Constructing a SUITE of Analytical Tools: A Case Study of Military Experimentation

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Abstract

The ability to conduct close combat, that is, to engage the adversary within his effective weapons range, is an enduring and important task for the land force. Close combat has proven difficult to analyse with any single analytical technique. High-level combat simulations have arguably been the most successful single technique used to date. This paper discusses how a recent study of close combat for the time frame of 2015-2030 was conducted using the principles of military experimentation, to provide advice to capability development at the concept exploration stage.

Using Flood and Jackson's classification the system under investigation is considered to be complex and adaptive, and the problem domain coercive and complex. The variables explored are the physical environment, the enemy force options and the friendly force structures. A matrix of analytical techniques was drawn from across disciplines to study the effectiveness of combat systems, and the interactions and interdependencies between the contributing systems. A hypothesis was proposed and explored to ascertain whether the current paradigm is appropriate or whether a new one emerges. The matrix of techniques includes mathematical models of combat, high- level combat simulations (both specific and generic scenarios), game theory and historical studies. This paper also addresses how emerging techniques such as agent-based distillations can be employed in the experimental process. Importantly the technique addresses the inter-relationship between process, equipment and the organisational aspects of a system.

Introduction

A military unit or formation consists of people and equipment, and the formation conducts activities through the application of its procedures. Using Flood and Jackson's classification, it follows then, that a military formation forms a system. The nature of military operations involving such systems produces extremely complex problems for three related reasons: the systems are an intricate mix of physical and psychological aspects, they are immersed in a complex physical environment, and they are usually in competition with other similar systems. Some aspects of military operations have been successfully reduced to tractable problems, for example the Lanchester models of combat (Taylor 1983, Przemieniecki 1994) various search models (Morse 1970) aspects of command and control such as information flows (Johnson, and Levis, Koh, DiCesare and Rubenstein 1991) and a range of logistics and transport problems. However, these approaches generally fail to take into account the dynamic interactions between the military unit, its environment and its adversaries, and they all fail to address the psychological issues. For this reason complex combat simulations have been developed and employed for force development since the late 1970s. Combat simulations are much more than complex combat models in that they usually represent of the three main physical activities of military entities: movement, engagement and detection, in complex representations of the physical world; command and control is often poorly modelled, if at all. Examples of Combat simulations are CAEn, Janus and CASTFOREM The complex coercive nature of combat and the weaknesses in any single modelling technique led to the development of a suite of analytical tools for the close combat study (Bowley, Castles and Ryan)

The Nature of the Problem

Flood and Jackson propose a system of systems approach, see Table 1, that describes problem domains in terms of two

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characteristics: the system's complexity and the nature of the interactions within the system under investigation. In the framework, complexity is described as either simple or complex, and the interactions as unitary, pluralist or coercive. Simple systems are defined as having few components and interactions, the elements of the system have predetermined attributes and their behaviour can be described by well-defined laws. Complex systems have components that display probabilistic behaviours, often have their own goals, and the system itself is often open and evolving. In unitary systems the components have 'interests' that are aligned and common objectives, pluralist systems have some divergence of 'interests' but with common goals, and coercive systems have no common 'interests' or objectives.

Table 1 Example Problem Domains

	Unitary	Pluralist	Coercive
Simple	machines	coalitions	prisons
Complex	organisms, cybernetics	cultures, commerce	warfare

Flood and Jackson claim that there is no single technique available to address complex coercive systems (or systems of systems), but these systems are relatively rare. They suggest that a prison system may be an example of a simple coercive system, because differences between groups are easy to identify, and may be addressed through the technique of critical systems heuristics, a technique that relies on this criteria. The objectives of the adversary, and how he intends to achieve them, form one of the most uncertain aspects of many problems in the military domain and therefore we suggest these are complex coercive systems. Problems in the military domain at the force level where the presence of the adversary and his actions are important are **all** in the complex/coercive category and this is the problem area addressed in the close combat study. The important aspects of this class of problems in the military domain are that the relationships between the components are not only coercive but adversarial, and the components adapt their behaviour in an attempt to defeat the adversary.

One popular way of analysing these types of problems in the military domain is through combat simulations, generally using a mix of live and computer moderated events. The problem with the computer moderated systems is that whilst they generally have excellent models of physical mechanisms, the impact of the humans within the systems are relatively poorly represented. For example complex command and control issues are normally reduced to relatively simple expert systems and decision trees. On the other hand live simulations compromise the fidelity of the combat resolution. The current development of entropy-based games and agent-based distillations (ABDs) (Ilachinski, 2000) are an attempt to better model the impact of human elements, surprise and tempo, and may therefore provide another tool for combat analysis.

The Battlelab Process

Recently a number of techniques have been developed to explore systems operating in the complex coercive domain, especially for military operations. The Army Experimental Framework (AEF) is one such technique developed through a number of army studies; in particular Army in the 21st Century (A21) 1995-96, Restructuring the Army (RTA) 1997-99 and finally through the AEF series (Australian Army, 2000) The key intellectual component of the AEF is the "Battlelab Process" which was developed for the analysis of Project Land 125 (the Soldier Combat System) and modified for RTA Phase 1 (1997-98) (Brennan *et al.* 2000). The process has evolved considerably since its initial implementation in Land 125 and a contemporary representation is shown in Figure 1 below.

There are two important stages in this process: systems synthesis and system testing.

Systems Synthesis The goal of systems synthesis is to construct an experimental force, including its equipment, organisation and doctrine. The inputs to this stage describe the context in which the force will be used and consist of strategic guidance in terms of the geo-political environment, government policy (in terms of defence outputs) and technology. The output from this stage is the experimental force construct which is described by a Concept of Operations, the missions the force must achieve, the capabilities the force requires, its vulnerabilities (expressed as critical areas) and the scenarios in which the force must operate.

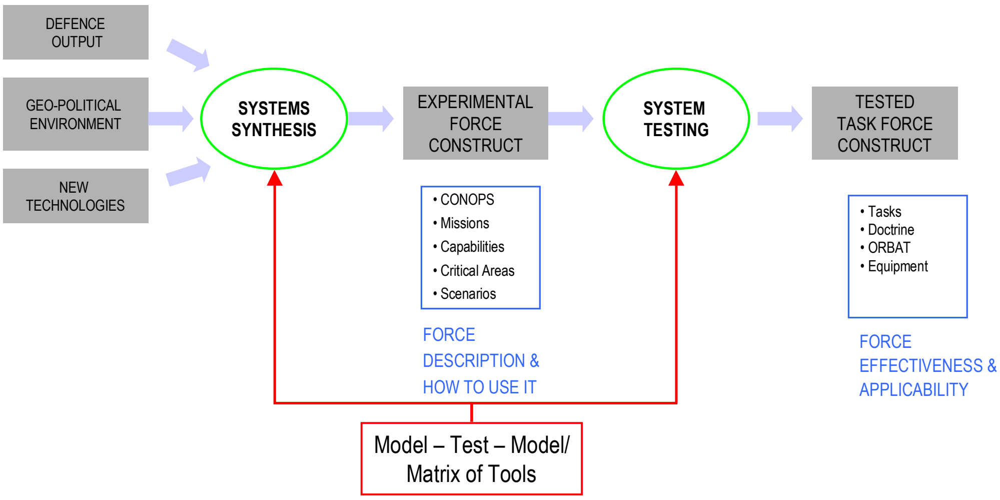

Figure 1 The "Battlelab Process" (Bowley, Davis and Brennan, 2000)

<u>System Testing</u> The goal of system testing is to evaluate the experimental force. The input to this stage is the experimental force and the output is a tested force described in terms of the tasks it must achieve and the relevant doctrine, organisation and equipment.

Both these processes are supported by the operational test and evaluation (OTE) "model – test – model" process modified for complex forces (Bowley and Lovaszy 1999) During RTA Phase 1 it became obvious that this process had serious limitations for force-level experimentation if it was attempted in a linear fashion. The process was modified so that instead of being iterative and linear, a suite of tools was developed and used in parallel.

The main analytical tools used in the BattleLab process have been seminars, wargames and simulations. A similar linear construct is proposed in the Operational Synthesis scheme. Operational synthesis seeks to integrate across a spectrum of existing methods of simulation and decision support and is, in essence an attempt to synthesise information from multiple tools to answer questions involving non-linearity, intangibles or coevolving landscapes (Horne) The principal tools in this scheme are mathematical modelling, wargames and simulations and ABDs

ABDs are low-resolution abstract models, used to explore questions associated with land combat operations in a short period of time. Being agent-based means that only simple behavioural rules need to be assigned. Being deliberately low-resolution means that the detailed physics of combat are largely ignored (or abstracted to simple constructs). Advances in computing power can then be exploited to produce a significant volume of data. This process is known as data farming (Brandstein, Horne) and allows extensive parameter excursions to be performed, both in terms of variations in platform capabilities and tactics (behavioural characteristics), from the baseline scenario. However the level of abstraction in ABDs implies that the results of a distillation should only be used to provide a focusing of ideas and that subsequent analyses be conducted to 'drill-down' with higher resolution modelling. There are a growing number of ABDs, including the U.S. Marine Corps' Irreducible Semi-Autonomous Adaptive Combat (ISAAC) model and the Enhanced ISAAC Neural Simulation Toolkit (EINSTein). The New Zealand Defence Technology Agency has also recently developed the Map Aware

Non-uniform Automata (MANA), and DSTO has initiated the development of the BactoWars ABD (Ilachinski 2000)

The Close Combat Study Overview

The close combat study falls within the "system test" domain, and as such a hypothesis was proposed and investigated. The hypothesis was of the form "In restricted terrain strategy 1 is preferred, in open terrain strategy 2 is preferred". Importantly the hypothesis was not formally tested through a repeatable experiment; it was tested using a series of generic CAEn scenarios[c], as seen in Figure 2. These confidences in scenarios was increased by correlating the results of simple CAEn scenarios with mathematical models of combat. The rationale used was that the solutions to the equations are widely accepted and therefore if the simulation results correlate, when the assumptions implicit in the equations are met, then we have some confidence in the validity of the simulation. The results of the CAEn generic scenarios were extrapolated through a series of more detailed studies. Extrapolation was required because of the number of assumptions that needed to be made about the nature of the interactions between the adversaries in order to make the problem amenable to simulation.[d] The variations run in the CAEn Generic Scenarios were guided by the interpretation of the specific and historic studies and the results of the initial generic scenario; hence a type of iterative search process was used to generate the insights. The errors inherent in the extrapolation process were minimised through developing subsequent variants of the CAEn generic scenario based on the specific study results and by correlating the results of all the studies to produce the insights.

Problem Definition

Assumptions and Definitions: A number of definitions were agreed with the sponsor of the study and assumptions were made to simplify the problem to the point where it could be modelled. A family of credible scenarios was developed from the agreed objectives and understanding of the close combat task. Similarly attrition and suppression were defined and related to area and aimed fire so as to correlate the Lanchester analysis with the CAEn simple scenario. Finally open and restricted terrains were defined so they could be modelled in CAEn and extreme cases of the terrain were then modelled

Once the scope of the problem was constrained the details of the CAEn modelling had to be determined. As it was assumed that the current doctrine for company attacks was relevant to both open and restricted terrain, the tactics were not changed between open and restricted terrain. Therefore the results of the CAEn generic scenarios have to be taken in this context, reinforcing the need to use other studies (the UK historical study and the CAEn and Janus specific studies) to extrapolate the results. If the tactics were modified the difference in outcome would be larger.

The Suite of Tools: The tools used in this study were a Lanchester model, the CAEn combat simulation (for a simple scenario, a series of generic scenarios and specific scenarios), Janus wargames and an historical analysis. The simple CAEn scenarios were used to correlate the results of CAEn with two Lanchester models: the area fire model and the aimed fire model. The generic CAEn games were analysed using game theory and the results of these two analyses were then compared to the results of the complex CAEn and Janus scenarios developed for RTA (Bowley and Brewer, 2001) and the UK historical analysis (Thornton, 1993)

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[c] Simple scenario: A CAEn scenario designed to satisfy the assumptions of Lanchester analysis.
Generic scenario: A CAEn attack/defence vignette conducted by doctrinal forces using doctrinal tactics on simplified terrain.
Specific Scenario: A CAEn scenario that models a specific vignette including the effect of terrain on tactics.

[d] A simple example is that both the defender and the attacker fight to the end, where there is historic evidence that one side or the other will break with as little as 10 – 30% casualties [Thornton].

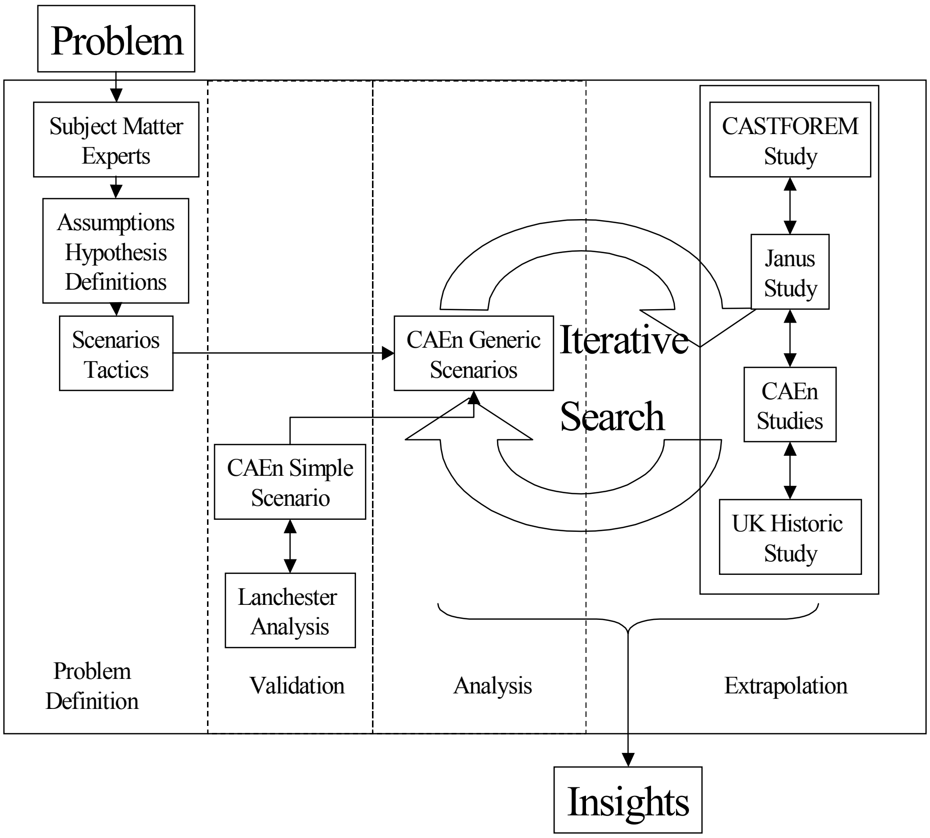

Figure 2 Outline of the Close Combat Study

<u>Correlation of results:</u> The output from the simple CAEn scenario replications was fitted to two Lanchester models, the firer/firer (F|F) model (aimed fire) and the firer target/firer target (FT|FT) (area fire) model to try and determine the mechanism of attrition in each situation (an earlier study comparing CAEn and Lanchester demonstrated that when most Lanchester assumptions are met then the results correlate (Bowley, Gaertner and Pepper)) Fitting the CAEn data to an aimed fire and an area fire Lanchester model should provide the basis for conclusions on the dominant attrition mechanism in each scenario.

<u>Firer | Firer (F|F) Lanchester Model:</u> A Firer | Firer (F|F) attrition process models an aimed fire battle where the rate of attrition of each side is proportional to the numerical strength of the opposing side. Mathematically this can be expressed as

$$\frac{dB}{dt} = -rR \qquad \text{with } B(0) = B_0 \quad (1)$$

and

$$\frac{dR}{dt} = -bB \qquad \text{with } R(0) = R_0 \quad (2)$$

with *B* and *R* representing the number of units, *dB/dt* and *dR/dt* representing the attrition rate and, *b* and *r* representing the attrition rate coefficients in the Blue and Red forces respectively. This is the most common Lanchester model. The assumptions of this model are (Taylor, 1983)

- Both forces must be continually engaged in combat.
- Each unit or individual weapon is within the maximum weapon range of all the opposing units.
- Collateral damage within the target area is negligible.
- The attrition coefficients *b* and *r* also include the probabilities of the target being destroyed when hit.

- The effective firing rates are independent of the opposing force level.

Each unit is aware of the location and condition of all opposing units so that its fire is directed only to live units or functioning weapons. When a target is destroyed, search begins immediately for a new target. Fire is uniformly distributed over surviving units.

Firer Target | Firer Target (FT|FT) Lanchester Model: The Lanchester Firer Target | Firer Target (FT|FT) model is an attrition model for area fire, and so it does not account for concentration of firepower. The differential equations for the FT|FT model are:

$$\frac{dB}{dt} = -BrR \quad \text{with } B(0) = B_0 \quad (3)$$

and

$$\frac{dR}{dt} = -RbB \quad \text{with } R(0) = R_0 . \quad (4)$$

The first five assumptions from the F|F model are also assumed under the FT|FT model. However, the remaining assumptions are different.

Each unit is aware of only the general area in which enemy forces are located and directs its fire into this area without receiving any feedback information about the inflicted damage.

Fire is uniformly distributed over the area in which enemy forces are located.

All units are uniformly distributed over the area.

A 99% confidence interval was constructed for the mean attacker force strength over time so as to ensure a sufficient number of CAEn replications were being conducted. The largest variance occurred in the “aimed fire open terrain” scenario for the attacker. The number of replications was large enough to make a normal approximation to the sample mean reasonable and to assume the sample variance is a good approximation to the population variance, since a measure for a terminating simulation across replications forms an independent and identically distributed (iid) variable. The maximum size of the confidence interval was (11.6, 16.8), which is a small enough interval for the purpose of Lanchester model parameter identification.

Two simple scenarios were developed in CAEn to closely approximate the assumptions of the Lanchester F|F model. All units were considered to be stationary and only used aimed fire. The terrain was flat, open vegetation. It was hypothesised that the Lanchester F|F model would have a low mean square error (MSE) when fitted to the CAEn results. This scenario was changed to indirect fire to fit the Lanchester FT|FT model. It was hypothesised that the Lanchester FT|FT model would have a low MSE when fitted to the CAEn results. The mean square errors are summarised in Table 2. The table also includes the optimal parameters *b* and *r* to two significant figures.

Scenario	F|F			FT|FT				
	MSE	*b*	*r*	MSE	*b*	$\hat{b}$	*r*	$\hat{r}$
Aimed Fire Simple Scenario	8.51	8.6×10^{-3}	2.6×10^{-3}	8.45	5.6×10^{-4}	2.8×10^{-4}	3.9×10^{-4}	3.1×10^{-4}
Area Fire Simple Scenario	71.85	3.8×10^{-3}	1.4×10^{-2}	12.63	4.1×10^{-4}	1.2×10^{-4}	3.1×10^{-4}	1.7×10^{-4}
Open Aimed	13.84	7.4×10^{-4}	5.8×10^{-3}	30.14	3.1×10^{-5}	2.4×10^{-5}	1.2×10^{-4}	6.8×10^{-5}
Open Area	10.65	6.7×10^{-4}	3.5×10^{-3}	18.45	4.4×10^{-5}	2.2×10^{-5}	6.0×10^{-5}	4.1×10^{-5}
Close Aimed	12.24	6.1×10^{-4}	2.4×10^{-3}	13.65	3.5×10^{-5}	2.0×10^{-5}	3.6×10^{-5}	2.8×10^{-5}
Close Area	32.81	2.6×10^{-3}	1.4×10^{-2}	15.12	2.0×10^{-4}	8.4×10^{-5}	2.8×10^{-4}	1.7×10^{-4}

Table 2 MSE and optimal parameters from Lanchester least-squares curve fits

These results require careful interpretation as in each scenario the null hypothesis is satisfied. However it is important to emphasise that a close curve fit, measured by a low MSE, does not imply that the Lanchester model is the correct model for the attrition process. In this case we can only say that the CAEn results converge with the Lanchester model at a rudimentary level. For poor curve fits a stronger conclusion may be drawn, in which case the model may be ruled out as an appropriate model for the scenario. For this study a threshold of 20 for the MSE was used.

Thus from Table 2 we can rule out a number of Lanchester models. The F|F model is clearly not the appropriate attrition mechanism for the Lanchester area fire scenario with a MSE of 71.85 or the close terrain area fire scenario with a relatively large MSE of 32.81. The FT|FT model can be ruled out for the open terrain aimed fire scenario with a MSE of 30.14.

The columns $\hat{b}$ and $\hat{r}$ in Table 2 show the estimated values for the FT|FT ability coefficients *b* and *r* calculated from the F|F model ability coefficients. Theoretically, the coefficients in the FT|FT model should be equal to the coefficients from the F|F model divided by the opposing force size (Taylor 1983)

Since the values of $\hat{b}$ and $\hat{r}$ are of the same order as *b* and *r* in the FT|FT model, this shows that the area fire model in CAEn is consistent with the Lanchester area fire model.

Analysis

The principal measure of effectiveness used to analyse the CAEn generic scenarios in this study was the loss exchange ratio (LER). This measure is analysed using a game theoretic approach. Other measures used are kills per weapon system, kills by type of fire (area and aimed), detection ranges, ratio of available targets and those detected and the conversion rate from detections to acquisitions to kill.

LER, often used in modelling combat, is usually the ratio of Red losses to Blue losses. A larger LER reflects a result where either greater Red casualties were inflicted or fewer Blue casualties sustained. For the scenarios used in this study Red was always the defender and Blue the attacker. In this study, the LER is therefore also the number of defender casualties divided by the number of attacker casualties.

Previously CAEn has been used to conduct studies (Hobbs, Castles and Rogers, 2000) to investigate the effects of changes in equipment (a new anti-armoured weapon, for example), with the analysis of the results limited to a difference of means test on the two statistics. The specific CAEn studies conducted for the RTA program looked at the difference in performance due to a change in a whole capability (the inclusion of armoured vehicles, for example). When investigating the impact of whole capabilities care must be taken because the tactics modelled may need to change to make effective use of the new equipment, hence it is normal to conduct constructive simulation runs to determine the appropriate tactics prior to the closed simulation runs.

In this study, tactics and the environment were varied, specifically the type of fire used by the attacker and the defender and open and restricted terrain; then representative capabilities were added to measure their impact in both environments.

Game Theory: Game theory is an approach to the study of conflict that provides a rational framework for determining the best strategies in a game. In this study the attacker and defender tactics were arranged in a series of two by two matrices, for example aimed or area fire for each side, tanks/no tanks supporting the attacker or defender and so on. The measure of effectiveness used to fill the matrices was the LER expressed as defender losses over attacker losses; the higher the value the better for the attacker and the lower the better for the defender. Although a game with loss exchange ratios as the payoff is not zero sum, it is easy to create a 1:1 mapping to a zero sum game. The attacker attempts to maximise the value in the columns and the defender to minimise the value in the rows. Using this logic it is possible to determine the best tactic for each player. When each player has a tactic that always provides a better solution regardless of the other's strategy then the solution is a single combination of strategies as a solution.

Game theory made it simple to identify the best tactics, and the base scenario is used to illustrate its utility in analysing combat simulation results. The results for the base scenario variants have been sorted into two matrices Figure 3 for open terrain and Figure

4 for restricted. Using the logic outlined above it can be determined by inspection that both matrices have a single optimum solution for both attacker and defender. In open terrain the optimum tactic is Strategy 2 and in restricted terrain it is Strategy 1.

Where one side uses the preferred tactic against the non-preferred there is a significant improvement in performance. The extreme case of this is in restricted terrain where the defender selects aimed fire and the attacker area fire. It is postulated that the defender had very few opportunities to acquire targets due to being continually suppressed by the weight of fire and the short assault distance.

Attacker / **Defender**	Strategy 1	Strategy 2
Strategy 1	0.76	1.0
Strategy 2	0.17	0.29

Figure 3 Light Infantry Matrices Scaled Loss Exchange Ratios in Open Terrain

Attacker / **Defender**	Strategy 1	Strategy 2
Strategy 1	0.11	0.04
Strategy 2	1.0	0.12

Figure 4 Light Infantry Matrices Scaled Loss Exchange Ratios in Restricted Terrain

Extrapolation

Five previous, independent studies were used to extrapolate the results of the analysis: a CASTFOREM Study, a Janus Study, two specific CAEn studies of infantry company attacks in open country and urban terrain, and an historical study of close combat. The results of the analysis were compared with the results of these studies. Importantly the specific CAEn studies were constrained to a far greater extent than the generic scenario so as to better capture the intricacies of combat, however only one baseline scenario and several variants were run due to the time required to build the model. The historic study is of course constrained by actual combat. The correlation of the various studies was based on the opinions of subject matter experts; there was no formal correlation conducted.

The essence of the technique is that there is a trade off between the number of factors that can be considered and the rigour of the technique. The historical studies are the least rigorous because of the difficulty in determining the cause and effect, whilst the Lanchester equations are the most rigorous. The problem with the Lanchester equations is that the assumptions made make the analysis almost irrelevant to real world combat if it is used in isolation. Hence a goal of the approach is to balance the depth and breadth of the tools used.

Conclusion

This work demonstrates how a suite of tools can be utilised to explore a complex coercive system and most importantly how a problem can be investigated using a balanced suite of analytical tools. The two stages required are the problem definition and analysis stages of the accepted operational analysis (OA) but expanded through the use of a suite of tools in both stages with the aim of synthesising and testing a system of systems in a complex environment against an adaptive adversary. The important aspects of the technique are to build the suite of tools and the iteration between the extrapolation studies and the core analytical study. The focus of future work is to determine the utility of operational synthesis, agent based distillations and human factors experiments within the process.

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