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Editorial

In this issue, S. Jain, P. Advani and M. Kumar have contributed a technical paper on *An inventory model for Weibull distribution deterioration with allowable shortage under cash discount and permissible delay in payments*. We are delighted to be publishing the paper here for Bulletin readers.

I am pleased to inform you that the electronic version of ASOR Bulletin is now available at the ASOR national web site: http://www.asor.org.au/. Currently, the electronic version is prepared only as one PDF. We like to thank our web-master Dr Andy Wong for his hard work in redesigning and smoothly managing our national web site. In September alone, our web site has about 950 visitors and 3400 page requests logged. Your comments on the new electronic version, as well as ASOR national web site, is welcome.

ASOR Bulletin is the only national publication of ASOR. I would like to request all ASOR members, ASOR Bulletin readers and OR organizations in the country to contribute to the ASOR Bulletin. The editorial policy is available either from the Bulletin web site or from the inside back cover of the Bulletin. The detailed instructions for preparing the manuscripts is available in the URL: http://www.asor.org.au/.

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An Inventory Model for Weibull Distribution Deterioration with Allowable Shortage under Cash Discount and Permissible Delay in Payments

Sanjay Jain[a], Priya Advani[b] and Mukesh Kumar[c]

Abstract
The inventory theory has undergone a profound structural transformation in the last few decades. The trade credit /cash discount scheme revolution has expanded well beyond the cutting-edge high-tech industrial sector redefining the rules of global competition. Numerous studies have been undertaken to explain inventory models with different features. While findings from earlier studies have been conflicting, recent industrial-level studies indicate that multi features inventory models has a positive impact on business scenario. We propose an inventory model with integration of many real features like two-parameter Weibull distribution deterioration allowing shortages under cash discount scheme and permissible delay in payments. A numerical example is taken to illustrate the application of developed models and to examine the sensitivity of model parameters.

Key Words: Weibull distribution, Permissible delay in payments, Trade credit, Cash discount, Deterioration, Shortage.

Introduction
The traditional inventory models were developed under the assumption that payment will be made to the suppliers immediately on receipt of the consignment. But in real life, suppliers allow some grace period / credit facilities before they settle account with the retailers. In such a case no interest is charged if the account is settled within the permissible delay period. Beyond this period the supplier will charge interest. Such a benefit motivates the retailers to order more quantity because delay in payment indirectly reduces the purchase cost of the items. However, if the items in the inventory system deteriorate, ordering large quantities would not be economical. Owing to this fact, during the past few years, many articles dealing with models under trade credit have appeared in various research journals.

Goyal (1985) developed a single-item inventory model under the condition of permissible delay in payments. Aggarwal & Jaggi (1995) extended Goyal's (1985) model by allowing constant rate of deterioration. Jamal et al. (1997) further generalized the model allowing shortages. Teng (2002) modified the Goyal's (1985) model by considering the fact that unit selling price is usually higher than the unit cost. Chung et al. (2005) proposed retailer's lot-sizing policy under permissible delay in payments depending on the order quantity. Recently Pal & Ghosh (2007) developed an inventory model for deteriorating items with stock-dependent demand under permissible delay in payments.

In the above mentioned models, cash discount factor is not taken into consideration. The supplier often provides its customers a cash discount so as to motivate payment as early as possible, stimulate sales, or reduce credit expenses. The retailer can obtain cash discount if the payment is

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made within cash discount period offered by the supplier. Otherwise, the retailer will have to pay full payment within the trade credit period.

Many inventory models under cash discount policy and delay in payments can be found in Chang (2002) & Huang and Chung (2003). Ouyang et al. (2003) extended Goyal's model by incorporating shortages and cash discount. Chang and Teng (2004) generalized Goyal's model by assuming that the retailer pays the supplier the sales revenue when items are sold. Huang et al. (2007) investigated the cash where the retailer's unit selling price and the purchasing price per unit are not necessarily equal within the economic production quantity (EPQ) framework under cash discount and permissible delay in payments.

In this paper we develop an inventory model for deteriorating items with two-parameter Weibull distribution deterioration allowing shortages under cash discount and permissible delay in payments. This paper is organized as follows. In section 2 assumptions and notations are presented. In section 3 the mathematical model is formulated. In section 4 algorithm is stated. In section 5 numerical examples are cited and sensitivity analysis of the optimal solution with respect to parameters of the system is carried out.

Assumptions and Notations

Inventory model is developed under following assumptions and notations:

Assumptions

- Replenishment rate is infinite.
- The lead-time is zero.
- The rate of deterioration at any time t follows the two-parameter Weibull distribution: $Z(t) = \alpha \beta t^{\beta-1}$, where $\alpha (0 < \alpha << 1)$ is the scale parameter and $\beta (> 0)$ is the shape parameter.
- Inventory level remains non-negative for a time t_1 in each cycle after which shortages are allowed and unsatisfied demand is backlogged at the rate δ.
- Supplier offers cash discount if payment is made within time M_1; otherwise the full payment is due within time M_2.

Notations

c_1 = set-up cost.
c_2 = per unit holding cost excluding interest charges.
c_3 = shortage cost per unit per unit time.
c_4 = opportunity cost due to lost sales per unit.
c_5 = deterioration cost per unit per unit time.
c = item cost per unit.
R = demand rate per unit per unit time.
Q = order quantity per cycle.
$I(t)$ = inventory level at time.
I_e = interest earned per unit time.
I_p = interest paid per unit time.
r = cash discount rate.
M_1 = period of cash discount.
M_2 = last time of permissible delay (for settling the accounts $M_2 > M_1$).

T = length of replenishment cycle.
T_1 = length of positive inventory period.
$(T_1^{\,a}, T^{a})$ = optimal value of (T_1, T) in case 1.1.
$(T_1^{\,b}, T^{b})$ = optimal value of (T_1, T) in case 1.2.
$(T_1^{\,c}, T^{c})$ = optimal value of (T_1, T) in case 1.3.
$(T_1^{\,d}, T^{d})$ = optimal value of (T_1, T) in case 1.4.

Model Formulation

The system starts with Q units of on-hand inventory. Depletion of inventory occurs due to combined effects of demand and deterioration in the interval $0 < t < T_1$. Demand is partially backlogged in the interval, $T_1 < t < T$. Variation of inventory level $I(t)$ at any time t is given by

$$\frac{d\,I(t)}{d\,t} + \alpha\,\beta\,t^{\beta-1} I(t) = -R \qquad ;0 \le t \le T_1 \qquad \text{- - - (1)}$$

$$\frac{d\,I(t)}{d\,t} = -\,\delta\,R \qquad ;T_1 \le t \le T \qquad \text{- - - (2)}$$

The solutions of (1) and (2) with the boundary condition are respectively

$$I(T) = R\left\{(T_1 - t) - \alpha\,(T_1\,t^{\beta} - t^{\beta+1}) + \frac{\alpha}{\beta+1}\left(T_1^{\,\beta+1} - t^{\beta+1}\right)\right\};0 \le t \le T_1 \qquad \text{- -- (3)}$$

$$I(t) = \delta\,R\,(T_1 - t) \qquad ;T_1 \le t \le T \qquad \text{- - - (4)}$$

Thus the order quantity per cycle is

$$Q = I(0) + \delta\,R\,(T - T_1)$$

$$= R\left\{(1-\delta)\,T_1 + \frac{\alpha}{\beta+1}T_1^{\,\beta+1} + \delta\,T\right\} \qquad \text{- - - (5)}$$

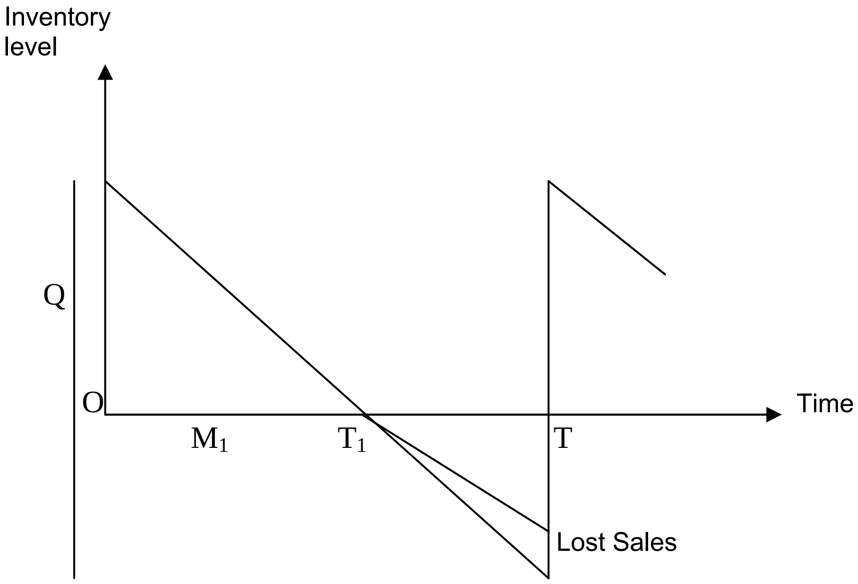
Since the supplier offers a premium of cash discount, there are two payment policies for the retailer:

(1) Payment is made at time M_1 to get the cash discount (Case 1)

(2) Payment is made at time M_2 not to get the cash discount (Case 2)

These two cases are as follows:
Case1. Payment is made at time M_1

Case 1.1 $M_1 < T_1$
In this case the length of the positive stock period is larger than the period of cash discount.



Case 1.1 $M_1 < T_1$

The components of total cost are calculated as follows.

(a) The setup cost per setup is fixed at c_1

(b) Holding cost during the interval $[0, T_1]$ is given by

$$\text{HC} = c_2 \int_0^{T_1} I(t)\, dt$$

$$= c_2 R \left\{ \frac{T_1^{\,2}}{2} + \frac{\alpha \beta}{(\beta+1)(\beta+2)} T_1^{\,\beta+2} \right\} \qquad \text{- - - (6)}$$

(c) Shortage cost during the interval $[T_1, T]$ is given by

$$\text{SC} = c_3 \int_{T_1}^{T} -I(t)\, dt = \frac{c_3\, \delta\, R}{2}\,(T - T_1)^2 \qquad \text{- - - (7)}$$

(d) Opportunity cost due to lost sales in the interval $[T_1, T]$ is given by

$$\text{OC} = c_4 \int_{T_1}^{T} R(1-\delta)\, dt = c_4 R(1-\delta)(T - T_1) \qquad \text{- - - (8)}$$

(e) Deterioration cost is given

$$\text{DC} = c_5 \left[I(0) - \int_0^{T_1} R\, dt \right]$$

$$= \frac{c_5\, \alpha\, R}{(\beta+1)} T_1^{\,\beta+1} \qquad \text{- - - (9)}$$

(f) Interest payable per cycle is given by

$$\text{I P} = c\, I_p \int_{M_1}^{T_1} I(t)\, dt$$

$$= c\, I_p\, R \left\{ \frac{(T_1 - M_1)^2}{2} + \frac{\alpha\beta}{(\beta+1)(\beta+2)} T_1^{\beta+2} + \frac{\alpha}{(\beta+1)} T_1 M_1 (M_1^{\beta} - T_1^{\beta}) - \frac{\alpha\beta}{(\beta+1)(\beta+2)} M_1^{\beta+2} \right\} \quad \text{--- (10)}$$

(g) Interest earned per cycle is given by

$$\text{I E} = c\, I_e \int_0^{M_1} R t \, dt$$

$$= \frac{c\, I_e R M_1^2}{2} \quad \text{- - - (11)}$$

(h) Since the payment is made at time M_1 , the retailer can get a r cash discount off the price of merchandise is given by $\text{CD} = r\, c\, Q$

$$= r\, c\, R \left\{ (1-\delta) T_1 + \frac{\alpha}{(\beta+1)} T_1^{\beta+1} + \delta T \right\} \quad \text{- - - (12)}$$

Therefore the total cost per unit time is

$$C_1(T_1, T) = \frac{c_1 + HC + SC + OC + DC + IP - IE - CD}{T} \quad \text{- - - (13)}$$

Using (6) to (12) in (13), we get

$$= \frac{1}{T} \left\{ c_1 + \frac{c_2 R T_1^2}{2} + \frac{\alpha\beta R}{(\beta+1)(\beta+2)} (c_2 + c\, I_p)\, T_1^{\beta+2} + \frac{c_3 \delta R}{2} (T - T_1)^2 \right.$$
$$+ c_4 R (1-\delta)(T - T_1) + c\, I_p\, R \left(\frac{(T_1 - M_1)^2}{2} + \frac{\alpha T_1 M_1^{\beta+1}}{(\beta+1)} - \frac{\alpha\beta}{(\beta+1)(\beta+2)} \right) M_1^{\beta+2}$$
$$\left. - \frac{\alpha R}{\beta+1} \left(c\, I_p M_1 + r\, c - c_5 \right) T_1^{\beta+1} - \frac{c\, I_e R M_1^2}{2} - r\, c\, R \left((1-\delta) T_1 + \delta T \right) \right\} \quad \text{- - - (14)}$$

For the minimization of cost we set,

$$\frac{\partial\, C_1(T_1, T)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial\, C_1(T_1, T)}{\partial T_1} = 0 \quad \text{- - - (15)}$$

$$\Rightarrow T = \frac{1}{c_3 \delta} \left\{ \frac{\alpha\beta}{(\beta+1)} (c_2 + c\, I_p) T_1^{\beta+1} - \alpha (c\, I_p M_1 + rc - c_5) T_1^{\beta} + (c_2 + c_3 \delta + c\, I_p) T_1 \right.$$
$$\left. - c_4 (1-\delta) - r\, c (1-\delta) - c\, I_p M_1 + \frac{c\, I_p \alpha}{(\beta+1)} M_1^{\beta+1} \right\} \quad \text{- - - (15-a)}$$

and

$$T^2 = \frac{2}{c_3\delta R}\left\{\frac{\alpha\beta R}{(\beta+1)(\beta+2)}(c_2+cI_p)T_1^{\beta+2} - \frac{\alpha R}{(\beta+1)}(cI_pM_1+rc-c_5)T_1^{\beta+1}\right.$$
$$+(c_2+c_3\delta)\frac{RT_1^2}{2}-(c_4+rc)R(1-\delta)T_1$$
$$+cI_pR\left[\frac{(T_1-M_1)^2}{2}+\frac{\alpha T_1M_1^{\beta+1}}{\beta+1}-\frac{\alpha\beta}{(\beta+1)(\beta+2)}M_1^{\beta+2}\right]$$
$$\left.-\frac{cI_eRM_1^2}{2}+c_1\right\}$$

--- (15-b)

To get the optimal values T^* and T_1^* of T and T_1 respectively we can proceed as follows: Eliminating T from above two equations 15(a) & 15(b) as $[T]^2 - T^2 = 0$, we get

$$\left[\frac{1}{c_3\delta}\left\{\frac{\alpha\beta}{(\beta+1)}(c_2+cI_p)T_1^{\beta+1}-\alpha(cI_pM_1+rc-c_5)T_1^{\beta}\right.\right.$$
$$\left.\left.+(c_2+c_3\delta+cI_p)T_1-c_4(1-\delta)-rc(1-\delta)-cI_pM_1+\frac{cI_p\alpha}{(\beta+1)}M_1^{\beta+1}\right\}\right]^2 -$$

$$\frac{2}{c_3\delta R}\left\{\frac{\alpha\beta R}{(\beta+1)(\beta+2)}(c_2+cI_p)T_1^{\beta+2}-\frac{\alpha R}{(\beta+1)}(cI_pM_1+rc-c_5)T_1^{\beta+1}\right.$$
$$+(c_2+c_3\delta)\frac{RT_1^2}{2}-(c_4+rc)R(1-\delta)T_1$$
$$+cI_pR\left[\frac{(T_1-M_1)^2}{2}+\frac{\alpha T_1M_1^{\beta+1}}{\beta+1}-\frac{\alpha\beta}{(\beta+1)(\beta+2)}M_1^{\beta+2}\right]$$
$$\left.-\frac{cI_eRM_1^2}{2}+c_1\right\}=0$$

(15-c)

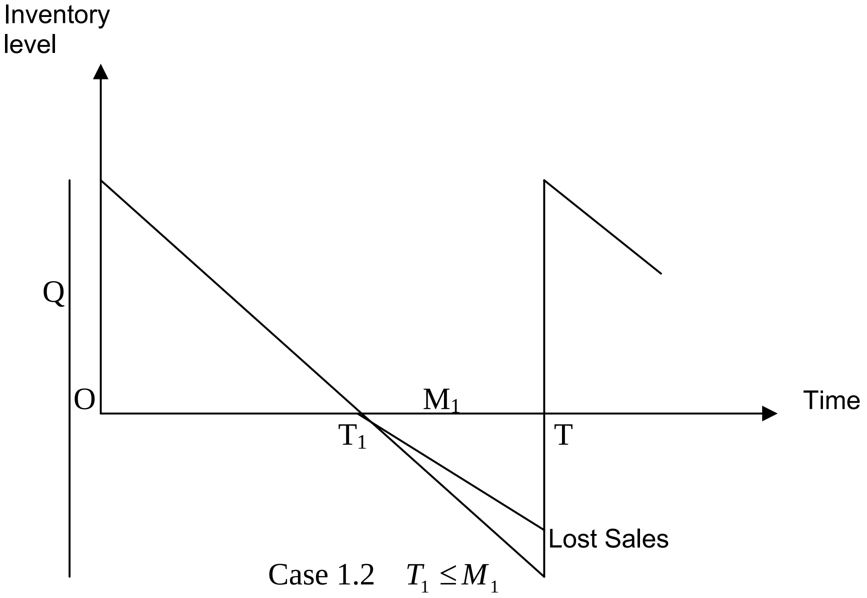
Equation (15-c) is reducing now in one nonlinear variable T_1, so cannot be solved analytically. Available software MS-Excel Solver is used here for solving T_1, T & C_1 from these equations as follows:

- Calculate the value of T from equation (15-a) and then square it. Calculate the value of T^2 from equation (15-b).
- Set the difference of above two calculated values equal to zero by changing the value of T_1. The value thus obtained will be T_1^* (Multiply this value by 365).
- The value of T_1^* is now substitute in either (15-a) or (15-b) to obtain the value of T^* (Multiply this value by 365).
- C_1 can be calculated by using optimal value of T_1 & T in equations (14) and (15-a) respectively.

It can be shown easily that second order sufficient conditions for a minimum value are satisfied.

Case 1.2 $T_1 \le M_1$

In this case the length of the positive stock period is not greater than the period of cash discount.



The setup cost, holding cost, shortage cost, opportunity cost due to lost sales and cash discount are identical to case 1.2. However since $T_1 \le M_1$, the retailer pays no interest during the period [0, M_1] and the interest earned is given by

$$\text{IE} = c I_e \left[\int_0^{T_1} R t \, dt + R T_1 \left(M_1 - T_1 \right) \right]$$

$$= \frac{c R I_e}{2} \left(2 M_1 T_1 - T_1^{\,2} \right) \qquad \text{- - - (16)}$$

Therefore the total cost per unit time is

$$C_2 (T_1, T) = \frac{c_1 + HC + SC + OC + DC - IE - CD}{T} \qquad \text{- - - (17)}$$

$$= \frac{1}{T} \left\{ c_1 + c_2 R \left(\frac{T_1^{\,2}}{2} + \frac{\alpha \beta}{(\beta+1)(\beta+2)} T_1^{\beta+2} \right) + \frac{c_3 \delta R}{2} (T - T_1)^2 + c_4 R (1-\delta)(T - T_1) \right.$$

$$\left. + \frac{\alpha R c_5}{\beta+1} T_1^{\beta+1} - \frac{c I_e R}{2} \left(2 M_1 T_1 - T_1^{\,2} \right) - r c R \left((1-\delta) T_1 + \frac{\alpha}{(\beta+1)} + \delta T \right) \right\} \qquad \text{- - - (18)}$$

For the minimization of cost we set,

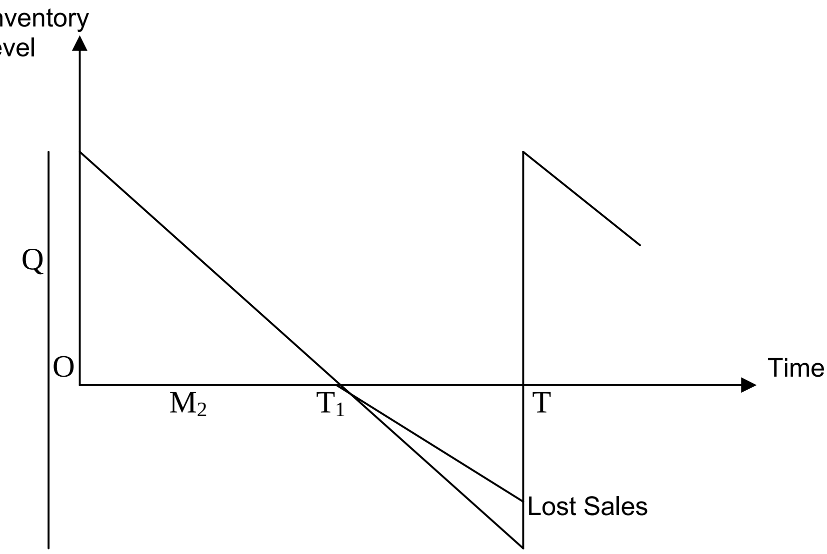
$$\frac{\partial C_2 (T_1, T)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial C_2 (T_1, T)}{\partial T_1} = 0 \qquad \text{- - - (19)}$$

The value of T_1, T & C_2 can be obtained similarly as described in case 1.1 also it can be shown easily that second order sufficient conditions for a minimum value are satisfied.

Case 2. Payment is made at time M_2

Case 2.1 $M_2 < T_1$

In this case the length of the positive stock period is greater than the last credit period.



Case 2.1 $M_2 < T_1$

The setup cost, holding cost, shortage cost, opportunity cost due to lost sales are identical to case 1.1.The retailer has no cash discount in this case and the interest payable is given by

$$\text{I P} = c I_p \int_{M_2}^{T_1} I(t)\, dt$$

$$= c I_p R \left\{ \frac{(T_1 - M_2)^2}{2} + \frac{\alpha \beta}{(\beta+1)(\beta+2)} T_1^{\beta+2} + \frac{\alpha}{(\beta+1)} T_1 M_2 (M_2^{\beta} - T_1^{\beta}) - \frac{\alpha \beta}{(\beta+1)(\beta+2)} M_2^{\beta+2} \right\} \quad \text{- - - (20)}$$

Interest earned per cycle is given by

$$\text{I E} = c I_e \int_0^{M_2} R t \, dt$$

$$= \frac{c I_e R M_2^{\,2}}{2} \quad \text{- - - (21)}$$

Therefore the total cost per unit time is

$$C_3(T_1, T) = \frac{c_1 + HC + SC + OC + DC + IP - IE}{T} \quad \text{- - - (22)}$$

$$= \frac{1}{T}\Bigg\{ c_1 + \frac{c_2 R T_1^{\,2}}{2} + \frac{\alpha \beta R}{(\beta+1)(\beta+2)} (c_2 + c I_p) T_1^{\beta+2} + \frac{c_3 \delta R}{2}(T - T_1)^2$$

$$+ c_4 R(1-\delta)(T-T_1) + c I_p R\left(\frac{(T_1 - M_2)^2}{2} + \frac{\alpha T_1 M_2^{\,\beta+1}}{(\beta+1)} - \frac{\alpha \beta}{(\beta+1)(\beta+2)}\right) M_2^{\,\beta+2}$$

$$- \frac{\alpha R}{\beta+1}\left(c I_p M_2 - c_5\right) T_1^{\beta+1} - \frac{c I_e R M_2^{\,2}}{2} \Bigg\}$$

--- (23)

For the minimization of cost we set,

$$\frac{\partial C_3(T_1, T)}{\partial T} = 0 \text{ and } \frac{\partial C_3(T_1, T)}{\partial T_1} = 0 \qquad \text{--- (24)}$$

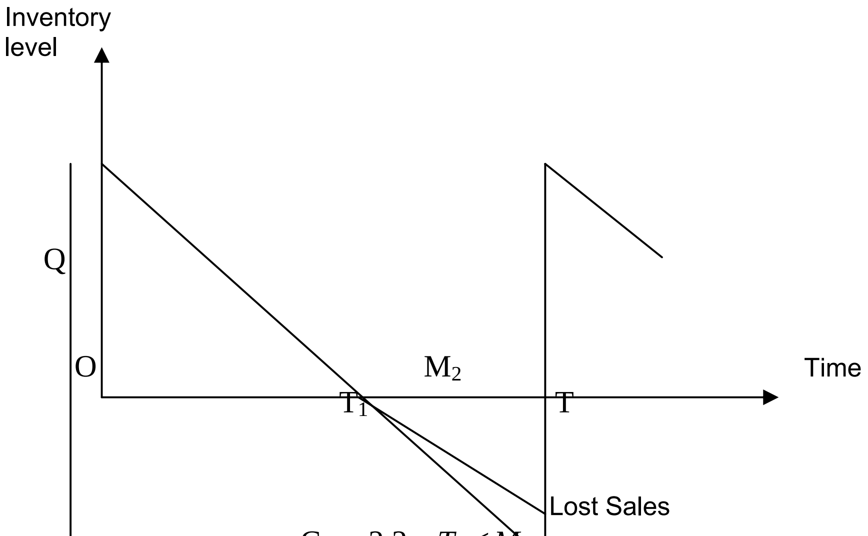
The value of T_1, T & C_3 can be obtained similarly as described in case 1.1 also it can be shown easily that second order sufficient conditions for a minimum value are satisfied.

Case 2.2 $T_1 \le M_2$

In this case the length of the positive stock period is not greater than the last credit period. The setup cost, holding cost, shortage cost, opportunity cost due to lost sales are identical to case 1.1. The retailer has no cash discount and no interest payable in this case and the interest earned is given by

$$\text{IE} = c I_e \left[\int_0^{T_1} R t \, dt + R T_1 \left(M_2 - T_1\right)\right]$$

$$= \frac{c R I_e}{2}\left(2 M_2 T_1 - T_1^{\,2}\right) \qquad \text{--- (25)}$$



Case 2.2 $T_1 \le M_2$

Therefore the total cost per unit time is

$$C_4(T_1,T) = \frac{c_1 + HC + SC + OC + DC - IE}{T} \qquad \text{- - - (26)}$$

$$= \frac{1}{T}\left\{ c_1 + c_2 R \left(\frac{T_1^2}{2} + \frac{\alpha\beta}{(\beta+1)(\beta+2)} T_1^{\beta+2} \right) + \frac{c_3 \delta R}{2} (T - T_1)^2 \right. \qquad \text{- - - (27)}$$

$$\left. + c_4 R (1-\delta)(T - T_1) + \frac{\alpha R c_5}{\beta+1} T_1^{\beta+1} - \frac{c I_e R}{2}\left(2 M_2 T_1 - T_1^2\right) \right\}$$

For the minimization of cost we set,

$$\frac{\partial C_4(T_1,T)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial C_4(T_1,T)}{\partial T_1} = 0 \qquad \text{- - - (28)}$$

The value of T_1, T & C_4 can be obtained similarly as described in case 1.1 also it can be shown easily that second order sufficient conditions for a minimum value are satisfied.

Algorithm

The optimal replenishment policies and minimum total cost per unit time can be obtained by using the following algorithm:

Step 1. Determine T_1^a and T^a from (15). If $M_1 < T_1^a$, obtain $C_1(T_1^a, T^a)$ from (14); otherwise (T_1^a, T^a) is infeasible.

Step 2. Determine T_1^b and T^b from (19). If $T_1^b \le M_1$, obtain $C_2(T_1^b, T^b)$ from from (18); otherwise (T_1^b, T^b) is infeasible.

Step 3. Determine T_1^c and T^c from (24). If $M_2 < T_1^c$, obtain $C_3(T_1^c, T^c)$ from (23); otherwise (T_1^c, T^c) is infeasible.

Step 4. Determine T_1^d and T^d from (27). If $T_1^d \le M_2$, obtain $C_4(T_1^d, T^d)$ from (28); otherwise (T_1^d, T^d) is infeasible.

Step 5. By comparing $C_1(T_1^a, T^a), C_2(T_1^b, T^b)$, $C_3(T_1^c, T^c)$ and $C_4(T_1^d, T^d)$, select the optimum replenishment cycle and optimal stock period (denoted by T^* and T_1^* respectively) with the least total cost per unit time (denoted by C^*). Once the optimal value T^* and T_1^* are obtained, the optimal order quantity, Q^*, can be obtained from (5).

Numerical Example

For the numerical illustration of the developed model, we consider the following values of the parameters

c_1 =\$ 200 per setup	c_2 =\$ 2 / unit / year	c_3 =\$ 8 / unit / year
c_4 = \$2/ unit / year	c_5 = 1.25 unit / year	R =1000 unit / year
c = \$20/ unit	I_e = 0.13/ year	I_p = 0.15/ year
δ = 0.8		

Sample calculation procedure for obtaining values in tabular format is as follows:
Substituting the above parameters with $M_1 = 5$, $M_2 = 30$, $\alpha = .02$ and $\beta = 1.5$ in equation (15-c) and solving by MS-Excel Solver software as described in case 1.1, we obtain $T_1{}^* = 0.248039$ and $T^* = 0.36732427$. Multiplying them by 365 we get $T_1^* = 90.53408$ and $T^* = 134.0734$. Substituting these values in equation (14) we get $C_1 = 1003.428$. Values of C_2, C_3 and C_4 can be obtained in the same way respectively in their cases. The optimal cost C^* can be obtained by using the algorithm given above.

The optimal solutions obtained by varying the values of r, M_1, M_2, α and β are shown in tables below

r = 0.01

M_1	M_2	A	β	C^*	T_1^* in days	T^* in days	Payment Time
5	30	0.02	1.5	1003.428	90.53408	134.0734	M_1
	60	0.04	1.5	879.5674	99.50439	126.846	M_2
	90	0.06	1.5	698.6566	103.9072	120.9178	M_2
10	30	0.02	1.5	975.7464	91.5139	133.4743	M_1
	60	0.04	1.5	879.4976	99.50439	126.846	M_2
	90	0.06	1.5	698.5339	103.9072	120.9178	M_2
15	30	0.02	1.5	947.8437	92.47791	132.8471	M_1
	60	0.04	1.5	879.4318	99.50476	126.8467	M_2
	90	0.06	1.5	698.4176	103.9072	120.9178	M_2

r = 0.015

M_1	M_2	α	β	C^*	T_1^* in days	T^* in days	Payment Time
5	30	0.02	1.5	909.8052	91.0145931	133.776983	M_1
	60	0.04	1.5	879.3887	99.5043912	126.846009	M_2
	90	0.06	1.5	698.3433	103.907209	120.917783	M_2
10	30	0.02	1.5	881.9129	91.9795893	133.15124	M_1
	60	0.04	1.5	879.319	99.5043909	126.846008	M_2
	90	0.06	1.5	698.2206	103.907209	120.917784	M_2
15	30	0.02	1.5	853.7974	92.9280967	132.496071	M_1
	60	0.04	1.5	852.9891	93.0386659	132.560669	M_1
	90	0.06	1.5	698.1044	103.907209	120.917783	M_2

r = 0.01

M_1	M_2	α	β	C^*	T_1^* in days	T^* in days	Payment Time
5	30	0.02	1.4	1003.615	90.5048652	134.0547862	M_1
	60	0.02	1.6	877.9349	99.7759595	127.0292776	M_2
	90	0.02	1.8	694.6428	104.450754	121.2487103	M_2
10	30	0.02	1.4	975.9333	91.4853843	133.4565872	M_1
	60	0.02	1.6	877.9049	99.7759595	127.0292776	M_2
	90	0.02	1.8	694.6169	104.450754	121.2487103	M_2
15	30	0.02	1.4	948.0314	92.4498511	132.8297723	M_1
	60	0.02	1.6	877.8765	99.7759592	127.029277	M_2
	90	0.02	1.8	694.5918	104.450754	121.2487103	M_2

r	M_1	M_2	α	β	C^*	T_1^* in days	T^* in days	Payment Time
r = 0.015	5	30	0.02	1.4	909.981	91.0145931	133.808674	M_1
		60	0.02	1.6	877.859	99.7759595	127.029278	M_2
		90	0.02	1.8	694.5781	104.451031	121.249205	M_2
	10	30	0.02	1.4	882.0887	91.9795893	133.182472	M_1
		60	0.02	1.6	877.8291	99.7759595	127.029278	M_2
		90	0.02	1.8	694.5521	104.451031	121.249205	M_2
	15	30	0.02	1.4	853.9736	92.9280967	132.526877	M_1
		60	0.02	1.6	852.8914	93.0386659	132.50605	M_1
		90	0.02	1.8	694.5271	104.451031	121.249205	M_2

Sensitivity Analysis

For fixed r and M_1 the larger the values of M_2 is, the smaller the total cost per unit time would be.

For fixed r and M_2 the larger the values of M_1 is, the smaller the optimal cost, the larger the value of M_1 is, the smaller the optimal total cost per unit time would be i.e. 1003, 975, 947 as the optimal payment time is M_2.

However if the optimal payment time is M_2 , the optimal total cost per unit time is independent of M_1.

For fixed r, M_1 and β as the value of scale parameter α increases the total cost also increases as compared to the model by Ouyang et al.

For fixed r, M_1 and α as the value of scale parameter β increases the total cost of the inventory system again increases as compared to the cost obtained by Ouyang et al. under same parametric values. Although the increase in cost is much significant with the increasing values of α as compared to the cost when the value of β increases.

Conclusion

An inventory model with Weibull distribution deterioration, shortages, cash discount and permissible delay in payment has been developed in the article. An algorithm is suggested to find the optimal replenishment policies and minimum total cost per unit time, which helps the inventory manager to decide whether it would be worthwhile to take advantage of a longer credit period for repaying the supplier by ordering a larger amount of the commodity. The sensitivity analysis reveals the importance of developed model. The model developed is much more realistic as it considers time-dependent deterioration.

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