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Editorial

In this issue, we are delighted to publish six papers. The contributions in these papers cover a range of topics such as soft operations research, optimisation of production systems, inventory modeling, queueing network modeling, and linear fractional transportation problem. We believe the Bulletin readers will enjoy reading these papers.

Please note that ASOR Bulletin will be published only electronocially from the next issue. The editorial policy and Bulletin format will still be the same. The electronic version of ASOR Bulletin is available (free of charge for all) at the ASOR national web site: http://www.asor.org.au/. Currently, the electronic version is prepared only as one PDF. We like to thank our web-master Dr Andy Wong for his hard work in redesigning and smoothly managing our national web site. Your comments on the new electronic version, as well as ASOR national web site, is welcome.

ASOR Bulletin is the only national publication of ASOR. I would like to request all ASOR members, ASOR Bulletin readers and OR organizations in the country to contribute to the ASOR Bulletin. The editorial policy is available either from the Bulletin web site or from the inside back cover of the Bulletin. The detailed instructions for preparing the manuscripts is available in the URL: http://www.asor.org.au/.

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"Tales of the Unexpected: Issues in Overseas Market Entry"

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Abstract:
The paper considers several examples of overseas market entry projects/deals, drawn from a range of different organisations, which went wrong to some degree. Explanations are offered as to why the projects, sited in three different countries, failed; and in discussing the examples suggestions are made of (accessible) models which can both help to explain the 'whys', and might also have helped the players to avoid or foresee, to some extent, the issues which arose.

A key conclusion to emerge from the discussion is that 'soft', evaluative models may be as important as, or even more important than, the hard, mainly financial models on which many organisations routinely focus their attention, when considering foreign entry projects. The use of these sorts of models can and should be seen as O.R. modelling at work to aid strategic planning and hence decision making. The importance of understanding the cultural, as well as the market, setting in which one makes an investment is highlighted.

Keywords: Overseas market entry; failure; evaluative models; planning; effectiveness; FDI; culture.

Introduction
The paper considers four examples of overseas market entry projects/deals, drawn from a range of different organisations, which went wrong to some degree. Explanations are offered as to why the projects, sited in three different countries, failed or faltered. In discussing the examples, we explain how a set of (relatively) 'soft', rather than 'hard', evaluative models can both help to explain why, with hindsight, the projects did fail or falter and how they might also have helped the players to avoid or foresee, to some extent, the issues which arose, had they been deployed prior to launching the projects.

The whole tenor of this paper is that planning does help and that it is sometimes the softer types of models, perhaps dealing with issues which may not all be readily quantified and their use programmed, which may throw light into otherwise dark corners. These soft models, alongside hard models, be they financial appraisal tools or evaluative, mathematical-modelling tools, help to deliver an holistic, evaluation package which is truly fit-for-purpose. As a precursor to what follows, consider the words of Sun Tzu, from his *Art of War* [1, p. 102] (written circa 500BC):

> *Now, if the estimates made before a battle indicate victory, it is because careful calculations show that your conditions are more favourable than those of your enemy; if they indicate defeat, it is because careful calculations show that favourable conditions for a battle are fewer. With more careful calculations one can win; with less, one cannot. How much less chance of victory has one who makes no calculations at all!*

Over the last fifteen years or more, there has been an overt discussion about the links between OR and strategy or the contribution OR can make to the overall strategy process. Indeed *JORS* [2, 3] , the house journal of *The Operational Research Society*, ran two special issues on the theme in 1999 and 2000, the former considering one particular OR approach, system dynamics, and the latter a wider range of approaches. The conclusion seems inevitable, that, provided one does not try to sterilise the argument by imposing rigid and unrealistic boundaries on the concepts, OR does have a role in strategy. This role may occur in a number of forms. It may relate to:

- option generation, using tools routinely called PSMs in OR;
- ‘scene setting’ techniques such as scenario planning and so-called strategic visioning;
- evaluative models;
- the use of technical modelling techniques, which are not usually viewed as strategic, to address problems which are themselves intrinsically strategic e.g. Robinson’s [4] use of discrete event simulation to explore major problems connected with the handling of nuclear waste; or indeed
- other approaches.

Thus we assume here that the question of whether OR does have a role to play in strategy is essentially yesterday’s debate and focus on the issue at hand; namely, soft models’ role in the overseas market entry decision, undoubtedly a strategic decision for all organisations.

One important caveat is in order here. There are those, e.g. Beasley [5], who allege that the proponents of soft-OR, (for details of some soft approaches see Rosenhead [6]), really believe that ‘soft’ is more important than or is superior in some way to ‘hard’. It is not at all our argument that soft-OR methods are superior or indeed inferior to hard methods. Rather we argue that soft methods, such as the multi-attribute frameworks described in the next section, do have their place in the OR-toolbag; and that, when such methods are used, it may be appropriate to use them in tandem with some complementary, hard tools for analysis (possibly financial). As for Rosenhead, he at least of soft-OR proponents, is more than happy to agree that hard methods have their place, as I know from talking to him about just that issue.

The paper continues with a section setting out the three evaluative tools which it is proposed can help and could have helped in the particular settings. There are almost certainly other models which could help similarly but we focus here on the three cited models. There follows then a section in which the four case examples are described and the uses of the respective tools is discussed. The four organisations and their market entry locations are: a UK technical support services company for the financial services sector, looking at China entry; the Kingfisher subsidiary B&Q’s China entry; the case of an articulation between a Malaysian private college and a UK university; and the now aborted infrastructure operation in Thailand of Hopewell Holdings, a Hong Kong based development company. Finally there is a conclusion which seeks to bring together the argument and present conclusions.

Three Evaluative Tools for Planning Enhancement

In this section we outline three evaluative tools to whose development we have been a party. They are in sequence the Foster-Dyson framework for evaluating effectiveness of strategic planning systems, from a process perspective (see Dyson and Foster [7]); Foster’s [8] FDI-screen for evaluating FDI projects; and, a 7-Forces model of country markets, based on Porter's [9] 5-forces model in some degree but changed so that the focus of the model is the company who is planning rather than the market segment, Tseng and Foster [10].

Reviewing these models here is, of course, firstly so that they may be clearly understood when deployed in the case problems which follow. However, in the light of a comment by a practitioner on an early draft, it also seems useful to note here the reasons why these models were derived or posited in the first place. In each case the answer was to fulfil a need which, while it may have been intuitively understood by some, was not obviously met in the available literature, at the time that the models were initially developed. Thus Dyson and Foster [7] began to develop their framework because there was no satisfactory, comprehensive process-oriented model of effectiveness in strategic planning in the literature and they wanted to use such a model in their research. In addition, the framework can be shown to have obvious utility for practising planners.

In the case of the FDI-screen, it was thought that such an holistic organising framework would be a commonplace but as Foster [8, 11] reports, he found that not only was the public literature sparse in this regard but companies, from the FTSE 100 and 250, whom he then asked about their practices, often had no such framework in-house, and indeed failed to assess some of the factors effectively. To be sure certain large corporations do have their own, comprehensive frameworks (for example, I am assured that Shell do) but the key point is the absence of such frameworks in many large and, it would be assumed by most, well-run companies. Thus creating the screen filled a gap.

In the case of the 7-Forces model the issue is simply one of extension and refocusing to improve the applicability and hence utility of the model.

The Foster-Dyson Framework

This framework's design emerged from the need for strategic planning (SP) to be effective. When the work commenced in the early 1980s, what was clear was that the field was open: there was little in the way of useful frameworks for assessing effectiveness. Most people relied on ends-oriented measures (see e.g. the 'does planning pay?' literature referred to above) and assumed that, if the financial results were poor, the SP must have been ineffective and vice versa, but this need not be so. It is possible that there is a perfectly good planning system, capable of delivering results, if implemented well, but which delivers poor results because of poor implementation or changing external environment over which the organisation has no control (e.g. government regulation or political crisis; September 11^{th} comes to mind as an example). Hence the Foster/Dyson framework for measuring effectiveness in terms of process (as compared to ends) was born. Effectiveness is characterised in terms of thirteen attributes of the SP process. Namely:

E_0 Clear statement of objectives
E_1 Integration of planning function
E_2 Catalytic action of planning function
E_3 Richness of formulation (of plans)
E_4 Depth of evaluation
E_5 Treatment of uncertainty in evaluation
E_6 Resources planned
E_7 Data used
E_8 Iteration in process
E_9 Assumptions made
E_{10} Quantification of goals

E_{11} Control measures (responsiveness to uncertainty)
E_{12} Feasibility of implementation.

It was further proposed that each factor could be measured or scored using a (typically 5 or 7 point) Likert scale, thus delivering a scoring profile of the organisation's planning effectiveness, or also allowing focus on individual attributes or a small subset, if desired.

There has been support from some authors since the time at which the Foster/Dyson framework was developed for the adoption of a multi-faceted, system-driven view of planning effectiveness. The most positive of these, *vis a vis* our position, have been King [12], Greenley [13], Ramanujam and Venkatraman [14] and Phillips and Moutinho [15]. The last named authors were particularly interested in the more detailed calibrations of the scales which Foster [16] developed to illustrate how individuals could take ownership of and develop the framework in their own settings. These more detailed descriptors can also help to make the framework useful as a planning as well as a diagnostic tool.

Foster's FDI-Screen

There are six factors (F_i) comprising the screen as follow:

F_1: Infrastructure Adequacy

F_2: Power Availability (a special case of F_1)

F_3: Labour Adequacy

F_4: Cultural Aspects of Projected Host (/Difference from investor's culture)

F_5: Market Potential

F_6: Country Risk

As in the case of the effectiveness framework, it was proposed that a subjective scoring scale could be attached to each of the attributes thereby creating a scoring profile which can be used in conjunction with other harder measures such as the IRR or whatever financial measures might be being used. Because two of the measures are measures of difficulty rather than positive presence, namely F_4 and F_6, it is normal to score the complements of those factors so that the resultant score-profile is readily interpreted as right-hand tendency is good and left is problematic.

In the case examples, it is factor F_6 which is the main focus of attention. One of the interesting points to emerge from the empirical work which underpinned the development of the screen was the scant modeling attention paid to country risk by many large public corporations. Hence the relatively soft scoring approach advocated in the original paper seems sensible because it makes the process accessible and hence real for the players in that organization; using somebody else's complex but closed black box won't illuminate to the same extent. In fact, we suggested using a subjective scoring scheme originally published by the *Economist* as the way to reach a final simple score out of five say. Their schema has a number of attributes, some political, some social, each of which attracts a weighted score leading to an overall score out of 67! Included in their scored attributes is corruption, which is often crucial but which some people find it 'embarrassing' to discuss publicly. In the cases of the Malaysian college and the Hopewell project described in the next section it is, or was, a critical factor.

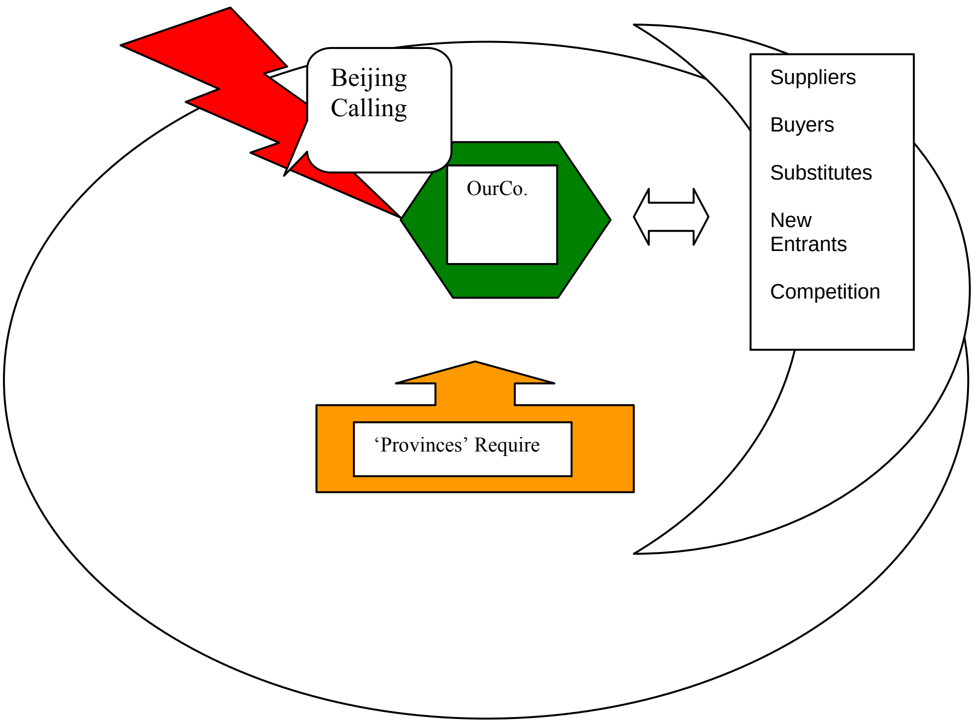
The FDI-screen, as originally proposed, was intended to be used by companies or organizations evaluating FDI opportunities in countries other than their home base. It became clear that in fact it could also be used “the other way round”; in other words, potential FDI hosts could appraise themselves, ‘looking through the lens from the opposite direction’. As a result of such a viewing, they might then seek to take proactive steps to improve their actual position and hence assessed status as potential “attractors” for (to them) inbound FDI. An example of this approach at work is given in Foster and Wang [17].

The 7-Forces Model

In the paper of Tseng and Foster [10], they were looking for a way to summarize the impact of the twin bureaucratic forces of central and local Chinese government on local market structures. One might almost say their dislocative impact.

In this regard at least, these twin aspects of government (provincial and central) could be seen to comprise a 6th-force in an industry segment model such as Porter’s [9] However, it should be noted that their model, while similar in broad intent to Porter’s, adopts a different focus. Porter’s focus is on the segment as a whole; their model installs the individual enterprise as the focal point of the model, as is shown in Figure 1 here.

Figure 1 The 7-Forces Model (for China)



This gives a model which shows that there are (at least) two major elements to be understood and dealt with by a company in their industry segment within a country such as China:

- The five forces of the original Porter model – shown as a group in the crescent which embraces the company: the double-headed arrow between the company and the crescent emphasizes the dynamic interaction.

- The strong presence of government at two major levels – both have to be dealt with; they may not always be in harmony (provinces/municipalities may try to 'go it alone' to a degree); but, especially for the foreign company, Beijing's central authority has the final say (whoever may be in the right legally).

This is not to say that there are not other, very important, environmental factors (social for example). The symbolism of the closed elliptical boundary is a soft systems representation of precisely the fact that this enhanced, industry-segment model sits in its own wider, contextual environment. The explicit inclusion of the government aspect simply highlights the great importance still of government in China, as compared say to the US or EU countries.

Having outlined the models which will be applied in the cases which follow we now consider them in sequence.

Four case histories

PAF Ltd

PAF Ltd (the company's identity is disguised for reasons of confidentiality) is a medium sized firm operating from a base in the North-East of England which offers a set of services relating to credit and debit cards and other specialist cards such as fuel cards. PAF offers transaction enabling services and real-time (at the time of a sale being transacted) fraud prevention systems for the types of cards noted. Since the delivery of the service can be effected remotely using the company's computer systems, international expansion is an obvious potential strategy for them to pursue. In 2006, the company already had extra-European clients on at least two other continents. As China and the rest of the Far East boomed, the CEO reasoned that these could be his next markets of opportunity.

Bearing that in mind, PAF hired a young woman as an in-company researcher and simultaneously entered into a consultancy agreement with one of their local universities, who had specialist knowledge in the area of China business and retailing in China in particular. The arrangement was that the academic consultants would offer expertise in issues of China/Asian market entry and co-supervise the work of the researcher. As work began the PAF team imagined that the huge numbers of lorries ferrying goods around China and the booming use of e-payment methods should offer them major opportunities. The initial brief was to assess and advise on the best way to enter the China market, perhaps using the small, westernised 'city-states' of Hong Kong and Singapore as pilot markets.

Initially, the project went quite smoothly, notwithstanding the CEO's propensity to make snap judgements which could result in unexpected redirections of 'trading strategy' at short notice – indeed like many companies, especially SMEs, PAF's strategy seems to be emergent in nature, and subject to incremental change to take advantage of opportunities as they occur. Fuel cards were ruled out early on: Chinese truckers pay cash for their fuel and the owners probably aren't about to go to fuel cards. This is not only to do with usability and issues of employee trust but probably also the desire to have fewer rather than more financial footprints available to the PRC taxman!

On the matter of providing fraud prevention services, PAF entered talks with one of the PRC's leading (bank) payment services companies with a view to forming a strategic partnership. The

Chinese party certainly want the kind of capability PAF can offer, so things looked promising until a serious problem emerged. The Chinese partners were clear that the system to help them would need dual-byte capability (DBC), that is the ability to read both western script and Chinese characters. Currently PAF do not have DBC. The cost of acquiring it and getting it up and embedded on their processing platform was estimated to be of the order of £300k-£400k. As a strategic investment – giving access to not only China but also Taiwan as well as possibly other smaller locations or market niches – the first order appraisal seems to be that it is a modest investment. To the surprise of both the researcher and the consultants, the PAF CEO ruled out making the investment on the grounds of cost. This in turn means that arguably the biggest, juiciest segment of the PRC market is off the agenda, for the present at least. And the next decision? "We'll look at India instead; start the market appraisal immediately, please."

At the time of writing, appraisal of the India market continues and, in parallel, some modest scale deals, not requiring DBC, have been signed in China, both with the bank mentioned and travel businesses. For all of these initial deals the task is related to screening transactions made by parties using overseas credit cards, for which DBC is not required. Indeed, having got some initial deals started seems to have stimulated more opportunities and the researcher has now been re-oriented to a primary focus on China, six months after the 'look to India' decision.

What models might have been used to avoid this unexpected problem or at least diagnose the matter as key at an earlier stage? As already noted, the PAF style of doing business seems not to involve a great deal of strategic planning (SP) – short term cash generation is the driver. The Foster-Dyson framework for planning effectiveness would have been helpful to PAF, given a different corporate ethos. One of the important factors in that framework is E_{12}, feasibility of implementation. Had PAF adopted a more overtly SP-oriented approach to its possible China entry, it seems likely that this barrier-to-entry would have shown itself sooner. It may well be that bumping in to E_0, clear statement of objectives, at an early stage might also have created beneficial pause for thought. As it was PAF nevertheless rated the project a clear success because, helped by a systematic approach to market evaluation and coaching in the demands of dealing with Chinese business culture, market entry was achieved within 18 months.

Kingfisher in China

The second case also relates to a UK company doing business in China, namely the development of the B&Q brand in China by Kingfisher plc. Kingfisher first entered the PRC market in 1999 and by the end of 2007 had 62 stores in first and second tier cities. In seeing a potential market to exploit in China, B&Q refined and adapted their UK-centric, core approach away from DIY to DFM (do-it-for-me). This is particularly well suited to the local market not only because the Chinese middles class are not used to doing decorating etc for themselves but also because flats (the main stock of housing units for sale) are sold in a fairly stripped-down, or unfitted, form. Thus, before moving in to a new flat there is more to be done than might be normal in western Europe. In the trading periods up to year end 2006/7 (in February 2007), it is claimed by local management that operating surpluses were generated by the stores albeit the group accounts show no profit for B&Q (China), presumably because development costs of the continuing expansion were being off-set against those retail operating profits.

However, in Autumn 2007, B&Q announced that their China subsidiary had incurred losses of £9.5m in the first half of 2007/8 (to 4 August 2007), a slippage of £4m compared to the previous year, due primarily to changes in PRC regulations governing the relationships of foreign retailers with their local supply chain contractors. In the Kingfisher 2007 Interim Statement [18], the CEO for B&Q (Asia), B&Q (China)'s immediate holding company based in Hong Kong, stated that, "Finalisation of B&Q (China)'s 2007 supplier agreements was delayed pending clarification by the authorities in August 2007 of new regulations covering trading between retailers and suppliers.” He went on to say that the company expected to conclude satisfactory agreements in 2008 but anticipated a further hit on the second half results for 2007/8 after which it was expected that the China subsidiary would return to profitable trading – the Kingfisher final year results announced an £11m loss. As far as one can see, this was, from the Kingfisher/B&Q point of view, an *unavoidable* problem driven by government officials in Beijing. In terms of the illustrative models outlined at the start of the paper, the 7-F model is clearly the obvious one which applies and, perhaps unsurprisingly, it is the “Beijing Calling” lightning flash which comes immediately into focus. In summary form it certainly helps the outsider to see what is going on. From a planning perspective, it may be that the quixotic nature of the PRC’s regulatory amendment process does indeed mean that the issue was not readily forecastable and hence could only really be dealt with once it had happened.

The events described can also be seen to involve a thread of cultural dissonance, which brings in factor F_4 from the FDI-screen.

The case of Unitek Kolej Malaysia

In the mid-1990s a number of UK universities including Kingston, in the form of its Business School (KBS), entered into 2+1/1+2 articulation agreements with a Malaysian private college in Kuala Lumpur, which became known soon after KU signed its agreement as Kolej Unitek Malaysia. Although a private college, Unitek benefited up until around 1998/9 from Malaysian government support, both directly and indirectly, in the form of study grants to Bumiputra (Malay) students studying at the college. These study grants not only helped to support the students in KL while they studied at Unitek but were also continued, at a higher value, to cover their study abroad period. The minority of ethnic Chinese students who came to KU paid their own way.

About three years into the deal, the Asian economic crisis hit and Malaysia suffered its share of problems. After the Ringgit (Malaysia’s currency) slumped to about two thirds of its previous value, the government sought to stem the tide by imposing a fixed exchange rate at the new lower level and imposing foreign currency movement restrictions on its citizens. Within the education budget, aggregate spending for student grants was cut from session 1998/9: this had two effects. First only ‘A-grade’ students received the more expensive grants to travel to the UK – ‘lesser’ students were funded but only to enable them to finish off a degree in Malaysia, possibly on a franchised ‘Top-Up’ course. This had the effect of cutting off the supply of students to institutions such as KBS.

One response to this problem might have been to seek to make a franchise agreement for the delivery of KU degrees at Unitek, but even had that been considered it would not have worked, because of the train of events which followed from a power struggle between Dr Mahathir, the Prime Minister and leader of the ruling UMNO party, and his Deputy Anwar Ibrahim. The box below sets out briefly the story of Anwar’s demise and fall from power. The immediate effect on KU, and presumably other partners was that by some point in 1999, Unitek had effectively ceased trading: why? Unitek had been able to access government support because of its links to the Malaysian government but to the Anwar faction within that government. As soon as Anwar had been removed to jail by the regime, their funding stream started to dry up.

In terms of understanding and modelling the issues in this case, all three of the models described earlier can apply to some extent. First, the Foster-Dyson effectiveness framework would suggest three particular elements which might possibly have been highlighted from the perspective of planning at KU: E_5, treatment of uncertainty, E_9,
assumptions made, and E_{12}, feasibility of implementation.

Brief chronology of events around the 'fall of Anwar':

2 Sept 1998: Anwar sacked as deputy prime minister and finance minister

3 Sept 1998: Anwar expelled from UMNO.

19 Sept 1998: Two close Anwar associates, his former speechwriter, Munawar Anees, 51, and Anwar's adopted brother, Sukma Dermawan, are sentenced to six months in prison after pleading guilty to engaging in "unnatural sex" and allowing Anwar to sodomize them.

20 Sept 1998: Anwar arrested at his home by police.

1999: sentenced, after a "highly controversial" trial, to six years in prison for corruption,

2000: sentenced to another nine years in prison for sodomy; however…

2004: Malaysia Federal Court reversed the second conviction and he was released

The issue of whether scrutiny of future uncertainty would have predicted the fall of Anwar is doubtful, unless the KU staff had happened to have an immediate conduit to the Malaysian political grapevine. Stresses in the UMNO party might have been 'knowable' but that the hierarchy would resort to what many observers considered to be trumped up charges, especially one as offensive as buggery, is another matter. Cutting the same cake from a different angle, the assumptions made were effectively 'stable government in Malaysia', which had largely been true for the preceding 20 years – not everyone liked the regime but it was seen as stable. As to the Asian 'flu', that frankly took most markets relatively unawares so it may be unfair to harshly criticise a university for failing to read its crystal ball accurately. That some problems might exist was probably predictable but it was the scale of the crisis, once started, which took many by complete surprise, see Haggard [19].

As far as effectiveness factor E_{12} is concerned, it can be argued that the project was feasible, albeit the inherent future risks were not fully appreciated, since some 70 'student fee years' resulted, generating revenues of over £400k (worth around £650k at 2008 prices). Since the set-up costs were minimal (less than £5k), the project had immediate payback.

In terms of the other two models, country risk from the FDI-screen applies but the truth is that KU failed to evaluate that element of the potential risk back in 1995: nevertheless they should have looked at it. And taking the 7-F model and replacing 'Beijing Calling' with 'UMNO calling', flags well the process which did occur. So in conclusion, with the benefit of hindsight, use of these types of models could have helped KU to foresee the type of problem/s which occurred but it would have required more expertise and time than was immediately to hand, within the University's External Affairs Directorate, to have realistically predicted the downside risks. What may be more important is that, as already noted, the result was an immediate pay-off so that one can argue that there was effectively zero risk, provided one had not allowed one's unit to be over dependent on the new incremental revenues. There were some other universities, not too far distant from Kingston, which did suffer significant short term embarrassment from over-exposure to the Malaysian, student, import market in the 1990s. Given the scale on which they were operating, all the factors noted should have been evaluated; use of these models would surely have improved the decision making.

Hopewell in Bangkok

In 1991, Hopewell began work on a proposed, mixed-mode, elevated road-rail system (BERTS), which was to run northwards from the main train station in the city centre in Bangkok out to the then sole airport at Don Muang (a route of about 25kms). There was also to be an eastward branch following the State Railways of Thailand single track line across the centre of the city out part-way towards the location of the new Suvarnabhumi airport. This was a much needed scheme and would have been built to a high specification – the same cannot be said of all Bangkok's infrastructure projects.

After a series of 'hiccups' over a period of years, work halted totally in January 1998 when the Thai government notified Hopewell (Thailand) Ltd. of its intention to terminate their agreement - Hopewell is a major, Chinese-owned developer/contractor from Hong Kong. At best one could say that there was a lack of mutual trust between Hopewell and the Thai authorities. Certainly the collapse in land/property prices in Thailand during the post-1997, economic problems was a factor but more interesting would be the reasons behind earlier delays to commencement of the project (during which time there were a number of changes of government and a military coup). The Thai government alleged breaches of contract by Hopewell, who for their part insisted that they had acted in good faith and were the victims of dilatory action over a lengthy period of time, at best, culminating in expropriation by the government party in violation of the terms of their concession, Hopewell [20]. Hopewell state that to the time of the breakdown they had invested some US$640m! What remain today are dozens of reinforced concrete pillars which would have supported the roads and rail beds, pointing poignantly skywards in silent gestures of hopelessness.

Meanwhile the Bangkok traffic congestion at that time was, and in some large measure remains, atrocious. Another elevated train project in the central zone was [re]scheduled to open to coincide with the King of Thailand's "sixth cycle birthday" (i.e. his 72nd) in December 1999, and did so, but it is only in the centre of the city. In addition a new subway/underground system (for which key work in the central area started in early 1999) caused, more local chaos during its construction phase, before eventually bringing some relief to the road situation. By the start of 2003, according to an industry insider, the tunnels for the underground had been drilled but the fitting out and testing took more than another year, and the scope of the network was small.

At the core of this project, the problem could be said to be essentially one of corruption. In the Foster [8] FDI-screen this comes under factor six (F_6), country and political risk. In terms of the modified, 7-forces (7-F) model, Tseng and Foster [10], it could be seen as coming within the scope of the lighting strike, factor of central government control, and/or interference – see Figure 1 again. Indeed, it illustrates perfectly why Tseng and Foster chose to use the lightning symbol to illustrate the influence of central governments in Asia on firms and their market segments. Lightning can be dangerous and can strike suddenly with devastating effect. Was the Thai government's attitude capable of prediction and were the eventual problems therefore avoidable? In one sense yes but why should it be this particular, much needed, project against which the Thai government machine launched its lightning strike? Thus the modified 7-F model helps us see the issues but, from a forward looking perspective in 1990, the project may have looked solid. Given that it is our understanding that corruption was a major issue, if not the sole issue, from a planning perspective, it may be that the problem, or some others which might have arisen, would only have been capable of planning and corrective action if a position of ethical relativism had been adopted and with it an attendant willingness to "solve" the problem by making a (large?) bribe. The very act of discussing ethical relativism also suggests that cultural dissonance was present to some significant degree.

Conclusion

In the four case histories it has been shown how three different, 'soft', evaluative planning models could shed light on problems which had arisen in the contexts of four, very different organisations, linked only by a common thread here of having made entries into international markets, using a variety of forms of entry, which had failed or encountered unforeseen problems – hence tales of the unexpected.

The first important conclusion to emerge from the discussion is that these 'soft', evaluative models may be as important as, or even more important than, the hard, mainly financial models on which many organisations routinely focus their attention, when considering foreign entry projects. For example, the payback and further return from the UK university-Malaysian college link was sound, as noted before, so in that way the project did not fail. However, it did fail in the sense that the cash flows came to an abrupt end which had not been anticipated. Had the effectiveness framework or the 7-forces model been deployed at the planning stage, or even during implementation, the abrupt ending may well have been foreseen, if not wholly avoided.

Secondly, the fourth factor in the FDI-screen, which relates to cultural distance between investors and hosts, was a common factor across all the stories, albeit not the main, direct, modelling focus in the episodes described. The importance of investing time and energy to bridging these cultural gaps cannot be over-estimated. Handling them is a necessary condition for success. Using the FDI-screen, or some similar model, as a part of the regular planning process would ensure that such an important variable would not be overlooked.

The final conclusion is that, in the spirit of the learning organization (from what DiBella [21] calls the "developmental perspective"), enquiry and scrutiny using exploratory models are never wasted time because they help us to develop more refined understandings of problems even if they do not deliver "definite solutions" in the way that optimisation routines do when they are appropriately applied. In this context, the type of evaluative, planning models described and used here should be seen as examples of O.R. at work; models helping, at a minimum, to improve the structuring or thinking through of problems and typically helping also to begin to generate usable solutions. Put another way, Sun Tzu made the case for a planning approach to business and O.R. as a key aid to that planning 2500 years ago: we have just been a little slow to understand his wisdom.

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Modeling of a Transfer-Line Production System with a Rework Machine

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Abstract
The purpose of this paper is to model transfer line production systems incorporating rework, using semi-regenerative processes, for the expected duration analysis. The system under study is provided with an initial buffer of unlimited capacity. That is stage I (Machine I) is never starved. All the products emerging out of machine I are inspected at the inspection station. The good ones are transferred out of the system while the products not conforming to specifications are reworked in a rework machine. Explicit expressions for some of the system characteristics have been obtained using state-space method and regeneration point technique. All the random variables involved in the analysis are assumed to be arbitrarily distributed (i.e. general). The work has been extended to include multi-type rework.

1. Introduction
Analysis of production systems is one of the oldest problems in industrial engineering. The literature on analysis of production systems is aplenty but the literature on analysis of production systems incorporating the concept of rework is scanty. Most of the available literature on analysis of production systems assumes that all the distributions involved follow exponential distribution. Therefore most of the existing models could be analyzed using Markovian Processes [1, 2].

The focus of analysis of this paper is discrete part manufacturing systems, where each item processed is distinct and the processing times being non-Markovian (i.e. general). Such systems are normal in mechanical, electrical and electronics industries making components for cars, refrigerators, electric generators, or even computers [3]. The analysis of productions systems, though not given the importance to the extent it deserves, is one of the most important problems in production operations, analysis and management.

Variation in the production rate of the transfer lines may be due to external causes such as power supply failures, material shortages or perhaps the way incoming orders arrive and production plans are prepared [4]. The efficiency of a transfer-line with no internal storages (inventories) can be substantially less than that of the efficiency of the transfer-lines with internal storages. Internal storages provide a means to improve the line efficiency so that it becomes closer to the efficiency of the worst stage, that is, the stage with the lowest throughput if it were operated on its own [5].

Systems without internal storages are frequently encountered in industry. In such cases, since there is no buffer in between the stages, the behaviour of each stage is highly dependent on one another due to the effect of blocking. Two types of behaviour are encountered in such transfer-line systems. They are synchronous behaviour and asynchronous behaviour. In the case of asynchronous behaviour, parts can move independently of each other, whereas in the case of synchronous behaviour, transfer of parts from one machine to the next one occurs simultaneously

[6]. This may be the case, for instance, when a rigid parts transfer system is used. It should however be noted that in the case of two-machine transfer lines, it is easy to show that the production rate obtained using asynchronous transfer is greater than that corresponding to a synchronous transfer line. Clearly, the production rate of a transfer line with synchronous behaviour provides a lower bound on the production rate of the same line with asynchronous transfer [7].

The basic causes of problems in production line are different production rates, variability of the service time due to randomness, and station breakdowns. Losses in line efficiency are evidenced in periods where a station is blocked or starved. A station is blocked if the service of an item in this station is completed and service in the next station is still going on so that it is not possible for the item to enter the next station. In this case, the station remains idle until the service in the next station is completed. A station is starved if there are no items either in the buffer, which is provided for the machine, or in service [8].

The analysis of two stage transfer-line production systems provides useful hints to describe generalized (i.e. n-stage) systems. This is because of the reason that any multi-stage transfer-line production system can be decomposed in to a series of two-stage models and be analyzed [9].

Several authors such as Avi-Itzhak [10], Avi-Itzhak and Yadin [11] have analysed production systems to find various measures of system performance. Muth [12] has considered variables service times. Rao [13] and Lau [14] have analyzed production systems of tandem type. But, in all their work, inspection was not taken into account or (all the) rejected items were scrapped. But, this may not be feasible always. This is particularly so, when the cost of an item is high. In fact, as it has been pointed out by Gupta and Chakraborty [15], rework is inevitable in many production systems. Not much work has been reported on rework. Few authors [16], [17] have suggested rework of rejected products, but their work is confined to deterministic models. Others have considered only Markovian approach. Most of the work in the literature is mainly concentrated on the analysis of steady-state characteristics of the system which may not be useful in reality, as most of the systems will breakdown or collapse before reaching the steady state. Shanthikumar and Tien [18] have analysed the transient behaviour of the system without taking into account the concept of rework. Gopalan and Kannan [19] have analysed a two stage transfer line production system wherein both processing and rework are done on the same machine itself. Several authors have analyzed two stage production systems by modeling them as a queueing system. Neuts [20] has provided solutions for a wide range of queueing systems with exponential processing times. Prabhu [21] analysed a queue of tandem type. Kumar [22] has obtained distributions of average idle time and queue lengths. An extensive and detailed survey of Markov renewal processes was carried out by Cinlar [23].

The present paper deals with the transient-state analysis of a two-stage transfer-line production system with rework. Without loss of generality, it can be said that this paper deals with a family of two-stage transfer-line production systems as all the processing times of machines (including that of rework) are assumed to be arbitrarily distributed. (i.e. no particular distribution is assumed for any of the processing times of machine I and the rework machine). The analysis is carried out by modeling the system using semi-regenerative process. The process is semi-regenerative because not all the one-step transitions are regenerative.

To start with, we consider only one type of defect. We extend the work, later, to multi type rework. That is, defect of a product need not be of single type. Defects could be of many types. Each type of defect warrants a different type of reprocessing type and hence rework processing times could be different. Therefore, the processing time of the rework is dependent on the type of defect. In a similar way, a product can have two or more defects. Without loss of generality, combination of two or more defects can be considered to be another type of defect. i.e., if we have, say, n types of defects then the combinations of two or more defects (which could occur) can be considered to be of $(n+1)^{st}$ type and so on.

The two-stage transfer-line production system under study is modeled using regenerative point technique. For details of this approach, we refer to Uematsu et al [24], Birolini [25]. Integral equations have been written for various state probabilities by identifying the system at suitable regenerations epochs. These equations, which are of convolution type, have been solved by successive approximation [26].

The following system characteristics have been obtained under the assumption that the distributions of all the processing times involved in the analysis are arbitrary:

1. Expected duration Machine I is busy in [0,t]
2. Expected duration Machine I is blocked in [0,t]
3. Expected duration Rework Machine is busy in [0,t]
4. Expected duration Rework Machine is idle in [0,t]
5. Expected duration Rework Machine is busy with rework of type 1 in [0,t]
6. Expected duration Rework Machine is busy with rework of type 2 in [0,t]
7. Expected duration Rework Machine is busy with rework of type "m" in [0,t], where $1 \le m \le n$

The contents of this paper are organized as follows: Section 2 gives a list of assumptions made, Section 3 presents the list of notations used. While Section 4 deals with system modeling and evaluation of system characteristics, numerical illustrations are presented in Section 5, for some particular cases. Section 6 is devoted to conclusion.

2. Assumptions

- Initial storage is assumed to be of infinite capacity (i.e. Machine I is never starved)
- Transfer of units from initial buffer to Machine I is instantaneous
- Transfer of units from Machine I to Rework Machine is instantaneous
- Inspections is instantaneous
- Processing times at both the machines are independent, random and arbitrarily distributed
- Rework Machine is never blocked
- Reworked jobs are always perfect
- Machine I/Rework Machine are reliable
- Setup is instantaneous

3. Notations

RM	: Rework Machine
pdf	: Probability density function
cdf	: Cumulative distribution function
sf	: Survivor function
$f(\cdot)$	: Pdf of processing time of Machine I

$F(\cdot)$	: Cdf of processing time of Machine I
$\overline{F}(\cdot)$	: Sf of processing time of Machine I
$g(\cdot)$	: Pdf of processing time of RM
$G(\cdot)$	: Cdf of processing time of RM
$\overline{G}(\cdot)$	: Sf of processing time of RM
$g_1(\cdot)/g_2(\cdot)$	: Pdf of processing time of rework of type 1/type 2 in RM
$G_1(\cdot)/G_2(\cdot)$	: Cdf of processing time of rework of type1/type 2 in RM
$\overline{G}_1(\cdot)/\overline{G}_2(\cdot)$	: Sf of processing time of rework of type 1/type 2 in RM
p_g	: Probability that a job completed by Machine I is good
p_r	: Probability that a job completed by Machine I is not good but can be reworked (single defect case)
p_{ri}	: Probability that a job completed by Machine I is not good and defective of type "i" (multiple defect case)
p_s	: Probability that a job completed by Machine I is neither good nor can be reworked (i.e. a scrap)
$*$	: Convolution (defined as follows)

$$f(t) * g(t) = \int_0^t f(u)g(t-u)du = \int_0^t g(u)f(t-u)du = g(t) * f(t)$$

4. Measures of System Performance

In this Section, we obtain mathematical expressions for various measures of system performance. The schematic diagram of the production system is given in Figure 1. Various possible states of the system (state space) are presented in Table 1.

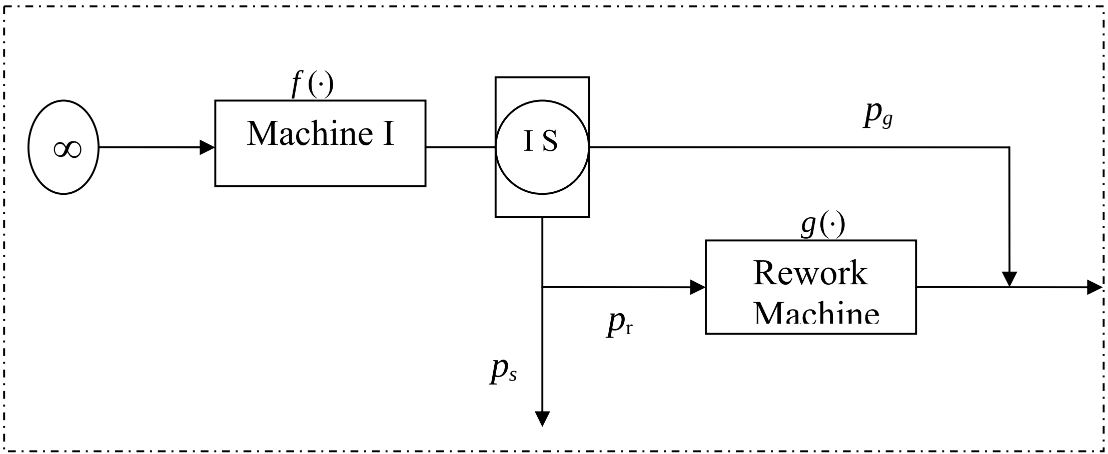

Figure 1. Schematic diagram of the production system (with single type of rework)

State	Machine I	Rework Machine
1	Busy	Free
2	Busy	Busy
3	Blocked	Busy

Table 1. State space (when there is only one type of defect)

4.1 Expected duration Machine I is busy in [0,t]

Let $Av_1^{\,I}(t)$ denote the probability that machine I busy at instant t given that the system was in state 1 at time t = 0. Starting with state 1, the various possible transitions are:

Machine I completes its processing time, and:

(i) the product from Machine I is good with probability p_g

(ii) the product from Machine I is not good but can be reworked with probability p_r

(iii) the product from Machine I is neither good nor can be reworked (i.e., a scrap) with probability p_s

The above three possible cases can be expressed in terms of integral equations of convolution type as follows:

(i) $Av_1^{\,I}(t) = f(t) * [(p_g) Av_1^{\,I}(t)]$

(ii) $Av_1^{\,I}(t) = f(t) * [(p_r) Av_2^{\,I}(t)]$

(iii) $Av_1^{\,I}(t) = f(t) * [(p_s) Av_1^{\,I}(t)]$

Combining all possible transitions in a single equation, we get

$$Av_1^{\,I}(t) = f(t) * [(p_g + p_s) Av_1^{\,I}(t) + p_r Av_2^{\,I}(t)] + \overline{F}(t) \ldots\ldots\ldots \quad \ldots\ldots..(1)$$

The term $\overline{F}(t)$ in the above equations refers to the state that Machine I is busy (irrespective of state(s) of other machine(s) in the system). This term is called *non-linear* term in the equation. The above equation explains that starting from state 1, the system makes transition to state 1 (again) with probability ($p_g + p_s$) and to state 2 with probability p_r.

Using similar logic, one can write equation for all possible transitions of the system starting from state 2.

$$Av_2^{\,I}(t) = [f(t)G(t) + g(t)F(t)] * [(p_g + p_s) Av_1^{\,I}(t) + p_r Av_2^{\,I}(t)] + \overline{F}(t) \ldots(2)$$

Now, we have integral equations for each and every state that appears in the equations thereby having a (closed or complete) system of integral equations. The system of equations obtained above is known as system of integral equations of convolution type.

The above set of integral equations can be written in the form of matrices as follows:

$$\mathbf{g}(t) - \int_0^t \mathbf{W}(t-u)\mathbf{g}(u)du = \mathbf{f}(t) \ldots\ldots\ldots\ldots\ldots\ldots.\ldots \quad \ldots\ldots(3)$$

where the matrix **W** is a square matrix of order *n* (*n* = the number of equations) consisting of the coefficients of $Av_i^{\,I}(t)$, **g** and **f** are column matrices of order *(n x 1)* consisting of $Av_i^{\,I}(t)$ and terms independent of $Av_i^{\,I}(t)$ respectively.

The above set of integral equations, being of convolution type, can be solved by the method suggested by Jones [26].

The expected duration, $\mu_1^{I}(t)$, when machine I is busy in [0,t] is given by

$$\mu_1^{I}(t) = \int_0^t Av_1^{I}(u)du \text{ ………………… .. …………(4)}$$

4.2 Expected duration Machine I is blocked in [0,t]

Let $Av_1^{BL}(t)$ denote the probability that machine I is blocked at instant t given that the system was in state 1 at time t = 0. Starting with State 1, the various possible transitions are:

$$Av_1^{BL}(t) = f(t) * [(p_g + p_s)Av_1^{BL}(t) + p_r Av_2^{BL}(t)] \text{ ……….…. ..(5)}$$

$$Av_2^{BL}(t) = [f(t)G(t) + g(t)F(t)] * [(p_g + p_s)Av_1^{BL}(t) + p_r Av_2^{BL}(t)] + F(t)\overline{G}(t) \text{ ….(6)}$$

The above set of integral equations can be written in the form of matrices as depicted in equation (3) where the matrix **W** is a square matrix of order *n* (*n* = the number of equations) consisting of the coefficients of $Av_i^{BL}(t)$, **g** and **f** are column matrices of order (*n x 1*) consisting of $Av_i^{BL}(t)$ and terms independent of $Av_i^{BL}(t)$ respectively.

The above set of integral equations, being of convolution type, can be solved by the method suggested by Jones [26].

The expected duration, $\mu_1^{BL}(t)$, when machine I is blocked in [0,t] is given by

$$\mu_1^{BL}(t) = \int_0^t Av_1^{BL}(u)du \text{ ………….……….. …… .(7)}$$

4.3 Expected duration Rework Machine is busy in [0,t]

Let $Av_1^{R(B)}(t)$ denote the probability that rework machine is busy at instant t given that the system was in state 1 at time t = 0. Starting with State 1, the various possible transitions are:

$$Av_1^{R(B)}(t) = f(t) * [(p_g + p_s)Av_1^{R(B)}(t) + p_r Av_2^{R(B)}(t)] \text{ ………(8)}$$

$$Av_2^{R(B)}(t) = [f(t)G(t) + g(t)F(t)] * [(p_g + p_s)Av_1^{R(B)}(t) + p_r Av_2^{R(B)}(t)] + \overline{G}(t) \text{….(9)}$$

The above set of integral equations can be written in the form of matrices as depicted in equation (3) where matrix **W** is a square matrix of order *n* (*n* = the number of equations) consisting of the coefficients of $Av_i^{R(B)}(t)$, **g** and **f** are column matrices of order *(n x 1)* consisting of $Av_i^{R(B)}(t)$ and terms independent of $Av_i^{BL}(t)$ respectively.

The expected duration, $\mu_1^{R}(t)$, when machine I is blocked in [0,t] is given by

$$\mu_1^{R}(t) = \int_0^t Av_i^{R}(u)du \text{ …………..……..…… …..(10)}$$

4.4 Expected duration Rework Machine is idle in [0,t]

Let $Av_1^{R(I)}(t)$ denote the probability that rework machine is idle at instant t given that the system was in state 1 at time t = 0. Starting with State 1, the various possible transitions are:

$$Av_1^{R(I)}(t) = f(t) * [(p_g + p_s)Av_1^{R(I)}(t) + p_r Av_2^{R(I)}(t)] + \overline{F}(t) \text{ ……….. ……(11)}$$

$$Av_2^{R(I)}(t) = [f(t)G(t) + g(t)F(t)] * [(p_g + p_s)Av_1^{R(I)}(t) + p_r Av_2^{R(I)}(t)] + \overline{F}(t)G(t) \text{ (12)}$$

The above set of integral equations, being of convolution type, can be solved by the method suggested by Jones [26].

The expected duration, $\mu_1^{R(I)}(t)$, when rework machine is idle in [0,t] is given by

$$\mu_1^{R(I)}(t) = \int_0^t Av_1^{R(I)}(u)du \text{ …………..………..…………(13)}$$

4.5 Expected duration Rework Machine is busy with rework of type 1 in [0,t]

Now, we shall extend the number of types of defects from one to two. i.e., after processing in machine I, the products are inspected at the inspection station and categorized as follows:

- The product is good with probability p_g
- The product is not good but can be reworked with probability p_r and is further categorized as follows:
  - The product is defective of type 1 with probability p_{r1}
  - The product is defective of type 2 with probability p_{r2}

  Clearly, $p_{r1} + p_{r2} = p_r$
- The product is neither good nor can be reworked with probability p_s

The schematic diagram for the situation where the number of types of defects is two is given in Figure 2. The state space is enumerated in Table 2.

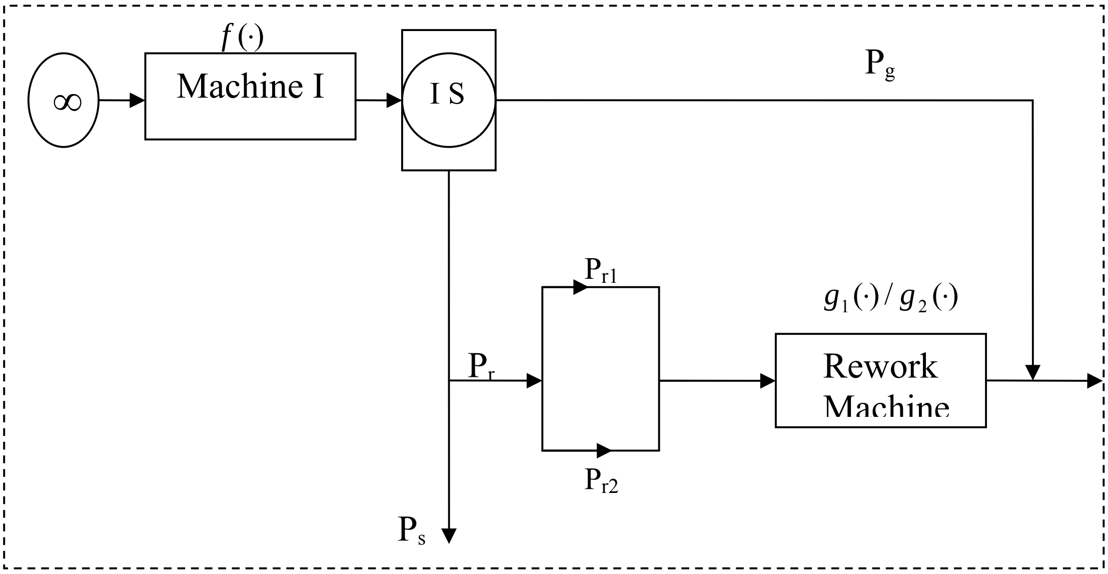

Figure 2. Schematic diagram of the production system with two types of rework

State	Machine I	Rework Machine
1	Busy	Free
2	Busy	Busy with rework of type 1
3	Busy	Busy with rework of type 2
4	Blocked	Busy with rework of type 1
5	Blocked	Busy with rework of type 2

Table 2. State space when there are two types of defects

In addition to various measures of system performance like expected duration machine I is busy and blocked, we have, in this case, measures of system performance such as expected duration rework machine is busy with rework of type 1 and expected duration rework machine is busy with rework of type 2. The expressions for the same are obtained as follows:

Let $Av_1^{R1}(t)$ denote the probability that rework machine is busy with rework of type 1 at instant t given that the system was in state 1 at time t = 0. Starting with State 1, the various possible transitions are:

$$Av_1^{R1}(t) = f(t) * [(p_g + p_s)Av_1^{R1}(t) + p_{r1}Av_2^{R1}(t) + p_{r2}Av_3^{R1}(t)] \text{……..} \quad (14)$$

$$Av_2^{R1}(t) = [f(t)G_1(t) + g_1(t)F(t)] * [(p_g + p_s)Av_1^{R1}(t) + p_{r1}Av_2^{R1}(t) + p_{r2}Av_3^{R1}(t)] + \overline{G}_1(t)$$
…………… (15)

$$Av_3^{R1}(t) = [f(t)G_2(t) + g_2(t)F(t)] * [(p_g + p_s)Av_1^{R1}(t) + p_{r1}Av_2^{R1}(t) + p_{r2}Av_3^{R1}(t)] \text{..} (16)$$

The expected duration, $\mu_1^{R1}(t)$, when rework machine I is busy with rework of type 1 in [0,t] is given by

$$\mu_1^{B1}(t) = \int_0^t Av_1^{R1}(u)du \text{ …………..………………} \quad .(17)$$

4.6 Expected duration Rework Machine is busy with rework of type 2 in [0,t]

Let $Av_1^{R2}(t)$ denote the probability that rework machine is busy with rework of type 2 at instant t given that the system was in state 1 at time t = 0. Starting with State 1, the various possible transitions are:

$$Av_1^{R2}(t) = f(t) * [(p_g + p_s)Av_1^{R2}(t) + p_{r1}Av_2^{R2}(t) + p_{r2}Av_3^{R2}(t)] \text{……} \quad (18)$$

$$Av_2^{R2}(t) = [f(t)G_1(t) + g_1(t)F(t)] * [(p_g + p_s)Av_1^{R2}(t) + p_{r1}Av_2^{R2}(t) + p_{r2}Av_3^{R2}(t)] \text{..} (19)$$

$$Av_3^{R2}(t) = [f(t)G_2(t) + g_2(t)F(t)] * [(p_g + p_s)Av_1^{R2}(t) + p_{r1}Av_2^{R2}(t) + p_{r2}Av_3^{R2}(t)] + \overline{G_2}(t)$$
… ……...……(20)

The expected duration, $\mu_1^{R2}(t)$, when rework machine I is busy with rework of type 2 in [0,t] is given by

$$\mu_1^{R2}(t) = \int_0^t Av_1^{R2}(u)du \text{ …………..……} \quad \text{…..…}(21)$$

4.7 Expected duration Rework Machine is busy with rework of type *k* in [0,t].

In this Sub-section, we shall generalize the number of types of defects to, say, "*m*". i.e. After processing in machine I, the products are inspected at the inspection station and are categorized as follows:

- A product is good with probability p_g
- A product is not good but can be reworked with probability p_r and is further categorized as follows:
    - The product is defective of type 1 with probability p_{r1}
    - The product is defective of type 2 with probability p_{r2}
    - … … …. ….
    - The product is defective of type n with probability p_{rn}
    - Clearly, $p_{r1}+ p_{r2} + p_{r3} \dots + p_{rm} = p_r$
- A product is neither good nor can be reworked with probability p_s

The schematic diagram for the situation where the number of types of defects is "*m*" is given in Figure 3. The state space is enumerated in Table 3.

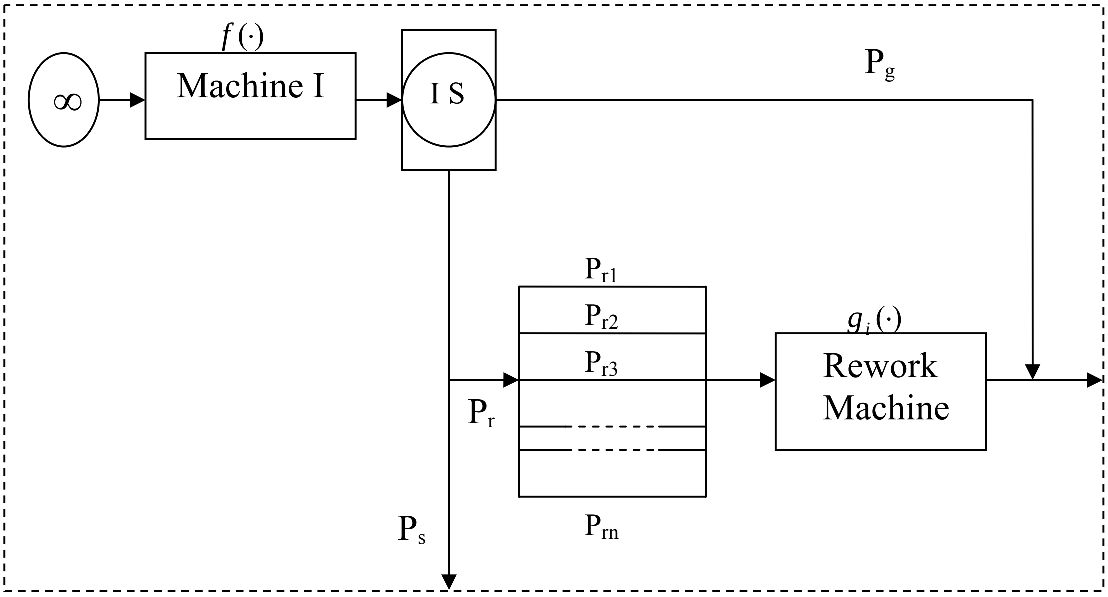

Figure 3. Schematic diagram of the production system with multiple type rework

State	Machine I	Rework Machine
1	Busy	Free
2	Busy	Busy with rework of type 1
3	Busy	Busy with rework of type 2
…	….	………….
…	….	………..
m+1	Busy	Busy with rework of type *m*
m+2	Blocked	Busy with rework of type 1
m+3	Blocked	Busy with rework of type 2
…	………	………..
2*m*+1	Blocked	Busy with rework of type *m*

Table 3. State space when there are "*m*" types of defects

The states with similar characteristics in the state space can be clustered together to minimize the number of states. Here, in this case, states 2 to *m+1* can be combined and be represented with a notation *2(i)* meaning that Machine I is busy and Rework Machine is busy with rework of type *"i"*. In a similar fashion, the states *m+2* to *2m+1* can be combined and represented as *3(j)* meaning Machine I is blocked while Rework Machine is busy with rework of type *"j"*. Clearly, the state space is reduced to a mere three states and equations be developed for various measures of system performances. The clustered state space is presented in Table 4. Only when the system of equations is to be solved, for a particular number of rework types, the equations can be exploded up and solved.

State	Machine I	Rework Machine
1	Busy	Free
2(*k*)	Busy	Busy with rework of type *"k"*
3(*k*)	Blocked	Busy with rework of type *"k"*

Table 4. Clustered state space when there are "*m*" types of defects $(0 \le k \le m)$

Let $Av_1^{R(k)}(t)$ denote the probability that Rework Machine is busy with rework of type '*k*' at instant t given that the system was in state 1 at time t = 0. Then,

$$Av_1^{R(k)}(t) = f(t) * [(p_g + p_s) Av_1^{R(k)}(t) + \sum_{i=1}^{n} p_{ri} Av_{2(i)}^{R(k)}(t)] \tag{22}$$

$$Av_{2(k)}^{R(k)}(t) = [f(t)G_k(t) + g_k(t)F(t)] * [(p_g + p_s) Av_1^{R(k)}(t) + \sum_{i=1}^{n} p_{ri} Av_{2(k)}^{R(k)}(t)] + \overline{G}_k(t) \ldots (23)$$

The expected duration, $\mu_1^{R(k)}(t)$, when rework machine I is busy with rework of type '*k*' in [0,t] is given by

$$\mu_1^{R(k)}(t) = \int_0^t Av_1^{R(k)}(u)du \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \ldots.(24)$$

5. Numerical Illustration

Computer programs have been devised using Pascal language to obtain the numerical values for particular cases. The numerical values for the expected duration machine I is busy, blocked in [0,t] and the expected duration Rework machine busy and idle in [0,t] for different values of parameters are given in Tables 5 and 6. The statistical distributions assumed for the purpose are

$$f(t) = \lambda_1^2 t \exp(-\lambda_1 t);\ g(t) = \lambda_2^2 t \exp(-\lambda_2 t);\qquad g_1(t) = \lambda_3^2 t \exp(-\lambda_3 t);\ g_2(t) = \lambda_4^2 t \exp(-\lambda_4 t)$$

Tables 5 and 6 show how sensitive are the numerical values with respect to changes in the machine 1 and rework machine processing rates. Extending the concept of rework to multi-type rework, two types of rework are assumed. For the given set of parameters (used in Table 1), the expected duration rework machine busy with rework of type 1 and with rework of type 2 are presented in Tables 7 and 8.

6. Conclusion

In this paper, the concept of rework with a separate rework machine is incorporated in the probabilistic modeling of transfer-line production systems with an initial buffer of unlimited capacity. A stochastic model subject to inter stage inspection and rework is developed by modeling the system using semi-regenerative stochastic processes. Analytical expressions for various measures of system performance such as expected duration machine I is busy, blocked, rework machine is busy, idle in a given interval of time have been obtained. Such a transient state analysis is more desirable than steady state approximations when it is desired to analyze the system over finite time durations.

$\lambda_2 = 4$; $p_g = 0.75$; $p_r = 0.20$; $p_s = 0.05$;					
λ_1	Time	Expected duration			
		Machine I is		Rework Machine is	
		Busy	Blocked	Busy	Idle
2.0	1.0	0.996	0.004	0.043	0.957
	2.0	1.981	0.019	0.135	1.865
	3.0	2.963	0.037	0.203	2.766
	4.0	3.944	0.055	0.331	3.668
	5.0	4.924	0.073	0.429	4.568
4.0	1.0	0.977	0.023	0.101	0.899
	2.0	1.911	0.089	0.282	1.717
	3.0	2.839	0.158	0.468	2.529
	4.0	3.764	0.227	0.653	3.339
	5.0	4.685	0.297	0.837	4.144

Table 5. Sensitivity of performance measures with respect to change in the processing rate of Machine I

$\lambda_2 = 5$, $p_g = 0.75$; $p_r = 0.20$; $p_s = 0.05$;					
λ_1	Time	Expected duration			
		Machine I is		Rework Machine is	
		Busy	Blocked	Busy	Idle
2.0	1.0	0.997	0.003	0.038	0.962
	2.0	1.987	0.013	0.115	1.885
	3.0	2.976	0.024	0.194	2.806
	4.0	3.964	0.035	0.273	3.726
	5.0	4.951	0.046	0.352	4.645
4.0	1.0	0.982	0.018	0.091	0.909
	2.0	1.936	0.064	0.241	1.758
	3.0	2.887	0.110	0.393	2.604
	4.0	3.835	0.157	0.545	3.447
	5.0	4.778	0.203	0.696	4.285

Table 6. Sensitivity of performance measures with respect to change in the processing rate of Machine I

$\lambda_2 = 4$; $\lambda_3 =$; $p_g = 0.75$; $p_{r1} = 0.15$; $p_{r2} = 0.05$; $p_s = 0.05$;				
λ_1	Time	Expected duration Rework Machine is		
		Busy	Busy with rework of type 1	Busy with rework of type 2
2.0	1.0	0.043	0.032	0.011
	2.0	0.135	0.101	0.034
	3.0	0.203	0.175	0.058
	4.0	0.331	0.248	0.083
	5.0	0.429	0.322	0.107
4.0	1.0	0.101	0.076	0.025
	2.0	0.282	0.212	0.071
	3.0	0.468	0.351	0.117
	4.0	0.653	0.490	0.163
	5.0	0.837	0.628	0.209

Table 7. Sensitivity of performance measures with respect to change in the processing rate of Machine I

$\lambda_2 = 5$; $\lambda_3 =$; $p_g = 0.75$; $p_{r1} = 0.15$; $p_{r2} = 0.05$; $p_s = 0.05$;				
λ_1	Time	Expected duration Rework Machine is		
		Busy	Busy with rework of type 1	Busy with rework of type 2
2.0	1.0	0.038	0.029	0.010
	2.0	0.115	0.086	0.029
	3.0	0.194	0.145	0.048
	4.0	0.273	0.205	0.068
	5.0	0.352	0.264	0.088
4.0	1.0	0.091	0.068	0.023
	2.0	0.241	0.181	0.060
	3.0	0.393	0.295	0.098
	4.0	0.545	0.409	0.136
	5.0	0.696	0.522	0.174

Table 8. Sensitivity of performance measures with respect to change in the processing rate of Machine I

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Deteriorating Inventory Model for Two – Level Credit – Linked Demand under Permissible Delay in Payments

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Abstract:
In Practice, the supplier offers a fixed credit period to the retailer but the retailer does not offer any credit period to the customers, which is unrealistic because of global competition. In real practice, retailer may offer a credit period to its customer in order to boost his own demand. In this paper, impact of credit period on demand is studied when units in inventory deteriorate at a constant rate. An easy – to – use algorithm is developed to determine the optimal credit period and replenishment policy for the retailer. Finally, numerical example is presented to illustrate the theoretical results followed by the sensitivity of various parameters on the optimal solution.

***Keywords*:** Inventory, deterioration, credit – linked demand, Two – level credit policy.

1. Introduction:
In Wilson's economical inventory model, it was assumed that the retailer must settle the account against the procured goods immediately. But in practice, the supplier offers a certain credit period to settle the account for stimulating retailer's demand. During this permissible credit period, the retailer can generate revenues on the sales and earn interest on the generated revenue, but beyond this period the supplier charges interest on the unsold stock. Thus, paying later indirectly reduces the inventory holding cost. Hence, trade credit is concerned to be effective promotional tool.

The concept of trade credit was first introduced by Haley and Higgins (1973). They developed model to determine economic order quantity under condition of permissible delay in payments with known constant deterministic demand, no shortages and zero – lead time. Goyal (1985) excluded penalty cost due to a late payment in Haley and Higgins model. Chung (1989) discussed inventory model under permissible delay in payments using the discounted cash flows (DCF) approach. Shah (1993) and Aggarwal and Jaggi (1995) extended the Goyal's model to incorporate deterioration of units in the inventory system. Jamal et. al. (2000) further generalized the model to allow shortages. Jaggi and Aggarwal (1994) extended Chung (1989) to formulate inventory model for determining the optimal procurement quantity of deteriorating items when permissible delay period is offered using the DCF approach. Hwang and Shinn (1997) jointly optimized retailer's sale price and lot size when the supplier offers delay in payments. Dye (2002) developed inventory model for stock – dependent demand for deteriorating items when partial backlogging is allowed and trade credit is offered. Teng (2002) argued that it is economically advantageous for a retailer to order less quantity and take benefits of trade credit more frequently. Chang et al. (2004) developed an inventory model for deteriorating items with instantaneous stock – dependent demand and time value of money when permissible delay in payments is offered. Teng et al. (2005) developed the optimal pricing and lot sizing under permissible delay in payments by considering the difference between selling price and purchase quantity and demand to be price sensitive. Goyal et al. (2007) formulated an EOQ model for a retailer when supplier offers a progressive interest scheme, and provided an easy to use closed form solution to make the decision. Shah and Soni (2008) computed optimal ordering policy for stock – dependent demand under scenario of progressive payments.

All the aforementioned citations assumed that the customer must pay for the items as soon as the items are purchased from the retailer. Now – a – days in most of the business transactions, the supplier offers a credit period to the retailer and retailer; in turn posses on some credit period to his/her customers. Huang (2003) analyzed an inventory model when retailer offers a credit period to his customer which is smaller than the credit period offered by the supplier, in order to increase the demand. In all above articles, the effect of credit period is studied on the objective function. The impact of credit period on demand is ignored. In practice, it is observed that demand of an item does depend upon the length of the credit period offered by the supplier to the retailer or retailer to the customer. Jaggi et al. (2008) gave idea of credit – linked demand function to determine the retailer's optimal credit and replenishment policy when both the suppliers as well as the retailer offers the credit period to stimulate end – user demand. In this article, we explore effect of deterioration on optimal policy when demand is dependent upon the allowable credit.

2. Assumptions and Notations:

The proposed mathematical model is based on the following assumptions.

1. The inventory system deals with a single item.
2. The supplier offers a credit period *M* to settle the accounts to the retailer and the retailer, in turn, offers a credit period *N* to his customers to settle the accounts.
3. The demand rate is a function of the customer's credit period, *N*; offered by the retailer. The demand function for any *N* can be represented as a differential difference equation:

   $$R(N+1) - R(N) = r\left[R_m - R(N)\right]$$

   where $R(N)$: demand for any N per unit time

   R_m : Maximum demand

   r : Rate of saturation of demand.

   The solution of the above differential difference equation, using initial condition that at $N = 0, R(0) = R_0$ (initial demand) is given by

   $$D(N) = R_0(1-r)^N + R_m\left(1-(1-r)^N\right)$$

   i.e. $D(N) = R_m - (R_m - R_0)(1-r)^N$
4. Replenishment rate is instantaneous.
5. Shortages are not allowed.
6. Lead – time is zero or negligible.
7. The units in inventory system deteriorate at a constant rate, θ where $(0 \le \theta \le 1)$. The deteriorated units can neither be repaired nor replaced during a cycle time.

In addition, following notations are used:

Q	Order quantity (a decision variable)
T	cycle time (a decision variable)
$I(t)$	the inventory level at any instant of time t, $0 \le t \le T$
A	ordering cost per order
C	unit purchase cost of the item
P	unit selling price of an item.
I	inventory carrying charge fraction (excluding interest charges) per $ per unit time.
I_e	the interest earned per $ per unit time.
I_c	the interest charged per $ per unit time
M	retailer's credit period offered by the supplier for settling the account
N	customer's credit period offered by the retailer for settling the account
$\Pi(T, N)$	retailer's profit per unit time which compromises (a) revenue from sales, minus (b) Cost of purchasing unit; (c) Cost of placing orders; (d) inventory holding cost (e) interest charged for the unsold items after the permissible trade credit ; plus (f) interest earned from the sales during the allowable trade credit.

3. Mathematical Model:

The inventory level depletes due to demand and deterioration. The inventory level at any time t during the cycle is governed by the differential equation,

$$\frac{dI(t)}{dt} + \theta I(t) = -R(N) \quad ; \quad 0 \le t \le T \tag{1}$$

with initial condition; $I(0) = Q$ and boundary condition $I(T) = 0$. Then solution of differential equation is,

$$I(t) = \frac{R(N)}{\theta}\left[e^{\theta(T-t)} - 1\right] \quad ; \quad 0 \le t \le T \tag{2}$$

and the order quantity is,

$$Q = I(0) = \frac{R(N)}{\theta}\left[e^{\theta T} - 1\right] \tag{3}$$

The retailer's profit per time unit time compromises of the following components:

(a) Sales revenue; $$SR = \frac{PQ}{T} \tag{4}$$

(b) Cost of purchasing; $$PC = \frac{CQ}{T} \tag{5}$$

(c) Cost of placing orders; $$OC = \frac{A}{T} \tag{6}$$

(d) Inventory holding cost; $$IHC = \frac{CIR(N)}{\theta^2 T}\left[e^{\theta T} - \theta T - 1\right] \tag{7}$$

The calculation for interest earned and charged will depend upon the lengths of T, N and M. The following three cases arise:

Case 1: $N \le M \le T+N$ (Figure 1)

Here, the retailer generates revenue in [*N*, *M*] and earns interest on sales revenue for the time period (*M*–*N*). At *M*, accounts are to be settled and during [*M*, *T*+*N*] interest charges are payable to the supplier by the retailer on the unsold stock. Hence,

(e) Interest charged per time unit time is

$$IC_1 = \frac{CI_c}{T}\int_0^{T+N-M} I(t)dt = \frac{CI_c R(N)}{\theta^2 T}\left[e^{\theta T} - e^{\theta(M-N)} - \theta(T+N-M)\right] \quad (8)$$

and

(f) Interest earned per unit time is

$$IE_1 = \frac{PI_e}{T}\int_0^{M-N} R(N)tdt = \frac{PI_e R(N)(M-N)^2}{2T} \quad (9)$$

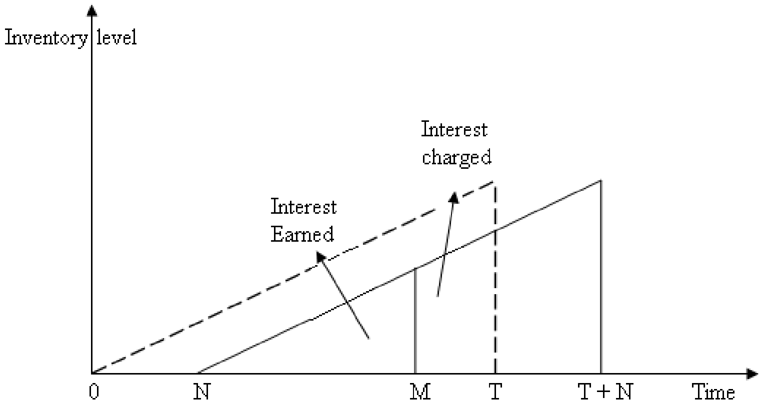

Figure 1 When $\boldsymbol{N \le M \le T+N}$

Using equations (4) – (9), the retailer's profit per unit time is,

$$\Pi_1(T,N) = SR - PC - OC - IHC - IC_1 + IE_1 \quad (10)$$

Case 2: $N \le T+N \le M$ (Figure 2)

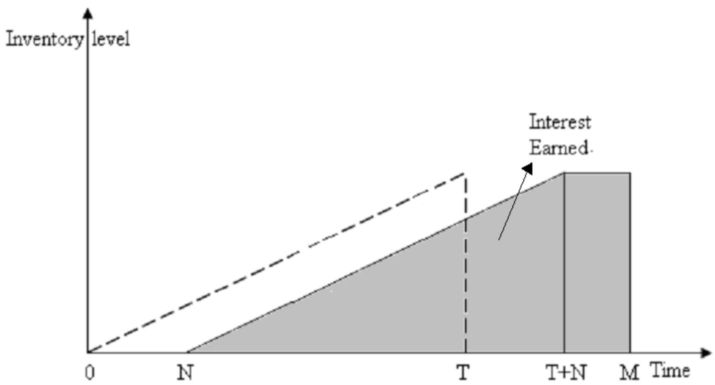

Figure 2 When $\boldsymbol{N \le M \le T+N}$

Here, the retailer earns interest on the revenue received during the period (N, $T+N$) and on total sales revenue; PQ for a period of (M–T–N),

(e) Total interest earned per time unit is,

$$IE_2 = \frac{PI_e}{T}\int_N^{T+N} R(N)tdt + \frac{PI_e}{T}(M-T-N)R(N)T$$

$$= PI_e R(N)\left(M - N - \frac{T}{2}\right) \qquad (11)$$

(f) Interest charges are zero. i.e. $IC_2 = 0$ (12)

As a result, using equation (4) – (7) and (11) – (12), the retailer's profit per time unit is,

$$\Pi_2(T,N) = SR - PC - OC - IHC - IC_2 + IE_2 \qquad (13)$$

Case 3: $M \le N \le T+N$ (Figure 3)

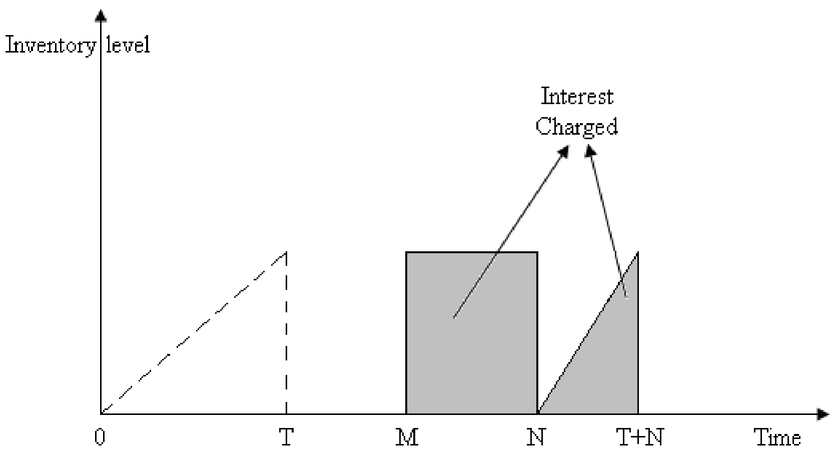

Figure 3 When $\boldsymbol{N \le M \le T+N}$

In this case,

(e) Interest charged per unit time is,

$$IC_3 = \frac{CI_c}{T}\left[(N-M)Q + \int_N^{T+N} I(t)dt\right]$$

$$= \frac{CI_c R(N)}{\theta^2 T}\left[(N-M)\theta(e^{\theta T}-1) + e^{\theta T} - \theta T - 1\right] \qquad (14)$$

Hence, using equation (4) – (7) and (14), the retailer's profit per time unit is,

$$\Pi_3(T,N) = SR - PC - OC - IHC - IC_3 \qquad (15)$$

Therefore, the retailer's profit per unit time $\Pi(T,N)$ is

$$\Pi(T,N)=\begin{cases}\Pi_1(T,N) & if \quad N\le M\le T+N\\ \Pi_2(T,N) & if \quad N\le T+N\le M\\ \Pi_3(T,N) & if \quad M\le N\le T+N\end{cases} \tag{16}$$

which is a function of two variables *T* and *N*, where, *T* is continuous and *N* is discrete.

To obtain closed form of the solution, we write series expansion containing term upto θ under the assumption that $0\le\theta\le 1$. Hence,

$$\Pi_1(T,N)=(P-C)R(N)+\frac{(P-C)R(N)\theta T}{2}-\frac{C(I+I_c)R(N)T}{2}-\frac{A}{T}-\frac{(CI_c-PI_e)R(N)(M-N)^2}{2T}$$

$$\Pi_2(T,N)=(P-C)R(N)+\frac{(P-C)R(N)\theta T}{2}-\frac{CIR(N)T}{2}-\frac{A}{T}+PI_eR(N)\left(M-N-\frac{T}{2}\right)$$

$$\Pi_3(T,N)=(P-C)R(N)+\frac{(P-C)R(N)\theta T}{2}-\frac{CIR(N)T}{2}-\frac{A}{T}-CI_cR(N)\left(\frac{T}{2}+N-M\right)$$

4. Solution Methodology:

Our aim is to determine the optimal values of *T* and *N* which maximizes $\Pi(T,N)$. For fixed *N*, take the first and second order derivatives of $\Pi_i(T,N)$, i =1, 2, 3 gives,

$$\frac{\partial\Pi_1}{\partial T}=\frac{(P-C)R(N)\theta}{2}-\frac{C(I+I_c)R(N)}{2}+\frac{A}{T^2}+\frac{(CI_c-PI_e)R(N)(M-N)^2}{2T^2} \tag{17}$$

$$\frac{\partial^2\Pi_1}{\partial T^2}=-\frac{2A}{T^3}-\frac{(CI_c-PI_e)R(N)(M-N)^2}{T^3} \tag{18}$$

$$\frac{\partial\Pi_2}{\partial T}=\frac{(P-C)R(N)\theta}{2}-\frac{CIR(N)}{2}+\frac{A}{T^2}-\frac{PI_eR(N)}{2} \tag{19}$$

$$\frac{\partial^2\Pi_2}{\partial T^2}=-\frac{2A}{T^3} \tag{20}$$

$$\frac{\partial\Pi_3}{\partial T}=\frac{(P-C)R(N)\theta}{2}-\frac{C(I+I_c)R(N)}{2}+\frac{A}{T^2} \tag{21}$$

and

$$\frac{\partial^2\Pi_3}{\partial T^2}=-\frac{2A}{T^3} \tag{22}$$

For fixed *N*, Equations (20) and (22) indicate that $\Pi_2(T,N)$ and $\Pi_3(T,N)$ are concave for all *T > 0*. However, $\Pi_1(T,N)$ is concave for all *T > 0* if $CI_c > PI_e$. Thus, there exists a unique value of $T = T_1$ which maximizes $\Pi_1(T)$. It is given by equating equation (17) to be zero. We get,

$$T_1 = \sqrt{\frac{2A + (CI_c - PI_e)R(N)(M-N)^2}{C(\theta + I + I_c)R(N) - P\theta R(N)}} \tag{23}$$

T_1 would satisfy the condition $0 \le M - N \le T$ provided,

$$2A - \left[C(\theta + I) + PI_e\right]R(N)(M-N)^2 \ge 0 \tag{24}$$

Similarly, there exists a unique value of $T = T_2$ which maximizes $\Pi_2(T)$. It is given by equating equation (19) to be zero. We get,

$$T_2 = \sqrt{\frac{2A}{(C(\theta + I) + PI_e)R(N)}} \tag{25}$$

T_2 would satisfy the condition $0 \le T \le (M - N)$ provided,

$$2A - \left[C(\theta + I) + PI_e\right]R(N)(M-N)^2 \tag{26}$$

and

$$T_3 = \sqrt{\frac{2A}{C(\theta + I + I_c)R(N)}} \tag{27}$$

maximizes profit $\Pi_3(T)$. T_3 would satisfy the condition $(M - N) \le 0 \le T$ provided

$$2A - \left[C(\theta + I + I_c)\right]R(N)(M-N)^2 \ge 0 \tag{28}$$

Combining the three possible cases, we have following theorem:

Theorem 1: For a fixed value of *N*

(i) If $2A - \left[C(\theta + I) + PI_e\right]R(N)(M-N)^2 \ge 0$ then $T^* = T_1$.

(ii) If $2A - \left[C(\theta + I) + PI_e\right]R(N)(M-N)^2 \le 0$ then $T^* = T_2$.

(iii) If $2A - C\left[\theta + I + I_c\right]R(N)(M-N)^2 \ge 0$ and $(M - N) < 0$ then $T^* = T_3$.

Proof: It immediately follows from (24), (26) and (28).

5. Computational Algorithm:

In order to optimize *T* and *N* simultaneously, we have following steps:

Step 1: Start with $N = 1$

Step 2: Determine the optimal value of *T* using Theorem 1

Step 3: If $0 \le M - N \le T$ then calculate $\Pi_1(T, N)$ otherwise go to Step 5.

Step 4: If $\Pi_1(T, N) > \Pi_1(T, N-1)$, increment *N* by *N+1* and go to Step 2 otherwise current value of *N* is optimal. Compute Q and $\Pi(T, N)$.

Step 5: If $0 \le T \le (M-N)$ then calculate $\Pi_2(T,N)$ otherwise go to Step 7.

Step 6: If $\Pi_2(T,N) > \Pi_2(T,N-1)$, increment N by $N+1$ and go to Step 2 otherwise current value of N is optimal. Compute T and $\Pi(T,N)$.

Step 7: If $(M-N) \le 0 \le T$ then calculate $\Pi_3(T,N)$.

Step 6: If $\Pi_3(T,N) > \Pi_3(T,N-1)$, increment N by $N+1$ and go to Step 2 otherwise current value of N is optimal. Compute T and $\Pi(T,N)$.

6. Numerical Example:

Let maximum demand (R_m) = 100 units/day, minimum demand (R_o) = 30 units/day, rate of saturation of demand (r)=12 %, A = \$1000/order, M = 45 days, C = \$ 30/unit, P = \$ 40/unit, I = 15 % per year, I_c = 15 % per year, I_e = 10 % per year (Jaggi et al. (2008)) and θ = 5 %.

Using algorithm, optimal cycle time is 29.35 days, optimal credit period (N) offered by the retailer to the customer is 43 days and profit per day is \$ 929.

The sensitivity analysis on r, M, A, I_c and θ is exhibited in Table 1–5, respectively.

Table 1
Effect of changes in r on the optimal solution.

r	T^*(days)	N^*(days)	$\Pi(T^*, N^*)$ in \$
0.09	29.25	45	922
0.12	29.35	43	929
0.15	30.89	17	936

Table 2
Effect of changes in M on the optimal solution.

M	T^*(days)	N^*(days)	$\Pi(T^*, N^*)$ in \$
30	29.43	33	918
45	29.35	43	929
60	29.33	65	932

Table 3
Effect of changes in A on the optimal solution.

A	T^*(days)	N^*(days)	$\Pi(T^*, N^*)$ in \$
500	20.75	45	949
1000	29.35	43	929
1200	32.16	42	922

Table 4
Effect of changes in I_c on the optimal solution.

I_c	T^*(days)	N^*(days)	$\Pi(T^*, N^*)$ in \$
0.12	31.03	45	933
0.15	29.35	43	929
0.18	27.89	43	925

Table 5 Effect of changes in θ on the optimal solution.			
θ	T^*(days)	N^*(days)	$\Pi(T^*, N^*)$ in \$
0.03	28.99	45	933
0.05	29.35	43	929
0.08	29.87	40	921

From Table 1, it is observed that the rate of saturation of demand is very sensitive parameter. It decreases credit period offered by the retailer to the customer. The retailer's profit increases.

The similar changes are observed in Table 2 when credit period offered to the retailer increases. The negative impact in optimal profit and credit period offered to the customer when ordering cost (Table 3) and interest charged (Table 4) increases. In Table 5, deterioration rate is varied. Increase in deterioration rate reduces retailer's profit.

7. Conclusion:

In this article, the effect of credit – linked demand on the retailer's optimal profit is studied. The retailer's profit is maximized with respect to the cycle time and the credit period offered by the retailer. It is observed that the credit period offered to customer has significant positive impact on the unrealized demand.

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An Analysis of a non-Markovian Queuing Network Model with Correlated Allocation using Simulation Technique

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Abstract

A non-Markovian queuing network model with correlated allocation has been studied. The analysis of the model has been performed by using simulation technique. Various measures of performance of the queues such as average queue length, average delay and average server utilization have been computed. The sensitivity analysis of the model has been carried out. The effect of the correlated allocation probabilities on different performance measures has also been studied. The present study may be useful in modeling the traffic flows in internet, in city road traffic and the patients' inflow in hospitals etc.

Key Words: Queuing Network, Correlation, Simulation, Sensitivity Analysis.

1. Introduction

Various queuing network models have been extensively used to analyze the complex communication networks. A variety of mathematical models [1] to explore some of the consequences of rapidly growing communication capacity for the evolution of Internet have been studied by F. P. Kelly. Routing plays a significant role in Internet communication. There are many routing algorithms available which find a path through one or more intermediate networks. The routing to multiple destinations in computer networks is studied by Kadaba and Jaffe in [2]. Gibbens and Kelly studied the dynamic routing in fully interconnected networks [3].

The study of Internet traffic is a challenging task. In Internet communication when a person wants to send a message to another person, that message is broken into small pieces called packets. Packets are one of the basic units of measurement in the Internet. These packets are all addressed to their final destination. Along the possible paths there are special purpose computers called Routers. These computers look at the network addresses and figure out the current best route available to their destination. Once these packets reach at their destination they are reassembled into their original massage.

In this paper, a non-Markovian queuing network model with correlated allocation has been developed. This model can be useful in studying the correlated allocation of messages over the Internet. A small internet-work has been considered as shown in figure-1, in which the flow of messages occurs from network A (LAN or WAN) in a queue to router O, where these packets are allocated either to network B or to network C depending upon the Internet Protocol (IP) addresses they possess.

The routers work at the network layer of OSI model. The network layer provides two types of services to the users: one is connection-oriented and the other is connectionless. In connection-oriented services, there are virtual circuits. In these circuits a route chosen is used for all traffic flowing over the connection, exactly the same way that the telephone system works. When the connection is released the virtual circuit is discarded. In connectionless service the independent packets are called datagrams in analogy with telegrams. In this service no routes are worked out in advance. Each packet is routed independently of its predecessors. Successive packets may follow different routes (see e.g. [4], pp-280). Owing to the independent routing of packets in connectionless service the allocation of packets at two consecutive transmission marks at router O is taken to be correlated in the sense that if a packet is allocated to the network B at a transmission mark then at the next transmission mark there is a probability of allocating a packet either to network B or to network C . Similarly if a packet is allocated to the network C at a transmission mark, then at the next transmission mark there is a probability of allocating a packet either to network B or network C.

The section wise classification of the paper is as follows: Section 2, development of the non-Markovian queuing network model; Section 3, Solution and sensitivity analysis of the model using simulation technique and in Section 4, Conclusion.

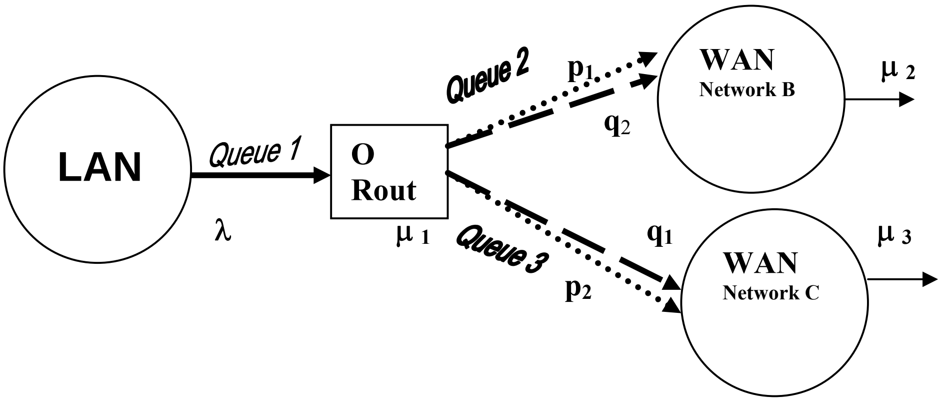

Figure-1.This figure shows the inter-network under consideration.

2. Queueing Network Model

The present queuing network model is based on the following assumptions:

(1) The packets arrive to the Router O to form queue1 in a Poisson stream with rate λ.

(2) The transmission times at queue1 are independently, identically and exponentially distributed with mean rate μ_1.

(3) After the transmission from Router O, the packets are either allocated to network B or to the network C on the basis of the IP addresses they possess. The allocation of packets at two consecutive transmission marks of Queue 1, is governed by the following transition probability matrix:

		To the latest transmission mark of Queue 1	
		Allocation to Network B	Allocation to Network C
From the latest but one transmission mark of Queue1	Allocation to Network B	p_1	q_1
	Allocation to Network C	q_2	p_2

Where $p_1 + q_1 = 1$ and $p_2 + q_2 = 1$. Thus, the allocation of packets at two consecutive transmission marks of Queue1 is correlated.

(4) The packets get queued in Queue 2 to network B and in Queue 3 to network C for their further transmission to final destination. The transmission times of packets at network B and network C are independently, identically and exponentially distributed with parameters μ_2 and μ_3 respectively.

(5) The capacities of the three queues i.e. Queue 1, 2 and 3 are infinite meaning thereby that any number of packets can be accommodated in these queues.

(6) The queue disciplines of these queues are FCFS.

3. Solution and Sensitivity Analysis of the Model using Simulation Technique:

The simulation analysis of the queuing network model under consideration has been done using a computer program written in C language [5-6]. The simulation results have been shown in the tables 1-5.From table-1 we see that with the increase in mean interarrival time to the network A, the measures of performance such as average delay, average queue length and average server's utilization decrease for queues 1, 2 and 3. From table-2, we find that the increase in mean service time in queue1 increases the measures of performance in queue 1 while in queues 2 and 3 there is no definite trend of variation. From table-3, we find that the increase in mean service time in queue 2 increases the measures of performance in queue 2 while in queues 1 and 3 there is no definite trend of variation. Also, from table-4, we find that the increase in mean service time in queue 3 increases the measures of performance in queue 3 while in queues 1 and 2 there is no definite trend of variation.

From table-5, we find that when we increase the two types of correlated allocation probabilities i.e. p_1 and p_2, the measures of performance in queues 2 and 3 increase while in queue 1 there is no particular trend of variation.

4. Conclusion:

In this paper, a non-Markovian queuing network model with correlated allocation has been studied. The solution and sensitivity analysis of the model have been done using simulation technique. The effect of correlated allocation probabilities on different performance measures has also been studied. This model can be useful in studying the correlated allocation of messages over the Internet.

Table-1: Variation of measures of performance with change in mean interarrival time in queue1 Mean service time in queue1 = 1.3, Mean service time in queue2=2.5 and Mean service time in queue3=2.6; $p_1 = 0.7$, $q_1 = 0.3$, $q_2 = 0.3$ and $p_2 = 0.7$

Mean interarrival time in queue1	Avg. delay in queue1	Avg. no. in queue 1	Avg. server's utilization in queue1	Avg. delay in queue2	Avg. no. in queue2	Avg. server's utilization in queue2	Avg. delay in queue3	Avg. no. in queue 3	Avg. server's utilization in queue3
1.1	986.028	895.017	1.000	15.284	11.645	0.938	79.794	60.957	0.995
1.2	331.818	273.993	0.999	77.180	59.420	0.979	134.129	102.921	0.995
1.3	56.569	42.711	0.978	16.771	12.508	0.926	27.236	20.461	0.964
1.4	15.647	10.949	0.907	10.055	7.170	0.854	25.183	17.923	0.941
1.5	8.056	5.406	0.870	10.861	7.283	0.855	15.942	10.671	0.869
1.6	4.601	2.841	0.797	4.151	2.563	0.742	8.686	5.363	0.838
1.7	4.214	2.478	0.762	5.289	3.110	0.742.	6.206	3.649	0.778
1.8	3.310	1.855	0.736	4.905	2.749	0.735	4.419	2.476	0.704
1.9	2.925	1.563	0.697	4.715	2.520	0.682	4.603	2.461	0.688
2.0	2.485	1.247	0.655	3.442	1.727	0.626	3.449	1.731	0.647

Table-2: Variation of measures of performance with change in mean service time in queue1
Mean interarrival time in queue1 = 2.5, Mean service time in queue2=2.5, and Mean service time in queue3= 2.6; $p_1 = 0.7$, $q_1=0.3$, $q_2= 0.3$ and $p_2 = 0.7$

Mean service time in queue1	Average delay in queue1	Average no. in queue 1	Average server's utilization in queue1	Average delay in queue2	Average no. in queue2	Average server's utilization in queue2	Average delay in queue3	Average no. in queue 3	Average server's utilization in queue3
1.1	0.811	0.322	0.440	1.504	0.598	0.479	2.037	0.809	0.536
1.2	1.172	0.475	0.497	1.936	0.785	0.515	1.752	0.710	0.521
1.3	1.142	0.558	0.521	1.958	0.776	0.492	1.646	0.650	0.501
1.4	1.926	0.761	0.568	1.669	0.660	0.478	1.810	0.715	0.506
1.5	2.008	0.802	0.595	1.750	0.699	0.500	2.012	0.803	0.514
1.6	2.533	1.007	0.641	1.756	0.698	0.503	2.037	0.810	0.502
1.7	4.189	1.705	0.706	1.668	0.679	0.503	1.997	0.813	0.526
1.8	4.843	1.946	0.730	1.804	0.725	0.514	1.755	0.707	0.512
1.9	6.252	2.494	0.773	1.747	0.697	0.496	1.850	0.738	0.507
2.0	7.526	3.016	0.802	1.918	0.769	0.510	1.849	0.741	0.537

Table-3. Variation of measures of performance with change in mean service time in queue2
Mean interarrival time in queue1 = 2.5, Mean service time in queue1=1.3, and Mean service time in queue3= 2.6; $p_1 = 0.7$, $q_1=0.3$, $q_2= 0.3$ and $p_2 = 0.7$

Mean service time in queue2	Average delay in queue1	Averag e no. in queue 1	Average server's utilization in queue1	Average delay in queue2	Average no. in queue2	Average server's utilization in queue2	Average delay in queue3	Average no. in queue 3	Average server's utilization in queue3
1.1	1.495	0.590	0.521	0.212	0.084	0.215	1.779	0.701	0.501
1.2	1.602	0.636	0.523	0.267	0.106	0.233	1.718	0.682	0.511
1.3	1.416	0.563	0.522	0.327	0.130	0.259	1.679	0.667	0.505
1.4	1.332	0.523	0.515	0.349	0.137	0.267	2.108	0.828	0.520
1.5	1.407	0.554	0.520	0.489	0.192	0.299	1.652	0.650	0.485
1.6	1.379	0.542	0.513	0.496	0.195	0.325	1.788	0.703	0.484
1.7	1.621	0.649	0.531	0.615	0.246	0.338	1.581	0.633	0.502
1.8	1.375	0.551	0.528	0.687	0.275	0.353	1.908	0.764	0.526
1.9	1.369	0.539	0.520	0.647	0.255	0.352	1.874	0.738	0.519
2.0	1.632	0.649	0.529	0.781	0.311	0.391	1.576	0.627	0.499

Table-4. Variation of measures of performance with change in mean service time in queue3
Mean interarrival time in queue1 = 2.5, Mean service time in queue1=1.3, and Mean service time in queue2= 2.5; p_1= 0.7, q_1=0.3, q_2= 0.3 and p_2 = 0.7

Mean service time in queue3	Average delay in queue1	Average no. in queue 1	Average server's utilization in queue1	Average delay in queue2	Average no. in queue2	Average server's utilization in queue2	Average delay in queue3	Average no. in queue 3	Average server's utilization in queue3
1.1	1.244	0.493	0.505	1.692	0.670	0.484	0.220	0.087	0.226
1.2	1.383	0.547	0.521	1.362	0.538	0.462	0.238	0.094	0.241
1.3	1.414	0.558	0.514	1.471	0.580	0.477	0.344	0.136	0.264
1.4	1.324	0.522	0.508	1.663	0.656	0.492	0.394	0.155	0.282
1.5	1.474	0.583	0.518	1.679	0.664	0.483	0.428	0.161	0.300
1.6	1.243	0.479	0.493	1.734	0.669	0.457	0.449	0.192	0.381
1.7	1.371	0.540	0.514	1.625	0.640	0.486	0.517	0.204	0.329
1.8	1.369	0.540	0.510	1.558	0.615	0.475	0.671	0.265	0.365
1.9	1.397	0.545	0.516	1.331	0.519	0.448	0.778	0.303	0.383
2.0	1.529	0.618	0.540	1.413	0.572	0.481	0.971	0.393	0.420

Table-5. Variation of measures of performance with change in probabilities of allocation. Mean interarrival time in queue1 = 2.5, Mean service time in queue1=1.3, and Mean service time in queue2= 2.5, Mean service time in queue3=2.6.

Probability P_1 and (q_1=1-p_1)	Probability P_2 and (q_2=1-p_2)	Avg. delay in queue1	Avg. no. in queue 1	Avg. server's utilization in queue1	Avg. delay in queue2	Avg. no. in queue 2	Avg. server's utilization in queue2	Avg. delay in queue 3	Avg. no. in queue 3	Avg. server's utilization in queue3
0.10	0.10	1.300	0.514	0.511	0.872	0.345	0.505	0.791	0.313	0.502
0.20	0.20	1.426	0.570	0.529	0.816	0.326	0.477	1.096	0.438	0.532
0.30	0.30	1.529	0.607	0.529	0.786	0.312	0.492	0.922	0.366	0.502
0.40	0.40	1.476	0.582	0.523	0.958	0.378	0.480	1.047	0.413	0.505
0.50	0.50	1.349	0.544	0.533	0.989	0.399	0.494	1.286	0.519	0.531
0.60	0.60	1.604	0.649	0.542	1.625	0.658	0.511	1.257	0.509	0.508
0.70	0.70	1.412	0.558	0.521	1.958	0.776	0.492	1.646	0.650	0.501
0.80	0.80	1.406	0.559	0.523	2.068	0.822	0.468	2.542	1.011	0.011
0.90	0.90	1.574	0.632	0.533	2.868	1.147	0.462	3.835	1.534	0.548
1.00	1.00	1.273	0.501	0.509	96.007	37.744	0.986	0.000	0.000	0.000

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A Heuristic for Obtaining a Better Initial Solution for the Linear Fractional Transportation Problem

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Abstract
A heuristic for obtaining an initial feasible solution (IFS) for the linear transportation problem with fractional objective function, is presented. The new heuristic gives superior performance over Vogel's Approximation Method (VAM)/Maximum Profit Method (MPM). It is faster in terms of total costs obtained, number of iterations required to reach the final solution. At the end two numerical examples are presented to explain the heuristic.

Keywords: linear fractional transportation programming, Vogel's method, Maximum profit method.

Introduction:
The fractional transportation problem constitutes a large portion of linear fractional programming applications. Various methods such as the stepping stone method and the modified distribution method (MODI) method have been developed to solve the fractional transportation problem provided the IFS is available. In order to proceed with these methods, it is necessary to obtain the IFS. A number of heuristics are used in this respect, among which are Vogel's approximation/maximum profit methods (minimization/maximization), the row minimum-cost method, the column minimum-cost method, North West corner method and many more. VAM usually results in a better initial solution compared with the initial solution found by the other methods. Thus, less iteration are required to reach to the final optimum solution.

For minimization problem VAM involves the calculation of the penalty parameter for each row and each column by subtracting the lowest cost of the associated row (column) in question from the next lowest cost in that row (column). Then an allocation is made to the lowest-cost cell of the row or column with the highest penalty parameter. The procedure continues until all the allocations are made, ignoring rows or columns where the supply from a given source is depleted from a destination point is met.

A new heuristic for finding an initial solution for the linear fractional transportation problem is presented in the next section of this paper.

Problem formulation
Mathematical formulation of linear fractional transportation problem (LFTP) as discussed by Swarup [10] is

Minimize/Maximized $Z = \frac{N(X)}{D(X)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} x_{ij}}$

where $$\sum_{j=1}^{n} x_{ij} = a_i$$

$$\sum_{i=1}^{m} x_{ij} = b_j$$

$$x_{ij}, a_i, b_j \geq 0, \quad i = 1,2....m, \quad j = 1,2....n$$

$$(x_{ij}) = X \subset S$$

Vectors (c_{ij}) and (d_{ij}) lie in $R^{m \times n}$ and X is a vector of mn decision variables, a_i being the availability at i^{th} supply and b_j the requirement at j^{th} demand point. $D(x) > 0$ for all $x \in S$, where S is a compact set of feasible points.

The heuristic

Step 1

Calculate the value of $\frac{c_{ij}}{d_{ij}}$ for each cell and prepare the new matrix of fractional values.

Step 2

For minimization calculate the penalty parameter for each row and each column by subtracting the lowest cost of the associated row (column) in question from the next lowest cost in that row/column (if both cost i.e. first and next are same then the penalty is zero). Then an allocation is made to the lowest-cost cell of the row or column with the highest penalty parameter. The procedure continues until all the allocations are made, ignoring rows or columns where the supply from a given source is depleted from a destination point is met.

For maximization we begin by finding the variable $x_{i_1 j_1}$ which corresponds to the highest profit (highest value of $\frac{c_{ij}}{d_{ij}}$). Then assign $x_{i_1 j_1}$ its largest possible value, i.e. $x_{i_1 j_1} = \min(b_{i_1}, a_{j_1})$. We mark (cross out) row i_1 and column j_1 and reduce the corresponding supply and demand by the value of $x_{i_1 j_1}$. Then we repeat the procedure using only those cells that do not lie in the crossed out rows and columns. We have to continue this process until there is only one cell in the transportation tableau that can be chosen.

The heuristic presented is based on the fraction cost concept and hence will be referred to as the "Fraction Cost Penalty Method" (FCPM) for minimization and "Fractional Cost Maximum Profit Method" for maximization.

This heuristic also has a computational advantage over VAM/MPM. We demonstrate this heuristic by two simple examples.

Numerical examples:

Example1

We take this minimization problem from M.C.Puri paper [3]. Its optimal answer is $\frac{381}{60} = 6.35$. LFTP is given in table 1. In table 3 the optimal solution of the LFTP is presented.

	B1	B2	B3	B4	Supply
A1	5 2	8 1	7 3	6 2	5
A2	6 1	10 4	5 2	5 3	3
A3	7 2	15 1	3 1	16 2	15
A4	15 3	21 1	8 1	18 1	12
Demand	5	14	10	6	35

Table 1 Problem formulation

we calculate the ratio of $\frac{c_{ij}}{d_{ij}}$ and using ratio find out the penalties corresponding to the row and column that are given in table 2.

	B1	B2	B3	B4	Supply	Penalties
A1	2.5	5(8)	2.3	3	5	.2
A2	6	3(2.5)	2.5	1.7	3	.8
A3	3.5	6(15)	3(3)	6(8)	15	.5
A4	5(5)	21	7(8)	18	12	3
Demand	5	14	10	6	35	
Penalties	1	5.5	.2	1.3		
	1	7	.7	5		
	1.5	6	5	10		

Table 2 Penalty matrix using FCPM

	B1	B2	B3	B4	Supply
A1	5 2	8 1	7 3	6 **5** 2	5
A2	6 1	10 **3** 4	5 2	5 3	3
A3	7 2	15 **11** 1	3 **3** 1	16 **1** 1	15
A4	15 **5** 3	21 1	8 **7** 1	18 1	12
demand	5	14	10	6	35

Table 3 Final optimal table

Initial feasible solution of minimization problem is $\frac{396}{60} = 6.6$ by our method and with Vogel's approximation method it is $\frac{418}{43} = 9.72$, so we can say that our method gives us a better result for minimization problem. For finding the final solution from IFS, we follow the procedure given in the book Erik Bajalinov [1]. If the above procedure is applied on the IFS obtained by our method only two iterations are required to reach the final optimum solution where as it needs five iteration to reach to the final optimum solution from IFS obtained by VAM.

Example2

This maximization problem has taken from book E. Bajalinov [1]. The final optimal answer of this problem is $\frac{7000}{5370} = 1.303538$. Fractional transportation problem with maximization condition is given in table 4. In table 6 the optimal solution of the FTP is presented.

	B1	B2	B3	B4	Supply
A1	10 15	14 12	8 16	12 8	150
A2	8 10	12 6	14 13	8 12	250
A3	9 13	6 15	15 12	9 10	200
Demand	150	250	50	150	600

Table 4 Problem formulation

In table 5 we calculate the $\frac{c_{ij}}{d_{ij}}$ and using the FCMPM procedure to find out the IFS.

	B1	B2	B3	B4	Supply
A1	0.67	1.17	0.5	150(1.5)	150
A2	0.8	250(2)	1.1	0.67	250
A3	150(0.69)	0.4	50(1.25)	0.9	200
Demand	150	250	50	150	600

Table 5 Allotment matrix using FCMPM

	B1	B2	B3	B4	Supply
A1	10 15	14 12	8 16	12 150 8	150
A2	8 10	12 250 6	14 13	8 12	250
A3	9 150 13	6 15	15 50 12	9 10	200
Demand	150	250	50	150	600

Table 6 Final optimal table

So our method gives the IFS as $\frac{7000}{5370} = 1.303538$, which is the final optimal answer. Thus when the method is applied no iteration is required to reach the maximum solution. By MPM we get IFS as $\frac{6700}{6870} = 0.975255$. We need one iteration to reach the final solution. Thus no iteration is used to reach to the final solution from our IFS where as it requires one iteration to reach the final solution from IFS obtained by MPM. In both cases we use the procedure given in the book by Erik Bajalinov [1].

Conclusion

The following conclusion is based on the findings presented in the previous section. Our method gives better performance as compare to VAM/MPM. By using our procedure less iteration is required to reach the final optimum solution. In other words it will save computer/otherwise time.

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An EOQ Model for Deteriorating Items with Allowable Shortage and Permissible Delay in Payment under Two-Stage Interest Payable Criterion

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Abstract:

In this article, an inventory model is developed under the condition of permissible delay in payment. Beyond the permissible delay period, the supplier charges interest at two different rates. The model also incorporates Weibull distribution deterioration and shortage. An illustrative example is also provided which support the developed model.

Keywords: inventory, deterioration, Weibull distribution, delay, shortage.

Introduction

In recent years, there is a spate of interest in studying the inventory systems with deteriorating items. Ghare and Schrader(1963) were the first to introduce the aspect of deterioration in the inventory models, they developed an inventory model for exponentially decaying in which inventory is not only depleted by demand alone but also by direct spoilage, physical depletion or deterioration. Later their work is extended by a number of researches taking two-parameter and three-parameter Weibull distribution to represent the time to deterioration and other innovative inventory aspects.

Among these researchers Covert and Philip (1973), Misra (1975), Elsayed and Teresi (1983), Jalan *et al* (1996) used two-parameter Weibull distribution and Philip (1974), Chakrabarty *et al* (1996) used three-parameter Weibull distribution to represent the time to deterioration.

While deriving the economic order quantity formula, it is generally assumed that the supplier must be paid for the items as soon as they are received. However, such a situation is not always observed in real world of transactions. In most situations purchaser is offered with a credit period (permissible delay period) for settling the account by the supplier.

Usually there is no charge if the amount is paid within this permissible delay period, but beyond this period interest is charged. Davis and Gaither (1985) developed an EOQ model for firms that are offered a one-time opportunity to delay payment for an order of a commodity. Goyal (1985) first established a single item inventory model under the condition of permissible delay in payments, assuming that the sales revenue is utilized to earn interest during the permissible delay period. Shah (1993) extended Goyal (1985) model for exponentially decaying items. Hwang and Shinn (1997) presented a lot sizing policy under the assumption of exponential decaying of products, permissible delay in payments and demand rate as a function of retail price. The inventory models developed by Goyal(1985), Shah(1993) and Hwang(1997) have a common feature that the sales revenue is utilized to earn interest only during the permissible delay period.

Later Aggarwal and Jaggi(1995) developed a model to determine the optimum order quantity for deteriorating items, with constant demand and permissible delay in payment; in which the customer utilizes the sales revenue and earns interest on it throughout the inventory cycle. Jamal *et al* (1997) extended Aggarwal and Jaggi's(1995) model incorporating shortages. Chung (2000) proposed an inventory model for items with constant rate of deterioration and permissible delay in payments, where the sales revenue is utilized to earn interest only during the permissible period. Dye and Chang(2003) formed a replenishment policy with linear trend in demand, deterioration, shortage and permissible delay in payment. Jain *et al* (2008) introduced an inventory model incorporating Weibull distribution with allowable shortage under cash discount and permissible delay in payments. The models discussed above consider a permissible delay period failing which an interest will be charged by the supplier on the outstanding balance.

In the competitive world of business, a supplier uses a number of marketing muscles to attract the consumer. One of these can be, instead of offering a single interest rate beyond the permissible period, the supplier can adopt a two-stage criterion. That is to say that beyond the permissible delay period, a certain rate of interest is charged for a fixed period and after this fixed period a comparatively higher rate of interest is charged for the rest of the cycle. This article is concerned with the development of an inventory model with constant demand, two-parameter Weibull distribution deterioration, shortage and a two-stage interest payable criterion.

The problem formulation

This study develops an inventory model with constant demand, Weibull distribution deterioration and permissible delay in payments. An entirely new concept of a two-stage interest payable criterion (offered by the supplier) is introduced. The model has two scenarios:

Scenario - I : The permissible delay period M is shorter than the period T_1 (the period with positive stock of the items). Again, scenario -I have two stages.

Stage I : An interest is charged by the supplier, at a certain rate beyond M up to an offered period on the outstanding balance.

Stage II : Interest is charged at a comparatively higher rate (than the stage I-interest rate) for the rest of the cycle.

Scenario - II : The permissible delay period M is greater than the period T_1. Under this scenario, the customer utilizes the sales revenue (on all the products bought) and earns interest on it throughout the inventory cycle.

The model is developed under the following notations and assumptions.

Notation

A The ordering cost (dollars/order)
c The unit cost per item (dollars/unit)
C_b Backorder cost (dollars/unit-year)
C_B Total backorder cost per cycle

C_D Total cost of deterioration per cycle

C_H Total holding cost per cycle

D The demand rate (units per unit time)

D_T Amount of units deteriorated during a cycle time, T

i The inventory carrying cost rate

I_e The interest earned per dollar per unit time

I_T Total interest earned per cycle

M Permissible delay period for settling the account

P_T Interest payable per cycle

Q The order quantity (units / order)

Q_1 Quantity consumed during time T_1

s Period with shortage

t_1 The time after which the interest (payable) rate changes

T The length of the inventory cycle (time units)

T_1 Length of the period with positive stock of the items

I_{p_1} The interest paid for the period $t_1 - M$, per dollar per unit time, dollars/dollar-year

I_{p_2} The interest paid for the period $(T_1 - t_1)$, per dollar per unit time, dollars/dollar- year, with the condition $I_{p_2} > I_{p_1}$

$(t_1 - M)$ Period of low interest

$(T_1 - t_1)$ Period of high interest

$b\ (= Q - Q_1)$ Maximum allowable shortage

$Z(t) = \alpha\beta t^{\beta-1}$ is the Weibull distribution function representing time to deterioration, where

$\alpha =$ scale parameter; $(0 < \alpha << 1)$

$\beta =$ shape parameter; $(\beta > 1)$

Assumptions:

(1) The inventory system deals with only one type of item.
(2) The replenishment occurs instantaneously.
(3) Lead-time is zero.
(4) There is no repair or replacement of the deteriorated items.

Model formulation

With the passage of time the inventory level gradually falls due to demand and deterioration up to time T_1 after which shortages begin to accumulate up to time T to a level b. At time T the inventory is replenished. The inventory system is depicted by figure 1.

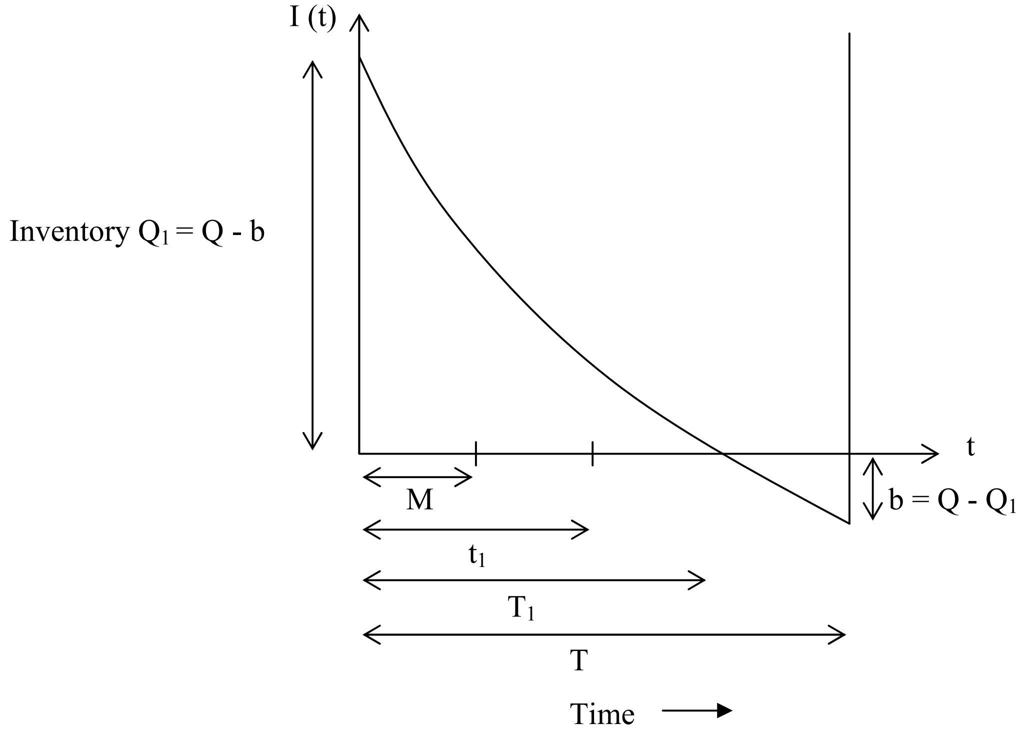

Figure 1. The deteriorating inventory system with delay in payments and shortages

The differential equation describing the instantaneous state of inventory $I(t)$, during the time period $(0 \le t \le T)$ is given by

$$\frac{d}{dt}I(t)+Z(t)I(t)=-D, \quad (0 \le t \le T)$$

or
$$\frac{d}{dt}I(t)+\alpha\beta t^{\beta-1}I(t)=-D \qquad \left(\because Z(t)=\alpha\beta t^{\beta-1}\right) \qquad \ldots(1)$$

Solution of (1), with $e^{\alpha t^{\beta}} = 1+\alpha t^{\beta}$ (as $\alpha << 1$) and $I(t) = I_0$ at $t = 0$ is,

$$I(t) = -De^{-\alpha t^{\beta}}\left[t+\left(\frac{\alpha}{\beta+1}\right)t^{\beta+1}\right]+I_0e^{-\alpha t^{\beta}} \qquad \ldots(2)$$

Using $I(t)=0$ at $t = T_1$ in (2), we get,

$$I_0 = DT_1\left[1+\left(\frac{\alpha}{\beta+1}\right)T_1^{\beta}\right] = Q-b \qquad \ldots(3)$$

where, b is the maximum shortage level permitted.

Substituting I_0 from (3) in (2) we get,

$$I(t) = De^{-\alpha t^{\beta}}\left[(T_1 - t) + \left(\frac{\alpha}{\beta+1}\right)\left(T_1^{\beta+1} - t_1^{\beta+1}\right)\right]; 0 \le t \le T_1 \qquad \ldots(4)$$

And $I(t) = 0$ when $T_1 \le t \le T$

The amount of materials deteriorates during one cycle is

$D_T = I_0 - DT_1$ (where DT_1 = demand during time T_1), using (3) we get,

$$D_T = D\left(\frac{\alpha}{\beta+1}\right)T_1^{\beta+1} \qquad \ldots(5)$$

Since the inventory model considers delay in payment, there arise two different cases:

Case I : Payment at or before the total depletion of inventory $(M \le t_1 \le T_1 < T)$

Case II : After depletion payment $(T_1 < M)$

Now both the cases will be studied separately.

Case I : $(M \le t_1 \le T_1 < T)$

In this case the total cost will have following components: the ordering cost, cost of material deteriorated, backorder cost, and the payable interest. From the sum of these costs interest earned will be subtracted. Now above costs are obtained as follows

Ordering cost = A dollars/orders

Cost of deterioration of D_T units

$$C_D = cD_T = cD\left(\frac{\alpha}{\beta+1}\right)T_1^{\beta+1} \qquad \text{(using (5))} \qquad \ldots(6)$$

Holding cost $C_H = ic\int_0^{T_1} I(t)dt = ic\int_0^{T_1} De^{-\alpha t^{\beta}}\left[(T_1 - t) + \left(\frac{\alpha}{\beta+1}\right)\left(T_1^{\beta+1} - t^{\beta+1}\right)\right]$

[using (4)]

simplifying above with $e^{-\alpha t^{\beta}} = 1 - \alpha t^{\beta}$ (as $\alpha << 1$) we obtain

$$C_H = DicT_1^2\left[\frac{T_1^{\beta}\alpha\beta}{(\beta+1)(\beta+2)} + \frac{1}{2}\right] \qquad \ldots(7)$$

Interest payable per cycle for the inventory not being sold after the due date M is

$$P_T = cI_{p_1}\int_M^{t_1} I(t)\,dt + cI_{p_2}\int_{t_1}^{T_1} I(t)\,dt + cI_{p_2}\int_{T_1}^{T} I(t)\,dt$$

$$\Rightarrow \qquad P_T = cI_{p_1}\int_M^{t_1} I(t)\,dt + cI_{p_2}\int_{t_1}^{T_1} I(t)\,dt \qquad (\because I(t) = 0 \text{ when } T_1 \le t \le T)$$

using (4) with $e^{-\alpha t^{\beta}} = 1 - \alpha t^{\beta}$ (as $\alpha << 1$), neglecting terms of $\alpha^2, \alpha^3, \ldots$ and simplifying we get

$$P_T = cD\left(T_1 + \frac{\alpha}{\beta+1}T_1^{\beta+1}\right)\left[I_{p_1}(t_1 - M) + I_{p_2}(T_1 - t_1)\right] - \frac{1}{2}cD\left[I_{p_1}\left(t_1^2 - M^2\right) + I_{p_2}\left(T_1^2 - t_1^2\right)\right]$$
$$- cD\left(\frac{\alpha}{\beta+1}\right)T_1\left[I_{p_1}\left(t_1^{\beta+1} - M^{\beta+1}\right) + I_{p_2}\left(T_1^{\beta+1} - t_1^{\beta+1}\right)\right]$$

$$+cD\frac{\alpha\beta}{(\beta+1)(\beta+2)}\left[I_{p_1}\left(t_1^{\beta+2}-M^{\beta+2}\right)+I_{p_2}\left(T_1^{\beta+2}-t_1^{\beta+2}\right)\right] \quad \ldots(8)$$

Interest earned per cycle is equal to the interest earned during the positive inventory, and is as follows,

$$I_T = cI_e\int_0^{T_1} Dt\,dt = \frac{cI_eDT_1^2}{2} \quad \ldots(9)$$

Backorder cost per cycle $C_B = C_b\int_0^{T-T_1} Dt\,dt = \frac{C_bD(T-T_1)^2}{2}$ …(10)

The total variable cost

$$TVC = A + C_D + C_H + P_T - I_T + C_B \quad \ldots(11)$$

$$= A + cD\left(\frac{\alpha}{\beta+1}\right)T_1^{\beta+1} + DicT_1^2\left[\frac{T_1^{\beta}\alpha\beta}{(\beta+1)(\beta+2)}+\frac{1}{2}\right]$$

$$+cD\left(T_1+\frac{\alpha}{\beta+1}T_1^{\beta+1}\right)\left[I_{p_1}(t_1-M)+I_{p_2}(T_1-t_1)\right]$$

$$-\frac{1}{2}cD\left[I_{p_1}\left(t_1^2-M^2\right)+I_{p_2}\left(T_1^2-t_1^2\right)\right]$$

$$-cD\left(\frac{\alpha}{\beta+1}\right)T_1\left[I_{p_1}\left(t_1^{\beta+1}-M^{\beta+1}\right)+I_{p_2}\left(T_1^{\beta+1}-t_1^{\beta+1}\right)\right]$$

$$+\frac{cD\alpha\beta}{(\beta+1)(\beta+2)}\left[I_{p_1}\left(t_1^{\beta+2}-M^{\beta+2}\right)+I_{p_2}\left(T_1^{\beta+2}-t_1^{\beta+2}\right)\right] -\frac{cI_eDT_1^2}{2}+\frac{C_bD(T-T_1)^2}{2}$$

The total variable cost per unit time $TC(T_1,T) = \frac{TVC(T_1,T)}{T}$

$$=\frac{A}{T}+cD\left(\frac{\alpha}{\beta+1}\right)\frac{T_1^{\beta+1}}{T}+DicT_1^2\left[\frac{T_1^{\beta}\alpha\beta}{(\beta+1)(\beta+2)}+\frac{1}{2}\right]\frac{1}{T}$$

$$+cD\left(T_1+\frac{\alpha}{\beta+1}T_1^{\beta+1}\right)\left[I_{p_1}(t_1-M)+I_{p_2}(T_1-t_1)\right]\frac{1}{T}-\frac{1}{2}cD\left[I_{p_1}\left(t_1^2-M^2\right)+I_{p_2}\left(T_1^2-t_1^2\right)\right]\frac{1}{T}$$

$$-cD\left(\frac{\alpha}{\beta+1}\right)T_1\left[I_{p_1}\left(t_1^{\beta+1}-M^{\beta+1}\right)+I_{p_2}\left(T_1^{\beta+1}-t_1^{\beta+1}\right)\right]\frac{1}{T}$$

$$+\frac{cD\alpha\beta}{(\beta+1)(\beta+2)}\left[I_{p_1}\left(t_1^{\beta+2}-M^{\beta+2}\right)+I_{p_2}\left(T_1^{\beta+2}-t_1^{\beta+2}\right)\right]\frac{1}{T}$$

$$-\frac{cI_eDT_1^2}{2}.\frac{1}{T}+\frac{C_bD(T-T_1)^2}{2}.\frac{1}{T} \quad \ldots(12)$$

Rather than evaluating the nature of the total cost function and ascertaining its convexity; we adopt the same procedure as adopted by Jamal *et al* [12]

The minimized total cost is obtained when

$$\frac{\partial TC(T_1,T)}{\partial T_1}=0 \qquad \text{and} \qquad \frac{\partial TC(T_1,T)}{\partial T}=0$$

Therefore using (12) we obtain

$$\begin{aligned}\frac{\partial TC(T_1,T)}{\partial T_1} &= cD\alpha\frac{T_1^{\beta}}{T}+\frac{DicT_1}{T}\left[\frac{\alpha\beta}{(\beta+1)}T_1^{\beta}+1\right]\\ &+\frac{cD}{T}\left[\left(1+\alpha T_1^{\beta}\right)\left\{I_{p_1}(t_1-M)+I_{p_2}(T_1-t_1)\right\}+I_{p_2}\left(T_1+\frac{\alpha}{\beta+1}T_1^{\beta+1}\right)\right]\\ &-\frac{cD\alpha}{(\beta+1)}\cdot\frac{1}{T}\left[I_{p_1}\left(t_1^{\beta+1}-M^{\beta+1}\right)+I_{p_2}\left(T_1^{\beta+1}-t_1^{\beta+1}\right)\right]-cDI_{p_2}\frac{T_1}{T}\left[1+\left(\frac{\alpha}{\beta+1}\right)T_1^{\beta}\right]\\ &-cI_eD\frac{T_1}{T}-C_bD\frac{(T-T_1)}{T}=0 \qquad \ldots(13)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial TC(T_1,T)}{\partial T} &= A+cD\left(\frac{\alpha}{\beta+1}\right)T_1^{\beta+1}+DicT_1^2\left[T_1^{\beta}\frac{\alpha\beta}{(\beta+1)(\beta+2)}+\frac{1}{2}\right]+cD\left[T_1+\left(\frac{\alpha}{\beta+1}\right)T_1^{\beta+1}\right]\\ &\left[I_{p_1}(t_1-M)+I_{p_2}(T_1-t_1)\right]-\frac{1}{2}cD\left[I_{p_1}\left(t_1^2-M^2\right)+I_{p_2}\left(T_1^2-t_1^2\right)\right]\\ &-\frac{cD\alpha}{(\beta+1)}T_1\left[I_{p_1}\left(t_1^{\beta+1}-M^{\beta+1}\right)+I_{p_2}\left(T_1^{\beta+1}-t_1^{\beta+1}\right)\right]\\ &+\frac{cD\alpha\beta}{(\beta+1)(\beta+2)}\left[I_{p_1}\left(t_1^{\beta+2}-M^{\beta+2}\right)+I_{p_2}\left(T_1^{\beta+2}-t_1^{\beta+2}\right)\right]-\frac{cI_eDT_1^2}{2}-\frac{C_bD}{2}\left(T^2-T_1^2\right)=0\\ &\qquad \ldots(14)\end{aligned}$$

By simultaneously solving (13) and (14) (using same solution procedure as used by Jamal *et al* (1997)), the values of T_1 and T are obtained. Using these optimal values of T_1 and T, the optimal ordering quantity and total cost of the inventory system is obtained.

Case II. $T_1 < M$

In this case our model reduces to an inventory model with constant demand, Weibull- distribution deterioration, shortages and permissible delay period greater than the length of the period with positive stock of the item. Therefore the customer pays no interest and uses sales revenue to earn interest upto the permissible period, i.e;

Interest payable $P_T = 0$

Interest earned is equal to the sum of interest earned during positive inventory period and interest earned during time period (T_1, M)

Therefore $$I_T = cI_e \int_0^{T_1} Dt\,dt + cDT_1 I_e (M - T_1) = cDT_1 I_e \left(M - \frac{T_1}{2} \right) \qquad \ldots(15)$$

The other costs i.e C_D, C_H and C_B remains unaltered and is given by (6), (7) and (10) respectively.

The total variable cost per unit time is obtained similarly as in case I

$$TC_1(T_1, T) = \frac{TVC_1(T_1, T)}{T} = \frac{A}{T} + cD\left(\frac{\alpha}{\beta+1} \right) \frac{T_1^{\beta+1}}{T} + Dic \frac{T_1^2}{T} \left[\frac{T_1^{\beta} \alpha\beta}{(\beta+1)(\beta+2)} + \frac{1}{2} \right]$$

$$-cDI_e \frac{T_1}{T} \left(M - \frac{T_1}{2} \right) + \frac{C_b D}{2} \frac{(T - T_1)^2}{T}$$

The minimized total cost is obtained when $\frac{\partial TC_1(T_1, T)}{\partial T_1} = 0$ and $\frac{\partial TC_1(T_1, T)}{\partial T} = 0$

$$\frac{\partial TC_1(T_1, T)}{\partial T_1} = 0 = Dc \frac{\alpha T_1^{\beta}}{T} + \frac{Dic\alpha\beta}{(\beta+1)} \frac{T_1^{\beta+1}}{T} + D(ic + cI_e + C_b) \frac{T_1}{T} - DcI_e M. \frac{1}{T} - C_b D \qquad \ldots(16)$$

and

$$\frac{\partial TC_1(T_1, T)}{\partial T} = 0 = A + Dc\left(\frac{\alpha}{\beta+1} \right) T_1^{\beta+1} + DicT_1^2 \left[\frac{T_1^{\beta} \alpha\beta}{(\beta+1)(\beta+2)} + \frac{1}{2} \right]$$
$$-DcI_e T_1 \left(M - \frac{T_1}{2} \right) - \frac{C_b D}{2} \left(T^2 - T_1^2 \right) \qquad \ldots(17)$$

Adopting the same procedure as in case I, (16) and (17) are solved simultaneously for optimal values of T_1 and T.

Numerical Illustration of the developed model:

Let D = 1000 units/years, A= 200 dollars/order, i = .12/year, I_{p_1} = .15/year, I_{p_2} = .17/year, I_e= .13/year, M= {0, 15, 30} days, t_1 = {15, 30, 45, 60, 75} days, scale parameter $\alpha = \{.02, .04, .06\}$, shape parameter β = {1.5, 2, 2.5}, C = {20, 40, 120, 150, 180, 200} dollars/unit and C_b= {10, 20, 50, 100, 1000} dollars/unit/year.

The economic ordering policies for the different combinations of above parameter are given below in tabular form.

Table 1-A Optimal solutions for payment before inventory depletion (Case-I), $C_b = 20$

				$\alpha = .02, \beta = 1.5$						$\alpha = .04, \beta = 1.5$						$\alpha = .06, \beta = 1.5$					
C	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	0	15	15	115	134	315	367	1063	100	110	130	301	356	1087	95	106	126	290	345	1110	91
20	15	30	15	118	135	323	370	952	88	113	131	310	359	977	83	109	127	299	348	1001	79
	30	45	15	122	138	334	378	854	77	117	133	321	364	881	72	113	130	310	356	907	68
	0	15	15	77	103	211	282	1395	62	74	100	203	274	1418	59	72	98	197	268	1439	57
40	15	30	15	82	104	225	285	1205	52	79	102	216	279	1231	49	76	99	208	271	1255	46
	30	45	15	88	107	241	293	1044	43	85	105	233	288	1075	40	80	100	219	274	1103	35
	0	15	15	28	67	77	184	2141	13	27	66	74	181	2153	12	26	65	71	178	2165	11
180	15	30	15	98	69	104	189	1703	8	37	69	101	189	1729	7	36	68	99	186	1753	6
	30	45	15	49	74	134	203	1365	4	47	72	129	197	1412	2	46	72	126	200	1455	1
	0	15	15	26	66	71	181	2185	11	25	65	68	178	2197	10	25	65	68	178	2208	10
200	15	30	15	36	67	99	184	1730	6	35	67	96	184	1756	5	34	66	93	181	1780	4
	30	45	15	47	72	129	197	1382	2	46	72	126	197	1429	1	45	72	123	197	1473	0

Table 1-B Optimal solutions for payment before inventory depletion (Case-I), $C_b = 20$

				$\alpha = .02, \beta = 1.5$						$\alpha = .04, \beta = 1.5$						$\alpha = .06, \beta = 1.5$					
C	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	0	15	15	115	134	315	367	1063	100	110	130	301	356	1087	95	106	126	290	345	1110	91
20	15	45	30	118	135	323	370	942	73	114	132	312	362	968	69	109	127	299	348	992	64
	30	75	45	124	139	340	381	839	49	119	135	326	370	867	44	114	130	312	356	893	39
	0	15	15	77	103	211	282	1395	62	74	100	203	274	1418	59	72	98	197	268	1439	57
40	15	45	30	83	105	227	288	1191	38	80	102	219	279	1217	35	77	100	211	274	1243	32
	30	75	45	90	109	247	299	1026	15	87	106	238	290	1058	12	84	104	230	285	1088	9
	0	15	15	28	67	77	184	2141	13	27	66	74	181	2153	12	26	65	71	178	2165	11
180	15	45	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	75	45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	0	15	15	26	66	71	181	2185	11	25	65	68	178	2197	10	25	65	68	178	2208	10
200	15	45	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	75	45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

where ' - ' represents infeasible solution.

Table 1-C Optimal solutions for payment before inventory depletion (Case-I), $C_b = 20$ (where ' - ' represents infeasible solution.)

				$\alpha = .02,\ \beta = 1.5$						$\alpha = .04,\ \beta = 1.5$						$\alpha = .06,\ \beta = 1.5$					
C	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	15	30	15	118	135	323	370	952	88	113	131	310	359	977	83	109	127	299	348	1001	79
20	15	45	30	118	135	323	370	942	73	114	132	312	362	968	69	109	127	299	348	992	64
	15	60	45	119	136	326	373	934	59	115	133	315	364	960	55	111	129	304	353	984	51
	15	30	15	82	104	225	285	1205	52	79	102	216	279	1231	49	76	99	208	271	1255	46
40	15	45	30	83	105	227	288	1191	38	80	102	219	279	1217	35	77	100	211	274	1243	32
	15	60	45	84	106	230	290	1181	24	81	103	222	282	1209	21	78	100	214	274	1234	18
	15	30	15	38	69	104	189	1703	8	37	69	101	189	1729	7	36	68	99	186	1753	6
180	15	45	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	15	60	45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	15	30	15	36	67	99	184	1730	6	35	67	96	184	1756	5	34	66	93	181	1780	4
200	15	45	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	15	60	45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 2-A Optimal solutions for payment before inventory depletion (Case-I), $C_b = 20$

				$\alpha = .02,\ \beta = 1.5$						$\alpha = .02,\ \beta = 2$						$\alpha = .02,\ \beta = 2.5$					
C	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	0	15	15	115	134	315	367	1063	100	117	136	321	373	1050	102	118	137	323	375	1043	103
20	15	30	15	118	135	323	370	952	88	120	137	329	375	938	90	121	138	332	378	931	91
	30	45	15	122	138	334	378	854	77	124	139	340	381	839	79	125	140	342	384	832	80
	0	15	15	77	103	211	282	1395	62	74	103	214	282	1380	63	79	104	216	285	1374	64
40	15	30	15	82	104	225	285	1205	52	83	105	227	288	1188	53	84	106	230	290	1181	54
	30	45	15	88	107	241	293	1044	43	89	108	244	296	1025	44	90	108	247	296	1017	45
	0	15	15	28	67	77	184	2141	13	29	68	79	186	2131	14	29	68	79	186	2128	14
180	15	30	15	38	69	104	189	1703	8	39	70	107	192	1683	9	39	70	107	192	1678	9
	30	45	15	49	74	134	203	1365	4	50	75	137	205	1334	5	50	75	137	205	1323	5
	0	15	15	26	66	71	181	2185	11	26	66	71	181	2176	11	26	67	71	184	2174	11
200	15	30	15	36	67	99	184	1730	6	37	68	101	186	1710	7	37	68	101	186	1704	7
	30	45	15	47	72	129	197	1382	2	48	72	132	197	1347	3	48	73	132	200	1336	3

Table 2-B Optimal solutions for payment before inventory depletion (Case-I), $C_b = 20$

				$\alpha = .02,\ \beta = 1.5$						$\alpha = .02,\ \beta = 2$						$\alpha = .02,\ \beta = 2.5$					
C	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	0	15	15	115	134	315	367	1063	100	117	136	321	373	1050	102	118	137	323	375	1043	103
20	15	45	30	118	135	323	370	942	73	120	137	329	375	928	75	122	139	334	381	921	77
	30	75	45	124	139	340	381	839	49	126	141	345	386	824	51	127	142	348	389	817	52
	0	15	15	77	103	211	282	1395	62	78	103	214	282	1380	63	79	104	216	285	1374	64
40	15	45	30	83	105	227	288	1191	38	84	105	230	288	1174	39	85	106	233	290	1167	40
	30	75	45	90	109	247	299	1026	15	92	110	252	301	1007	17	93	111	255	304	998	18
	0	15	15	28	67	77	184	2141	13	29	68	79	186	2131	14	29	68	79	186	2128	14
180	15	45	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	75	45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	0	15	15	26	66	71	181	2185	11	26	66	71	181	2176	11	26	67	71	184	2174	11
200	15	45	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	75	45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

where ‘ **-** ’ represents infeasible solution.

Table 2-C Optimal solutions for payment before inventory depletion (Case-I), $C_b = 20$

				$\alpha = .02,\ \beta = 1.5$						$\alpha = .02,\ \beta = 2$						$\alpha = .02,\ \beta = 2.5$					
C	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	15	30	15	118	135	323	370	952	88	120	137	329	375	938	90	121	138	332	378	931	91
20	15	45	30	118	135	323	370	942	73	120	137	329	375	928	75	122	139	334	381	921	77
	15	60	45	119	136	326	373	934	59	121	138	332	378	920	61	123	140	337	384	913	63
	15	30	15	82	104	225	285	1205	52	83	105	227	288	1188	53	84	106	230	290	1181	54
40	15	45	30	83	105	227	288	1191	38	84	105	230	288	1174	39	85	106	233	290	1167	40
	15	60	45	84	106	230	290	1181	24	85	106	233	290	1164	25	86	107	236	293	1157	26
	15	30	15	38	69	104	189	1703	8	39	70	107	192	1683	9	39	70	107	192	1678	9
180	15	45	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	15	60	45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	15	30	15	36	67	99	184	1730	6	37	68	101	186	1710	7	37	68	101	186	1704	7
200	15	45	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	15	60	45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

where ‘ **-** ’ represents infeasible solution.

Table 3-A Optimal solutions for payment before inventory depletion (Case-I), C = 20

				$\alpha = .02,\ \beta = 1.5$						$\alpha = .04,\ \beta = 1.5$						$\alpha = .06,\ \beta = 1.5$					
C_b	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	0	15	15	115	134	315	367	1063	100	110	130	301	356	1087	95	106	126	290	345	1110	91
20	15	30	15	118	135	323	370	952	88	113	131	310	359	977	83	109	127	299	348	1001	79
	30	45	15	122	138	334	378	854	77	117	133	321	364	881	72	113	130	310	356	907	68
	0	15	15	120	128	329	351	1113	105	115	123	315	337	1141	100	111	120	304	329	1168	96
50	15	30	15	122	129	334	353	991	92	117	124	321	340	1020	87	113	121	310	332	1048	83
	30	45	15	125	131	342	359	885	80	120	127	329	348	915	75	116	123	318	337	994	71
	0	15	15	122	126	334	345	1131	107	117	121	321	332	1161	102	113	117	310	321	1189	98
100	15	30	15	123	127	337	348	1006	93	118	122	323	334	1036	88	114	118	312	323	1065	84
	30	45	15	126	129	345	353	896	81	121	124	332	340	928	76	117	121	321	332	957	72
	0	15	15	123	123	337	337	1150	108	119	119	326	326	1182	104	114	114	312	312	1212	99
1000	15	30	15	124	134	340	340	1020	94	120	120	329	329	1052	90	116	116	318	318	1083	86
	30	45	15	127	127	348	348	907	82	122	122	334	334	940	77	118	118	323	323	971	73

Table 3-B Optimal solutions for payment before inventory depletion (Case-I), C = 20

				$\alpha = .02,\ \beta = 1.5$						$\alpha = .04,\ \beta = 1.5$						$\alpha = .06,\ \beta = 1.5$					
C_b	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	0	15	15	115	134	315	367	1063	100	110	130	301	356	1087	95	106	126	290	345	1110	91
20	15	45	30	118	135	323	370	942	73	114	132	312	362	968	69	109	127	299	348	992	64
	30	75	45	124	139	340	381	839	49	119	135	326	370	867	44	114	130	312	356	893	39
	0	15	15	120	128	329	351	1113	105	115	123	315	337	1141	100	111	120	304	329	1168	96
50	15	45	30	122	129	334	353	980	77	117	124	321	340	1010	72	113	121	310	332	1037	68
	30	75	45	127	133	348	364	868	52	122	129	334	353	899	47	117	124	321	340	929	42
	0	15	15	122	126	334	345	1131	107	117	121	321	332	1161	102	113	117	310	321	1189	98
100	15	45	30	124	128	340	351	994	79	119	123	326	337	1025	74	115	119	315	326	1054	70
	30	75	45	128	131	351	359	879	53	123	126	337	345	911	48	119	122	326	334	942	44
	0	15	15	123	123	337	337	1150	108	119	119	326	326	1182	104	114	114	312	312	1212	99
1000	15	45	30	125	125	342	342	1009	80	120	120	329	329	1041	75	116	116	318	318	1071	71
	30	75	45	129	129	353	353	890	54	124	124	340	340	933	49	120	120	329	329	955	45

Table 3-C Optimal solutions for payment before inventory depletion (Case-I), C = 20

				$\alpha = .02,\ \beta = 1.5$						$\alpha = .04,\ \beta = 1.5$						$\alpha = .06,\ \beta = 1.5$					
C_b	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	15	30	15	118	135	323	370	952	88	113	131	310	359	977	83	109	127	299	348	1001	79
20	15	45	30	118	135	323	370	942	73	114	132	312	362	968	69	109	127	299	348	992	64
	15	60	45	119	136	326	373	934	59	115	133	315	364	960	55	111	129	304	353	984	51
	15	30	15	122	129	334	353	991	92	117	124	321	340	1020	87	113	121	310	332	1048	83
50	15	45	30	122	129	334	353	980	77	117	124	321	340	1010	72	113	121	310	332	1037	68
	15	60	45	123	130	337	356	972	63	118	125	323	342	1001	58	114	121	312	332	1029	54
	15	30	15	123	127	337	348	1006	93	118	122	323	334	1036	88	114	118	312	323	1065	84
100	15	45	30	124	128	340	351	994	79	119	123	326	337	1025	74	115	119	315	326	1054	70
	15	60	45	124	128	340	351	985	64	120	124	329	340	1016	60	116	120	318	329	1045	56
	15	30	15	124	124	340	340	1020	94	120	120	329	329	1052	90	116	116	318	318	1083	86
1000	15	45	30	125	125	342	342	1009	80	120	120	329	329	1041	75	116	116	318	318	1071	71
	15	60	45	126	126	345	345	999	66	121	121	332	332	1032	61	117	117	321	321	1062	57

Table 4-A Optimal solutions for payment before inventory depletion (Case-I), C = 20

				$\alpha = .02,\ \beta = 1.5$						$\alpha = .02,\ \beta = 2$						$\alpha = .02,\ \beta = 2.5$					
C_b	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	0	15	15	115	134	315	367	1063	100	117	136	321	373	1050	102	118	137	323	375	1043	103
20	15	30	15	118	135	323	370	952	88	120	137	329	375	938	90	121	138	332	378	931	91
	30	45	15	122	138	334	378	854	77	124	139	340	381	839	79	125	140	342	384	832	80
	0	15	15	120	128	329	351	1113	105	122	130	334	356	1097	107	123	131	337	359	1090	108
50	15	30	15	122	129	334	353	991	92	124	131	340	359	976	94	125	132	342	362	968	95
	30	45	15	125	131	342	359	884	80	127	133	348	364	869	82	128	134	351	367	861	83
	0	15	15	122	126	334	345	1131	107	123	127	337	348	1115	108	125	129	342	353	1107	110
100	15	30	15	123	127	337	348	1005	93	125	129	342	353	989	95	126	130	345	356	981	96
	30	45	15	126	129	345	353	895	81	128	131	351	359	879	83	129	132	353	362	871	84
	0	15	15	123	123	337	337	1150	108	125	125	342	342	1133	110	126	126	345	345	1125	111
1000	15	30	15	124	124	340	340	1020	94	126	126	345	345	1003	96	128	128	351	351	995	98
	30	45	15	127	127	348	348	907	82	129	129	353	353	890	84	130	130	356	356	882	85

Table 4-B Optimal solutions for payment before inventory depletion (Case-I), C = 20

				$\alpha = .02,\ \beta = 1.5$						$\alpha = .02,\ \beta = 2$						$\alpha = .02,\ \beta = 2.5$					
C_b	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	0	15	15	115	134	315	367	1063	100	117	136	321	373	1050	102	118	137	323	375	1043	103
20	15	45	30	118	135	323	370	942	73	120	137	329	375	928	75	122	139	334	381	921	77
	30	75	45	124	139	340	381	839	49	126	141	345	386	824	51	127	142	348	389	817	52
	0	15	15	120	128	329	351	1113	105	122	130	334	356	1097	107	123	131	337	359	1090	108
50	15	45	30	122	129	334	353	980	77	124	131	340	359	965	79	125	132	342	362	957	80
	30	75	45	127	133	348	364	868	52	129	135	353	370	852	54	130	136	356	373	844	55
	0	15	15	122	126	334	345	1131	107	123	127	337	348	1115	108	125	129	342	353	1107	110
100	15	45	30	124	128	340	351	994	79	126	130	345	356	978	81	127	131	348	359	970	82
	30	75	45	128	131	351	359	879	53	130	133	356	364	862	55	131	134	359	367	854	56
	0	15	15	123	123	337	337	1150	108	125	125	342	342	1133	110	126	126	345	345	1125	111
1000	15	45	30	125	125	342	342	1009	80	127	127	348	348	992	82	128	128	351	351	983	83
	30	75	45	129	129	353	353	890	54	131	131	359	359	872	56	132	132	362	362	864	57

Table 4-C Optimal solutions for payment before inventory depletion (Case-I), C = 20

				$\alpha = .02,\ \beta = 1.5$						$\alpha = .02,\ \beta = 2$						$\alpha = .02,\ \beta = 2.5$					
C_b	M	t_1	$(t_1 - M)$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$	T_1	T	Q_1	Q	TC	$T_1 - t_1$
	15	30	15	118	135	323	370	952	88	120	137	329	375	938	90	121	138	332	378	931	91
20	15	45	30	118	135	323	370	942	73	120	137	329	375	928	75	122	139	334	381	921	77
	15	60	45	119	136	326	373	934	59	121	138	332	378	920	61	123	140	337	384	913	63
	15	30	15	122	129	334	353	991	92	124	131	340	359	976	94	125	132	342	362	968	95
50	15	45	30	122	129	334	353	980	77	124	131	340	359	965	79	125	132	342	362	957	80
	15	60	45	123	130	337	356	972	63	125	132	342	362	956	65	126	133	345	364	948	66
	15	30	15	123	127	337	348	1006	93	125	129	342	353	989	95	126	130	345	356	981	96
100	15	45	30	124	128	340	351	994	79	126	130	345	356	978	81	127	131	348	359	970	82
	15	60	45	124	128	340	351	985	64	126	130	345	356	969	66	128	132	351	362	961	68
	15	30	15	124	124	340	340	1020	94	126	126	345	345	1003	96	128	128	351	351	995	98
1000	15	45	30	125	125	342	342	1009	80	127	127	348	348	992	82	128	128	351	351	983	83
	15	60	45	126	126	345	345	999	66	128	128	351	351	982	68	129	129	353	353	974	69

Table 5 -A Optimal for payment after inventory depletion (Case II), $C_b = 10$

		$\alpha = .02, \ \beta = 1.5$					$\alpha = .04, \ \beta = 2$					$\alpha = .06, \ \beta = 1.5$				
C_b	M	T_1	T	Q_1	Q	TC	T_1	T	Q_1	Q	TC	T_1	T	Q_1	Q	TC
	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
120	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	45	35	72	96	197	1053	35	75	96	205	1066	34	73	93	200	1079
	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
150	30	28	77	77	211	1325	27	74	74	203	1334	27	76	74	208	1342
	45	32	67	88	184	992	32	70	88	192	1006	31	68	85	186	1020
	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
180	30	26	75	71	205	1310	25	72	68	197	1319	25	74	68	203	1328
	45	30	63	82	173	919	30	66	82	181	935	29	63	79	173	950
	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
200	30	25	75	68	205	1298	24	71	66	195	1306	24	72	66	197	1315
	45	29	60	79	164	864	29	64	79	175	883	28	60	77	164	897

Table 5-B Optimal for payment after inventory depletion (Case II), $C_b = 10$

		$\alpha = .02, \ \beta = 1.5$					$\alpha = .02, \ \beta = 2$					$\alpha = .02, \ \beta = 2.5$				
C_b	M	T_1	T	Q_1	Q	TC	T_1	T	Q_1	Q	TC	T_1	T	Q_1	Q	TC
	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
120	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	45	35	72	96	197	1053	36	75	99	205	1042	36	74	99	203	1039
	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
150	30	28	77	77	211	1325	28	75	77	205	1318	29	80	79	219	1317
	45	32	67	88	184	992	32	70	90	192	1006	33	69	90	189	1020
	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
180	30	26	75	71	205	1310	26	73	71	200	1303	26	73	71	200	1301
	45	30	63	82	173	919	31	66	85	181	907	31	65	85	178	903
	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
200	30	25	75	68	205	1298	25	73	68	200	1289	25	72	68	197	1287
	45	29	60	79	164	864	29	58	79	159	852	30	63	82	173	849

where ‘ - ’ represents infeasible solution.

Sensitivity Analysis

For C_b = constant (= 20) (Table 1-A, 1-B, 1-C, 2-A, 2-B, 2-C)

Tables 1-A, 1-B and 1-C depict the sensitivity of scale parameter α, (shape parameter β = constant) with various combinations of M and t_1 (for a constant C_b). Table 1-A reveals that (for less expensive items and for particular value of c and α) in case of simultaneous increase in the values of M and t_1 (such that the difference $(t_1 - M)$; the period of payable low interest is constant); the positive stock period (T_1), cycle time (T), Q_1 (quantity consumed in time T_1), Q (the order quantity) increases, while the total cost (TC) and $(T_1 - t_1)$ period of payable high interest decreases. As the value of α increases, the values of T_1, T, Q_1, Q and $(T_1 - t_1)$ decreases and TC of the inventory system increases. For more expensive items, the values of T_1, T, Q_1, Q and $(T_1 - t_1)$ decreases and the cost of the inventory system increases considerably.

Table 1-B is obtained for an increasing trend in the value of $(t_1 - M)$. It is evident that greater the value of $(t_1 - M)$, greater the value of T_1, T, Q_1, Q is obtained, also for high values of $(t_1 - M)$, TC and $(T_1 - t_1)$ falls sharply. For high values of c no feasible solution is obtained.

Table 1-C is obtained for a constant M and increasing value of t_1. It is observed that will the increase in the value of t_1 (M kept constant), TC and $(T_1 - t_1)$ decreases, although the values of T_1 and T does not alter considerably. In this case also, no feasible solution is obtained for high values of c.

A similar sensitivity analysis is carried out for shape parameter β, in table 2-A, 2-B and 2-C. A trend similar to that for the sensitivity of α is observed in the values of T_1, T, Q_1, Q and TC with the same combinations of M and t_1. It can also be observed that as β increases the inventory system becomes less sensitive.
For C = constant (= 20) (Table 3-A, 3-B, 3-C, 4-A, 4-B, 4-C)

Tables 3-A, 3-B and 3-C show sensitivity analysis for α (β = constant), with the same combinations of M and T_1 (as for the case C_b = constant). It is clear from table 3A that (for low backorder cost and particular value of C_b and α), values of T_1, T, Q_1, Q increase, while TC and $(T_1 - t_1)$ decreases. As the value of α increases, the values of T_1, T, Q_1, Q and $(T_1 - t_1)$ decreases and TC increase. For high backorder cost of items the values of T_1 and T becomes identical. Also for high values of backorder cost the period $(T_1 - t_1)$ increases as compared to low backorder cost of items. Table 3-B shows that (for an increasing $(t_1 - M)$), greater the value of $(t_1 - M)$, greater the value of T_1, T, Q_1, Q is obtained, also TC in this case decreases.

In contrast to Table 1-B, it can also be noticed that the solution obtained is feasible even for large values of C_b. Table 3-C also shows the same trend as in table 1-C, with the only difference that the solution obtained is feasible even for the large values of C_b. A similar sensitivity analysis is carried out for shape parameter β in table 4-A, 4-B and 4-C. A trend similar to that in table 3-A, 3-B and 3-C is observed. Also the inventory system is less sensitive as β increases.

Table 5-A, 5-B is concerned with case II when $M_1 \geq T_1$, it reveals that a purchaser prefers to buy goods more frequently in smaller lots as the permissible delay period M increases. So that the revenue from sales can be invested for a longer period.

Conclusion

We have obtained an inventory model with constant demand, Weibull distribution deterioration, and shortages and with two-stage interest payable criterion. The result shows that for constant backorder cost as the value of unit cost per item increases the cycle time, period of high interest decreases and total cost of the inventory system increases for different combinations of t_1 and M. Also for constant value of unit cost per item as the value of backorder cost increases the cycle time, total cost and the period of high interest increases.

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Other Information

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Last date of submission of the full paper: December 30, 2010.

Date of communication for finalists: March 31, 2011.

Date of presentations: July 10, 2011.

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