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A Mathematical Programming Approach to Defence Logistics Funding

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Abstract

Defence decision-makers are faced with the problem of allocating funds to logistics support for the Australian Defence Force every year. This may be viewed as a complex, multi-criteria decision problem that is amenable to mathematical approaches directed at priority assessment and balance of investment. Our proposed framework for objective assessment of the operational impact of various levels of logistics funding and a mathematical programming model of the logistics allocation problem are presented in this paper. The implementation of the framework and model indicates a more effective funding profile over the 'status-quo' situation of purely baseline funding which could lead to enhancements in capability.

Introduction

The readiness of the Australian Defence Force (ADF) is strongly dependent on the availability of its inventory of defence platforms. Higher levels of readiness translate into higher costs associated with the logistical support required to achieve higher levels of availability. Not all defence platforms are at the same level of readiness and the costs of logistical support vary also with platform type. Defence decision-makers, therefore, need to make informed decisions as to the levels of logistics funding allocated to each defence platform. This problem is considered to be one of Balance of Investment (BoI). That is, to determine the most appropriate allocation of limited resources to achieve the best 'value for money' in terms of ADF capability.

One way to examine the implication of the levels of logistics funding allocated to each defence platform under financial constraints and other business rules is to formulate a classical mathematical program (MP) of the form of the capital allocation or Knapsack problem (Winston 1994), or some extension of this basic model (Haynes et al. 2005, Brown et al. 2004, Greiner et al. 2003, Radulescu and Radulescu 2001). Prioritisation is usually a necessary step to determine the relative importance of the logistics bids. A particularly important aspect of the prioritisation step is the choice of measurement method. Some prioritisation measurement methods for defence are described in Nguyen 2003 and references therein. Our MP model differs from the above studies by the nature of the objective function, which is defined via a set of piecewise-linear functions.

Although our problem fits within the class of multi-activity, multi-period resource allocation problems (see for example Reeves and Sweigart 1982, Luss and Smith 1988, Klein et al. 1995, MirHassani et al. 2000), we will not consider the uncertainty in the resources or activities over time periods. This means that our model is a deterministic linear or mixed-integer program and is therefore computationally easier to solve.

Platform P	Y01	Y02	Y03	Y04	Y05	Y06	Y07	Y08	Y09	Y10
Baseline	20	20	30	30	30	0	0	0	0	0
Bid	0	0	0	5	5	5	5	0	0	0
Total	20	20	30	35	35	5	5	0	0	0

Table 1: Sample of Funding and Bid Data ($M) for a Platform.

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Platform Q	Target No.	Y01	Y02	Y03	Y04	Y05	Y06	Y07	Y08	Y09	Y10
Band 1 (High)	4	**3**	**2**	**3**	**2**	5	6	**3**	6	4	**3**
Band 2 (Medium)	5	**4**	5	7	7	7	5	**3**	5	**3**	**4**
Band 3 (Low)	13	**10**	**9**	**6**	19	**10**	**9**	**9**	**11**	19	18

Table 2: Sample of Readiness and Justification Data (numbers of platforms) for a Platform.

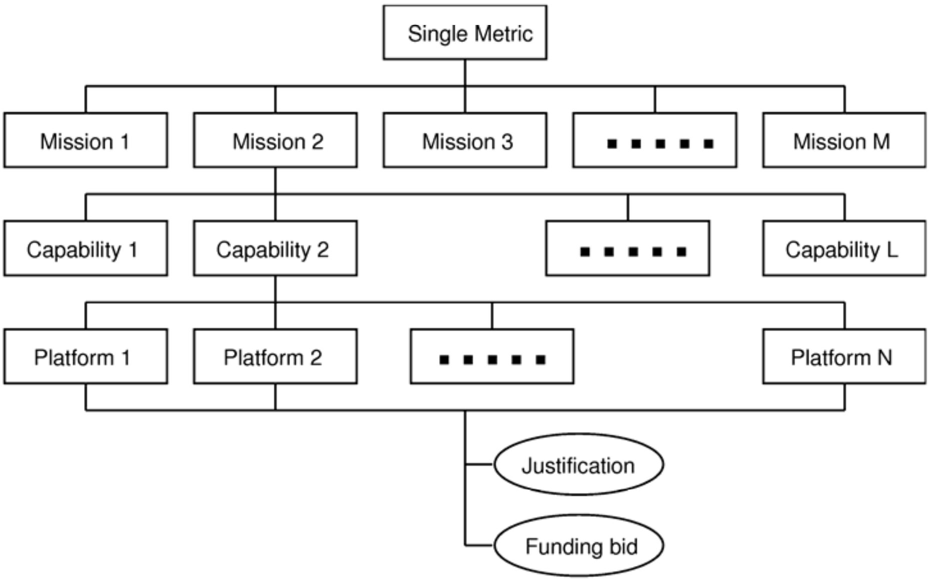

Figure 1: Assessment hierarchy to compute platform importance

Logistics Allocation Problem

Each year, all of the Defence platform (e.g. aircraft, ships, vehicles) logistics managers submit a bid for logistics funding. The bids include (among other things) the baseline funding spread over the next 10 years that the platform has been allocated; the funding spread over the next 10 years that the platform calculates as being required to bring the capability to its expected level; and a justification which is typically presented as a difference in estimated platform numbers against set availability targets. These numbers and targets are generally specified at different availability bands (e.g. high, medium and low availability). Table 1 illustrates hypothetical funding and bid data for a platform, and Table 2 illustrates hypothetical readiness and justification data for a platform.

Defence has the difficult problem of analysing a large set of platform bids in order to choose portfolios which meet limited budgets while managing the overall capability. If no additional supplementation funding is to be provided, funding for a particular platform must be taken from some other platform's baseline. Defence collects all the bids that are prioritised within the Service Group (Army, Navy, Air-Force), and performs a whole-of-Defence analysis to produce funding prioritisation recommendations from the viewpoint of operational requirements.

Our work in support of this problem consists of developing and applying: (1) a framework for more objective prioritisation of logistics bids; and (2) a mathematical formulation of the allocation problem with a decision support tool for automation of the process[1].

Prioritising Logistics Bids

Assessment Hierarchy

To assess the importance of a platform's funding bid, an assessment hierarchy (Figure 1) with 3 layers is proposed. This

[1]Note that the framework and mathematical model presented in this paper do not represent an agreed or endorsed position by Defence. Furthermore, all data presented here is fictitious and used for illustrative purposes only.

Level	Criticality Rating	Description
1	Supplementary	This capability supplements the mission, and is of secondary importance. Loss of this capability would not cause mission failure. However the failure of several supplementary capabilities in conjunction would compromise the mission.
2	Significant	This capability is of significant importance to the mission. Failure or deficiency of this capability will result in noticeable degradation of the mission.
3	Primary	This capability is of primary importance to the mission. Failure or deficiency of this capability will result in a severe degradation of the mission.
4	Indispensible	The mission is critically dependent upon this capability. Deficiency of this capability will result in complete mission failure.

Table 3: Rating Scheme for Capability Contribution to Missions

Level	Impact Rating	Description
1	Minor	The role of this platform has a minimal effect on the overall ability to achieve that capability.
2	Significant	The role of this platform has a significant effect on the overall ability to achieve that capability.
3	Major	The role of this platform has a major effect on the overall ability to achieve that capability.
4	Severe	The role of this platform has a severe effect on the overall ability to achieve that capability.
5	Critical	The role of this platform has a critical effect on the overall ability to achieve that capability.

Table 4: Rating Scheme for Platform Contribution to Defence Capabilities

approach accounts for the contribution of a platform's role (layer 3), defence capabilities (layer 2) and various missions (layer 1) against which the services prepare their respective forces. The justification and funding bid tags are not part of the assessment hierarchy but are used for each platform in estimating the potential capability enhancement and the cost of capability, respectively (see Equation (2)).

This is a simplified version of the Bayesian network of Platforms to Missions used in Chisholm and Asenstorfer 2006. By contrast, in order to keep the amount of input data manageable and to allow the formulation of the logistics allocation problem as a MP model, the assessment hierarchy employs the most widely used Multi-Attribute Decision-Making (MADM) method, which is the simple additive weighting (also known as the weighted sum) method (Edwards 1977).

This hierarchy needs to be populated with data that represents the relative contribution/importance of elements in one layer to those elements in the next layer above. Working from the top, we need to represent the relative importance of each of the missions to produce a single metric. This is preferred because it avoids 'multi-valued' criteria. The values used at this layer of the hierarchy could represent the relative likelihood of each mission, relative consequences of each mission, or (combining these two) relative risk of each mission. One could also use an equal weighting scheme, or choose to focus on only a single mission. The next two layers require data for the relative contribution of each of the defence capabilities to each of the missions and the relative contribution of each of the platforms to each of the defence capabilities. Subject Matter Advisors can be used to generate this information. The rating scheme used for these layers are given in Tables 3 and 4.

To obtain the platform importance index, we first normalise all rating values of the three layers. Let $W_{MIS}(k)$, $W_C(k,l)$, $W_P(l,i)$ be the corresponding normalised value (weighting) of the rating values $R_{MIS}(k)$, $R_C(k,l)$, $R_P(l,i)$ from the Mission, Capability and Platform layer respectively, where $k \in \{1, \ldots, M\}$, $l \in \{1,\ldots,L\}$, $i \in \{1,\ldots,N\}$ and M, L and N are the number of Missions, Capabilities and Platforms respectively. The platform importance index, I_i, is then given by

$$I_i = \sum_{k=1}^{M} W_{MIS}(k) \left(\sum_{l=1}^{L} W_C(k,l) \, W_P(l,i) \right), \quad (1)$$

with
$$W_{MIS}(k) = \frac{R_{MIS}(k)}{\sum_{q=1}^{M} R_{MIS}(q)},$$

$$W_C(k,l) = \frac{R_C(k,l)}{\sum_{q=1}^{L} R_C(k,q)}, \text{ and}$$

$$W_P(l,i) = \frac{R_P(l,i)}{\sum_{q=1}^{N} R_P(l,q)}.$$

Estimating Overall Priorities

The above importance figures are used to represent the generic importance of the platforms - irrespective of the criticality of the logistics bid. To take this latter aspect into account, we use the readiness and justification data which was illustrated in Table 2. Evidence of a logistics induced shortfall occurs if there is a difference in the numbers in the ‘target’ column and those in the ‘forward plan’. In Table 2, the **boldface type** in the forward year numbers indicates these occurrences. The shortfall is stated as the relative (or percentage) loss in capability, so that this can be more reliably compared across different platforms. Note also that there are different ‘availability bands’. Band 1 may indicate that platforms need to be ‘ready to go’ in under 48 hours. Other bands may indicate longer warning times. These are ‘rolled-up’ by assigning a weighting scheme. One could use an equal-weighting scheme (no preference among bands) or any other (for example, giving short notice bands more weight than those with longer warning times).

Let us denote w_b to be the weighting of availability bands, t_{bi} to be the required target and n_{bij} to be the estimated platform number, where $b \in \{1, \ldots, B\}$, $i \in \{1, \ldots, N\}$, $j \in \{1, \ldots, T\}$ and B, N and T are the number of availability bands, platforms and forward years respectively. The shortfall value s_{ij} of platform i in year j is calculated by the expression

$$s_{ij} = \sum_{b=1}^{B} w_b \, \frac{t_{bi} - n_{bij}}{t_{bi}} \; .$$

Finally, to produce a single value s_i for platform i, each of the years in the forward plan are given a weighting, which we denote by w_j^T. The values for s_{ij} are then ‘rolled-up’ into one number using the equation

$$s_i = \sum_{j=1}^{T} w_j^T \; s_{ij} \; . \quad (2)$$

Now we calculate the overall priority, denoted by P_i, of platform i using the equation

$$P_i = I_i \; s_i \; .$$

The above computation of platform priorities is a useful first step in the two-step process of the logistics funding problem. The more difficult step is to determine the best allocation (or in the case of no supplementation, the best reallocation) of funds across the various platforms. To solve this decision problem, we now develop a mathematical programming model.

Mathematical Programming Model

Determining an optimal logistics allocation solution has features in common with the well-characterised knapsack problem (Winston 1994, page 468). The knapsack problem involves maximizing the benefit from the contents of a knapsack, given a range of possible items that can be selected. Each item has a defined benefit and weight, while the knapsack itself has a total weight limit.

Here the logistics allocation problem involves minimising ‘the total level of residual capability shortfall’ by funding some platform bids from all bid submission, while subject to a budget constraint and various departmental business rules. The logistics allocation problem has an additional temporal dimension which is not present in the knapsack problem. In the knapsack problem, a resource is selected and used during only one time period. However, in the logistics allocation problem, the resource has been funded for multiple years. The situation is also complicated by allowing cuts to be made from baseline funding of other platforms. The decision variable (level of funding) may not, therefore, be suitably represented by a binary number, and the benefit (level of capability) must, therefore, be adjusted to the funding level.

Decision Variables

The decision variables of concern can be defined as ‘the fraction of the total funding bid to fund in each year j for each platform i’. Denote this by x_{ij}, where $i \in \{1, 2, \ldots, N\}$ (N is the number of platforms) and $j \in \{1, 2, \ldots, T\}$ (T is the number of years out from the current financial year). By definition, x_{ij} must be in the range $[0, 1]$.

Let $X_{ij}^0 \stackrel{\text{def}}{=} \frac{F_{ij}^{BL}}{F_{ij}^{BL}+F_{ij}^{SF}}$, where F_{ij}^{BL} is the baseline funding and F_{ij}^{SF} is the shortfall funding bid (see Table 1), then the value of x_{ij} can be one of the followings:

- $x_{ij} = 0$: no funding (neither baseline nor shortfall) for Platform i in year j.
- $0 < x_{ij} < X_{ij}^0$: Platform i receives less than its baseline funding in year j (i.e. its baseline funding is cut). This can happen if there is no additional supplementation funding and funds are needed elsewhere.
- $X_{ij}^0 < x_{ij} < 1$: Platform i receives its baseline funding and some of its shortfall bid in year j.
- $x_{ij} = 1$: Platform i is completely funded in year j (if platform i submitted a bid, this means it receives its baseline funding and its entire bid funding in that year. If platform i did not submit a bid, this means it receives its entire baseline funding in that year).

Allowing x_{ij} to range between zero and one, therefore, allows greatest flexibility in the funding options, from partial funding of the baseline amount, to a baseline and partial funding through to completely funding the platform.

Objective Function

Recall, the objective is to minimise 'the total level of residual capability shortfall' expressed mathematically by

$$\min \sum_{i=1}^{N} \sum_{j=1}^{T} I_i \; w_j^T \; S(x_{ij}). \tag{3}$$

This expression contains a product of three terms:

- I_i represents the importance of platform i, which was calculated in Equation (1).
- w_j^T represents the weight afforded to elements that occur in year j in the T-year forward plan. One schema may assign equal weights. Another schema may use a monotonically decreasing set of weights, indicating a preference or concern with the 'inner' years over those in the 'outer' years. The values for w_j^T can be chosen by the user.
- $S(x_{ij})$ represents the capability shortfall associated with platform i in year j if funding is provided according to the decision variable x_{ij}. The values for $S(x_{ij})$ are computed from the following theoretical model.

In Equation (2), we 'rolled-up' the data over the forward plan years, so that a single metric s_i could be computed (to allow prioritising platforms). For the decision problem, we maintain the temporal variation. The platform capability must be also dependent on the level of funding. Hence, the term $S(x_{ij})$ is used.

Figure 2 graphically describes the 'cost of capability' model we are using.

- The horizontal axis denotes the funding level for platform i in year j, by way of the decision variable x_{ij}. The point at $X_{ij}^0 = \frac{F_{ij}^{BL}}{F_{ij}^{BL}+F_{ij}^{SF}}$ represents funding of the baseline only. To the left and right of this point represents cutting the baseline and partially funding the bid, respectively. The vertical axis denotes the percentage of shortfall for platform i in year j, i.e. $S(x_{ij})$.
- There is one data point in this model. Recall that the readiness and justification data in Table 2 details the projected platform numbers across the forward plan assuming baseline funding only is provided. Thus, when $x_{ij} = X_{ij}^0$ we have from this data, s_{ij}, on the vertical axis, i.e $S(X_{ij}^0) = s_{ij}$.

We assume here that the capability is degraded completely ($S(x_{ij}) = 1$) if no funding at all is provided ($x_{ij} = 0$) and the capability is restored ($S(x_{ij}) = 0$) if funding is completely provided ($x_{ij} = 1$). Between these points, we assume a linear change. Two forms are possible, which represent either a situation of $s_{ij} < 1 - X_{ij}^0$ or $s_{ij} > 1 - X_{ij}^0$. Which of these exists depends on the data for the specific platform. A third case is also depicted in Figure 2, represented by the dotted line. This line is used to model the situation for platforms that did not submit a logistics bid, and is consistent with the other two cases (i.e. the point X_{ij}^0 becomes 1).

Mathematically, we define $S(x_{ij})$ as follows:

- For a platform with no bid,

$$S(x_{ij}) = 1 - x_{ij} \; .$$

- For a platform with a bid,

$$S(x_{ij}) = \begin{cases} \frac{s_{ij}-1}{X_{ij}^0} \, x_{ij} + 1 & \text{if } 0 \le x_{ij} \le X_{ij}^0, \\ \frac{s_{ij}}{X_{ij}^0 - 1} \, (x_{ij} - 1) & \text{if } X_{ij}^0 \le x_{ij} < 1. \end{cases}$$

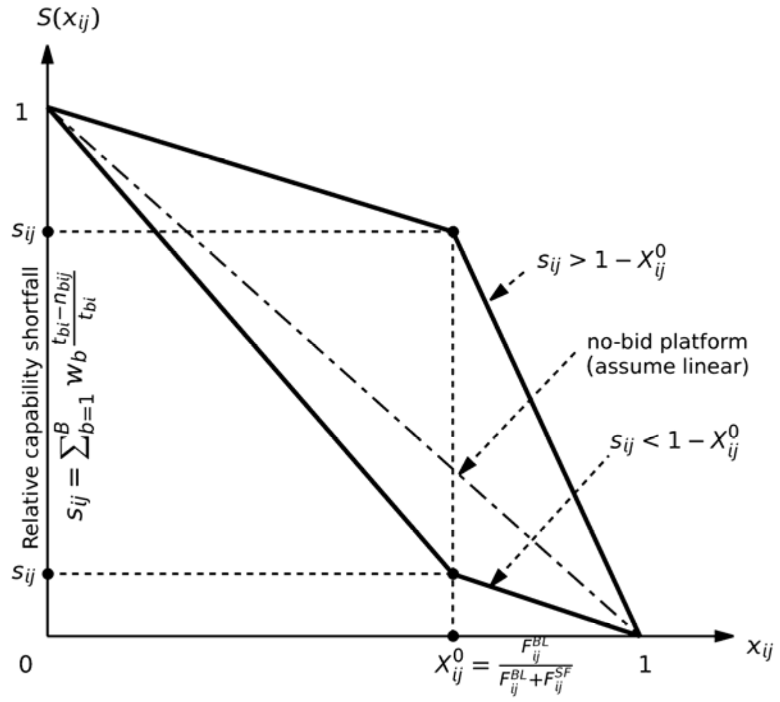

Figure 2: Cost of Capability Model for Relating Funding Profiles with Shortfall Value

Constraints

The minimisation of the objective function by determining the appropriate funding profiles may be constrained by various business rules. The most obvious is a budget constraint:

$$\sum_{i=1}^{N}\Big[(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-F_{ij}^{BL}\Big] \le 0,\ \forall j \in \{1,\dots,T\}.$$

One may wish to use various forms or relaxations of this budget constraint. For example, one budget profile might stipulate that the solution be 'within budget across the 4-year forward estimate (FE), but allow over/under spends in the outer years provided the total across the ten-year spread is within budget'. These can be modelled by the use of similar linear inequalities

$$\sum_{i=1}^{N}\Big[(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-F_{ij}^{BL}\Big] \le 0, \quad \forall j \in \{1,2,3,4\}, \tag{4}$$

and

$$\sum_{i=1}^{N}\sum_{j=1}^{T}\Big[(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-F_{ij}^{BL}\Big] \le 0. \tag{5}$$

Another common business rule might constrain the level of baseline cutting to platforms, or equivalently to provide a 'funding guarantee' to platforms. This avoids the problem of cutting too deeply into one (or more) platform and causing a 'catastrophic' degradation in capability. Again, this can be modelled with simple linear inequalities, such as for any $i \in \{1,\dots,N\}$

$$\sum_{j=1}^{T}\Big[(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-0.8\,F_{ij}^{BL}\Big] \ge 0. \tag{6}$$

This states that the total funding profile (over the T-year spread) for platform i is guaranteed to be at least 80% of its (total) baseline funding.

Other business rules might include things such as a 'service balanced' portfolio, whereby the total funding profile for each of the three services match a specified distribution (e.g. 1/3 each). Although the 'service balanced' portfolio is not explored in this study, we consider some very restricted constraints (that sometimes derive from safety, political imperatives, strategic priorities, etc.). For example, some platform baseline funding can not be 'touched',

$$\sum_{i\in \text{Untouched}}\sum_{j=1}^{T}\Big[(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-F_{ij}^{BL}\Big] \ge 0, \tag{7}$$

or some must be fully funded,

$$x_{ij}=1, \quad \forall i \in \text{Funded Set}, \quad \forall j \in \{1,\dots,T\}.$$

Notice that the class of our (MP) problem is obviously a *piecewise-linear program* (P-LP)

Portfolio	*'Status Quo'*	*'Funding Delay'*	*'Funding Guarantee'*	*'Baseline Untouched'*
Capability Shortfall Value	0.50	0.06	0.13	0.26
Capability Shortfall Reduction(%)	—	88%	75%	49%
Outstanding Shortfall ($M)	1500	675	758	876
Outstanding Shortfall Reduction (%)	—	55%	49%	42%
Fully Funded Platforms	0	11	11	9
Partially Funded Platforms	0	9	9	10

Table 5: Summary of Example Optimal Funding Allocation

as follows from the definition of our objective function using the piecewise terms $S(x_{ij})$. Also, it is known that any P-LP can be converted, by several transformations, to an equivalent MP problem in which the piecewise linear terms are replaced by linear terms (Fourer 1992). For every linear piece, such a transformation adds a few constraints or variables, including a zero-one integer variable. The resulting linear or mixed-integer formulation is then readily expressed and solved with any solver available. We use the algebraic modelling language, AMPL (Fourer et al. 2002, Chapter 14), in the implementation of our decision support tool as it automatically handles all these transformations.

Numerical Experiments

Optimal Funding Allocation Results

An illustrative example consisting of 20 missions, 50 capabilities and 120 platforms was used, and the resulting MP problem is solved with 1742 variables and 602 constraints. Optimal solutions are produced within seconds on a standard desktop machine using the CPLEX Solver (Version 8.1) and AMPL (Version 20021031) (cf. ILOG 2002). This allows various what-if analyses to be easily and quickly conducted. Table 5 presents the summary results of the optimal logistics funding allocation. In this table there are 4 types of solutions.

- *'Status Quo'* is to simply give all platforms their baseline funding. No MP problem needs to be solved in this case.
- *'Funding Delay'* allows the baseline to be cut totally in some years but the total funding profile over the 10-year horizon is guaranteed to be at least 80% of its total baseline. The P-LP (3)–(6) is solved:

$$\min \sum_{i=1}^{N}\sum_{j=1}^{T} I_i\, w_j^T\, S(x_{ij}),$$

subject to

$$0 \le x_{ij} \le 1,\ \forall i = \{1,\dots,N\},\ \forall j = \{1,\dots,T\},$$

$$\sum_{i=1}^{N}\Big[(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-F_{ij}^{BL}\Big] \le 0, \forall j \in \{1,2,3,4\},$$

$$\sum_{i=1}^{N}\sum_{j=1}^{T}\Big[(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-F_{ij}^{BL}\Big] \le 0,$$

$$\sum_{j=1}^{T}\Big[(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-0.8\,F_{ij}^{BL}\Big] \ge 0.$$

- *'Funding Guarantee'* provides all platforms with 80% baseline funding in every year. The last constraint of the previous P-LP (*'Funding Delay'* case) is replaced by

$$(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-0.8\,F_{ij}^{BL} \ge 0,$$
$$\forall i \in \{1,\dots,N\},\ \forall j \in \{1,\dots,T\}.$$

- *'Baseline Untouched'* is to ask 'what if the baseline of some platforms cannot be cut?' We again solve the previous P-LP (*'Funding Guarantee'* case) with the extra constraint (7).

In this hypothetical example the model suggests that the capability shortfall can be substantially reduced (by 49% − 88% relative to the 'Status Quo' situation) by redistributing some baseline funding (to reduce 42%−55% of the outstanding shortfalls). We see that imposing the additional constraints (Baseline Untouched) means that the solution found is now somewhat inferior. Here, the model suggests only 49% reduction in capability shortfall and only 42% of outstanding shortfalls can be reduced. More partially and less fully funded platforms are also observed.

Budget Line Analysis

Another type of analysis possible is to investigate the situation where there 'is a budget line' (i.e. there is supplementation available for logistics shortfall funding, but obviously not large enough to fund all bids). This is easily handled by adding terms B_j to the budget constraints (4)

$$\sum_{i=1}^{N}\Big[(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-F_{ij}^{BL}\Big] \le B_j, \forall j \in \{1,2,3,4\},$$

Extra fund per year ($M)	Capability Shortfall Reduction	Fully Fund Number	Fully Fund Total SF($M)	Partially Fund Number	Partially Fund Total SF($M)	Shortfall Reduction (%)
0	75%	11	587	9	155	49%
25	90%	11	587	9	320	60%
50	95%	12	987	10	250	82%
100	99%	12	987	11	364	90%

Table 6: Summary of Example Budget Line Analysis

and (5)

$$\sum_{i=1}^{N}\sum_{j=1}^{T}\left[(F_{ij}^{BL}+F_{ij}^{SF})\,x_{ij}-F_{ij}^{BL}\right]\leq B_j,$$

and then resolving the P-LPs. A summary of this type of analysis with the *'Funding Guarantee'* case is presented in Table 6.

The model suggests that the residual capability shortfall can be progressively reduced by additional supplementation. The approximate cost of effectively removing this is about $100M per year. The model also appears to suggest a 10% 'mark-up' on bids (i.e. with $100M per year supplementation the capability shortfall is reduced by almost 100%, but only 90% of the total shortfall funding is required to be given).

Conclusion

This paper presents the development and illustration of a mathematical approach for priority assessment and balance of investment for the logistics allocation problem. Where possible, it utilises existing numerical performance metrics to allow quantitative analysis. The subsequent model can be used to suggest a possibly more effective funding profile over the 'Status Quo' and can quantify the estimated improvement.

Note that in order to more fully appreciate the value of the proposed framework and model, the analysis should be compared with the current approach adopted by Defence and the funding solutions it produces. What is clear, however, is that the framework and model assists in making explicit the assumptions, constraints and value-functions associated with the decisions as well as a tool for automating some of the currently time-consuming manual processes. The formulation of the decision problem as a MP also allows a quick what-if capability, for example quarantining certain platforms from any funding cuts or investigating the impact of supplementation. Additional business rules (e.g. service-balancing) may also be incorporated into the model.

However, the current version of the model is restricted to assuming independence between platforms and significant verification and validation of the data model are required. Extension to incorporate the synergistic nature of defence platforms (e.g. using a Bayesian Belief Network as in Chisholm and Asenstorfer 2006 or Constraint Programming Models as in Hentenryck 2002) may further enhance the models applicability.

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