEDGEMENTS

The first author was supported by Conicyt (Chile) through Fondecyt project #1120587. The third and forth authors were supported by the Deutsche Forschungsgemeinschaft through projects SCHW548/5-1 and BA1189/4-1.

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