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Operations Research is the application of the methods of science to complex problems arising in the direction and management of large systems of men, machines, materials and money in industry, business government and defence. The distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as chance and risk, with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management determine its policy and actions scientifically.

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Guest Editorial

The 20th National Conference of Australian Society for Operations Research incorporating the 5th International Intelligent Logistics System Conference was held on the Gold Coast, Australia, in September 2009. It is our honour, on behalf of the Australian Society for Operations Research to present these special post-conference issues, which provide a unique opportunity to maintain currency with Operations Research issues in Australia and other parts of the world. An encouraging feature of the papers is the breadth they cover in both theory and application. These special issues contain a range of papers dealing with different areas relating to the theme of the conference "Making the Future better by Operations Research". The majority of them deal with application and analysis. Some of the papers are theoretical and discuss the techniques required to analyse real life applications. As a result, the topics covered in these papers highlight the diversity of the applications of Operations Research techniques.

In this final issue, Paul Corry and Erhan Kozan extend their previous study on Machine Layout Problem. Ant Colony Optimisation is adapted in solving more complex industry problem. Sujan Piya, Katsuhiko Takahashi and Katsumi Morikawa investigate the order acceptance decision in a make-to-order system, and propose a methodology in assisting the manufacturer's offer and the customer's counter-offer. Yumi Tadano, Hidenori Kawamura, Keiji Suzuki and Azuma Ohuchi study the analytic hierarchy process (AHP) and develop the "comparison support method" for evaluating alternatives. Wahyudi Sutopo, Senator Nur Bahagia, Andi Cakravastia and T.M.A. Ari Samadhi investigate the staple food distribution problem in agricultural industry and the market intervention by government. A buffer stock model is developed by combining both econometrics and non-linear programming formulation.

The editors of the special issues wish to express their appreciation to all authors for the contribution of their latest findings to Operations Research. We would also like to thank Dr Andrew Higgins, Dr Azharul Karim, Professor Charles Newton, Dr Gaurav Singh, Dr John Betts, Dr Layna Groen, Dr Leonid Churilov, Dr Lorey Marquez, Dr Monica Barbu, Dr Paul Corry, Dr Robert Burdett, Dr Rodolfo García-Flores, Professor Roger Braddock, Dr Shi Qiang (Samuel) Liu, Dr Thang Cao, Dr Wayne Philip, Dr Yarkov Zinder, Dr Yi Yue, and anonymous reviewers for the involvement of the reviewing process in ensuring the maintenance of the highest scientific standards for these special issues. The reader is reminded that the contents prepared by the author were electronically reproduced for publication. Therefore, the views and opinions are those of the authors. Anyone with questions about a paper should contact the authors.

The editors thank

Guest Editors
Erhan Kozan and Andy Wong

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Adapting Ant Colony Optimization to a Real Machine Layout Problem

Paul Corry[1] and Erhan Kozan[2]
[1]AECOM, Australia
[2]School of Mathematical Sciences, Queensland University of Technology, Australia
(Email: Paul.Corry@aecom.com; e.kozan@qut.edu.au)

Abstract

Machine layout problems describe the arrangement of machines with fixed geometries to minimise material handling costs. Previous work has applied an ant colony optimisation (ACO) algorithm to machine layout problems and found ACO to be superior to an existing technique. This study adapts the ACO layout algorithm to a more complex machine layout problem originating from industry. The problem is particularly difficult because of additional geometric and technical constraints. Computational experiments determine good values for some of the control parameters and compares variations of the algorithm.

Introduction

Material handling costs contribute a significant portion of total operating expenses in any manufacturing facility. This is particularly true in a general job shop where many product types are manufactured. A job shop consists of a number of machines and each product requires processing on several of these machines. In this environment it is possible to minimize material handling by arranging machines in an appropriate way. The problem of determining such an appropriate arrangement is called the *machine layout problem* (MLP). A good MLP solution has groups of machines with high rates of product transfer in close proximity.

For some applications of MLP it is appropriate to use an assignment problem formulation. Given M machines, and M possible locations determine the best machine-location assignments that minimise material handling costs. Although this problem is NP-complete (Sahni and Gonzalez 1976), the simple structure means that optimal solutions can be obtained in reasonable time. More recently Wu et al (2007) have applied a genetic algorithm to this problem and integrate with the cell formation and scheduling problems. Two studies by Solimanpur et al (2005) and Hani et al (2007) both apply ant colony optimisation to machine location assignment problems. The former study considers a single row configuration whilst the latter considers a variety of configurations and handling equipment. Samarghandi and Eshgi (2010) also consider a single row configuration facility and propose a tabu search algorithm.

Yang and Peters (1998) proposed a machine layout model that included the geometric constraints that are important considerations in many applications. Because the mixed-integer formulation is computationally prohibitive for real-sized problems, a heuristic was used to reduce the original problem. Corry and Kozan (2003) proposed an ant colony optimisation (ACO) algorithm for the machine layout problem (MLP-ACO). Their study compared the performance of MLP-ACO with the heuristically reduced integer programming of Yang and Peters. ACO was found to generate improved layouts. Solimanpur and Jafari (2008) formulate a mixed-integer nonlinear model for determining the optimum layout of machines in a two dimensional area and propose an algorithm based on branch-and-bound to find the optimal solution. Bock and Hoberg (2007) address the MLP with fixed and irregular machine geometries and propose several heuristic approaches. Xie and Sahinidis (2008) developed a branch and bound algorithm for the classical 2D facility layout problem and demonstrate a significant improvement in performance against the Yang and Peters (2008) method on the same problem instances. Solimanpur and Jafari (2008) also solve the classical 2D facility layout problem using their own branch-and-bound approach to find optimal solutions which performed well on small and medium sized problems.

The present study adapts MLP-ACO of Corry and Kozan to a particular problem originating from industry. Unlike the layout problems previously solved by MLP-ACO, this study must consider additional physical realities. MLP-ACO was chosen to for this study because of its flexibility and demonstrated ability to find good-quality solutions.

The next section describes a mathematical model for the considered problem and relates it to previous studies. This is followed by a description the MLP-ACO algorithm of and how it is adapted to for this study. After this, MLP-ACO is evaluated in a series of numerical experiments. The paper concludes with some closing remarks.

Model Formulation

The concept behind the model is to represent machines as rectangular objects to be arranged within a rectangular grid. All machine boundaries must be parallel to the boundaries of the rectangular containment area. Without this restriction, the geometric constraints of the problem are profoundly more complicated. Within this geometric framework, machines are to be arranged to minimise material handling costs.

An additional consideration new to this study is to ensure that removal of waste material is possible. To enable fork-lift access, the waste bin of a machine must be adjacent to an aisle. This constraint is implemented as a penalty function to the objective. Other environmental considerations include roof supports, storage areas, aisles and unmovable machines, which are all obstacles in the layout. The problem can be modelled as a non-linear mixed-integer-programming problem for minimising total distance-flow cost.

The following assumptions have been made for the machine layout problem:

i. all machines are of fixed rectangular geometry;
ii. all machines must be contained within a fixed rectangular area called the *floor*;
iii. each machine can assume one of four orientations 0°, 90°, 180° and 270° such that machine boundaries are parallel to the boundaries of the floor;
iv. floor dimensions, machine dimensions and coordinates of machine vertices must be measured in integral units.

For a given floor with dimensions W by H, Figure 1 describes the parameters and variables associated with geometric aspects of the problem. The diagram shows a machine denoted by i in its four possible orientations represented by binary variables z_i^0, z_i^{90}, z_i^{180} and z_i^{270} corresponding to orientations of 0°, 90°, 180° and 270° respectively. Parameters relating to machine geometry are all given relative to an orientation of 0°. These include the width w_i and height v_i, location of the part loading point (a_i, b_i) and the location of the waste bin (a_i^S, b_i^S). Both of these coordinates are given relative to the lower left vertex of the machine.

Figure 1 also describes the problem variables. These include coordinates of the machine origin, loading point and waste bin. Unlike the parameters mentioned previously, these points are described in absolute coordinates (that is, relative to lower left vertex of the floor). Figure 1 demonstrates how these points change when the machine is rotated. Note that the machine origin does not rotate but always remains the lower left vertex of the machine. Conversely, the loading point and waste bin do rotate with the machine.

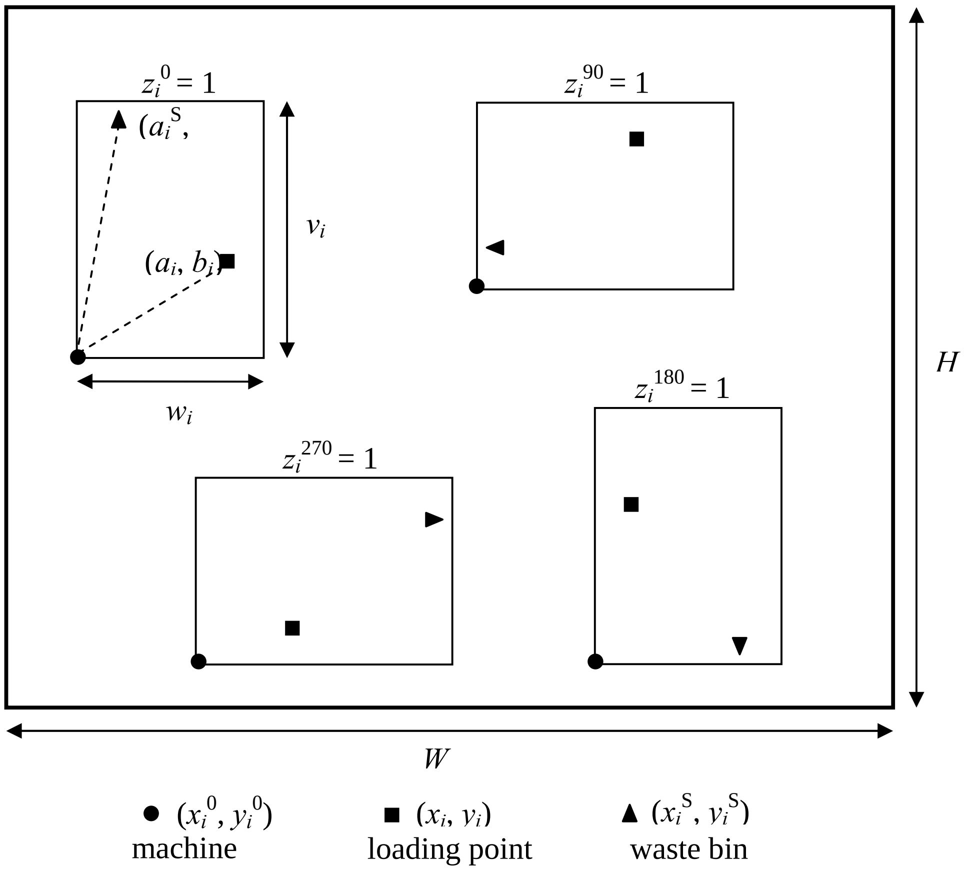


FIG. 1. *Parameters and variables associated with layout geometry.*

A formal list of variables and parameters used to describe the machine layout problem is given below.

<u>Input Parameters</u>

n	number of machines in layout
W	width of floor (x – direction)
H	height of floor (y – direction)
w_i	width of machine i (x – direction)
v_i	height of machine i (y – direction)
(a_i, b_i)	position of loading point relative to machine origin when $z_i^0 = 1$
s_i	= 1 if machine i requires waste removal, 0 otherwise
(a_i^S, b_i^S)	position of machine waste bin relative to machine origin when $z_i^0 = 1$

F_{ij} amount of material flow between machines i and j

$S(x_i^{\mathrm{S}}, y_i^{\mathrm{S}})$ waste removal penalty function for machine i

M arbitrarily large number

$\Lambda = \{(i, j) \mid i = 1, \ldots, n \wedge j = i+1, \ldots, n \wedge i \neq j\}$

Decision Variables

z_i^0 = 1 if machine i orientation at 0° (shorter side at bottom), 0 otherwise

z_i^{90} = 1 if machine i orientation at 90° (longer side at bottom), 0 otherwise

z_i^{180} = 1 if machine i orientation at 180° (shorter side at bottom), 0 otherwise

z_i^{270} = 1 if machine i orientation at 270° (longer side at bottom), 0 otherwise

(x_i^0, y_i^0) origin of machine i (vertex closest to the floor origin (0,0))

Other Variables

(x_i, y_i) coordinates of the part loading point of machine i

$(x_i^{\mathrm{S}}, y_i^{\mathrm{S}})$ coordinates of the waste bin of machine i

$\delta_{ij}^{\mathrm{lft(rht)}}$ = 1 if machine i is left (right) of machine j, 0 otherwise

$\delta_{ij}^{\mathrm{blw(abv)}}$ = 1 if machine i is below (above) of machine j, 0 otherwise

Model

$$\min \sum_{(i,j)\in\Lambda} F_{ij}\left[\left|x_i - x_j\right| + \left|y_i - y_j\right|\right] + \sum_{i=1}^{n} s_i S(x_i^{\mathrm{S}}, y_i^{\mathrm{S}}) \quad (1)$$

where

$$x_i = (x_i^0 + a_i)z_i^0 + (x_i^0 + v_i - b_i)z_i^{90} + (x_i^0 + w_i - a_i)z_i^{180} + (x_i^0 + b_i)z_i^{270} \qquad i = 1, \ldots, n \quad (1a)$$

$$y_i = (y_i^0 + b_i)z_i^0 + (y_i^0 + a_i)z_i^{90} + (y_i^0 + v_i - b_i)z_i^{180} + (y_i^0 + w_i - a_i)z_i^{270} \qquad i = 1, \ldots, n \quad (1b)$$

$$x_i^{\mathrm{S}} = (x_i^0 + a_i^{\mathrm{S}})z_i^0 + (x_i^0 + v_i - b_i^{\mathrm{S}})z_i^{90} + (x_i^0 + w_i - a_i^{\mathrm{S}})z_i^{180} + (x_i^0 + b_i^{\mathrm{S}})z_i^{270} \qquad i = 1, \ldots, n \quad (1c)$$

$$y_i^{\mathrm{S}} = (y_i^0 + b_i^{\mathrm{S}})z_i^0 + (y_i^0 + a_i^{\mathrm{S}})z_i^{90} + (y_i^0 + v_i - b_i^{\mathrm{S}})z_i^{180} + (y_i^0 + w_i - a_i^{\mathrm{S}})z_i^{270} \qquad i = 1, \ldots, n \quad (1d)$$

s.t.

$$x_i^0 + (z_i^0 + z_i^{180})w_i + (z_i^{90} + z_i^{270})v_i \leq x_j^0 + M\delta_{ij}^{\mathrm{rht}} + M\delta_{ij}^{\mathrm{blw}} + M\delta_{ij}^{\mathrm{abv}} \qquad (i,j)\in\Lambda \quad (2)$$

$$x_j^0 + (z_j^0 + z_j^{180})w_j + (z_j^{90} + z_j^{270})v_j \leq x_i^0 + M\delta_{ij}^{\mathrm{lft}} + M\delta_{ij}^{\mathrm{blw}} + M\delta_{ij}^{\mathrm{abv}} \qquad (i,j)\in\Lambda \quad (3)$$

$$y_i^0 + (z_i^{90} + z_i^{270})w_i + (z_i^0 + z_i^{180})v_i \leq y_j^0 + M\delta_{ij}^{\mathrm{lft}} + M\delta_{ij}^{\mathrm{rht}} + M\delta_{ij}^{\mathrm{abv}} \qquad (i,j)\in\Lambda \quad (4)$$

$$y_j^0 + (z_j^{90} + z_j^{270})w_j + (z_j^0 + z_j^{180})v_j \leq y_i^0 + M\delta_{ij}^{\mathrm{lft}} + M\delta_{ij}^{\mathrm{rht}} + M\delta_{ij}^{\mathrm{blw}} \qquad (i,j)\in\Lambda \quad (5)$$

$$\delta_{ij}^{\mathrm{lft}} + \delta_{ij}^{\mathrm{rht}} + \delta_{ij}^{\mathrm{blw}} + \delta_{ij}^{\mathrm{abv}} = 1 \qquad (i,j)\in\Lambda \quad (6)$$

$$z_i^0 + z_i^{90} + z_i^{180} + z_i^{270} = 1 \qquad i = 1, \ldots, n \quad (7)$$

$$0 \leq x_i^0 \leq W - (z_i^0 + z_i^{180})w_i - (z_i^{90} + z_i^{270})v_i \qquad i = 1, \ldots, n \quad (8)$$

$$0 \leq y_i^0 \leq H - (z_i^{90} + z_i^{270})w_i - (z_i^0 + z_i^{180})v_i \qquad i = 1, \ldots, n \quad (9)$$

$$z_i^0, z_i^{90}, z_i^{180}, z_i^{270} \in \{0,1\} \qquad i = 1,...,n \quad (10)$$

$$\delta_{ij}^{\mathrm{lft}}, \delta_{ij}^{\mathrm{rht}}, \delta_{ij}^{\mathrm{blw}}, \delta_{ij}^{\mathrm{abv}} \in \{0,1\} \qquad (i,j) \in \Lambda \quad (11)$$

$$x_i^0, y_i^0 \text{ integers} \qquad i = 1,...,n \quad (12)$$

In the objective function, equation (1), the first term measures the total flow-weighted-distance between machines. The second term is a problem specific function designed to penalise layouts that hinder waste removal. In a later section, the waste removal penalty function used for this study is defined. All distances are measured using a rectilinear norm, since parts will not travel in straight lines with many rectangular obstacles present.

Loading points and waste bin points for each machine are determined by equations (1a) through (1d). The binary orientation variables ensure the correct calculations are initiated for loading and waste bin points. Equations (2) to (5) are disjunctive constraints that prevent every pair of machines from overlapping. A machine *i* can be either left of, right of, above or below a machine *j* and there is a constraint for each case. Equation (6) ensures that only one disjunctive constraint is activated for any given machine pair *ij*. There is some overlap between the feasible regions defined by the binary variables. For example, it is possible for a machine *j* to be left of and below another machine *i*. In this case, either $\delta_{ij}^{\mathrm{lft}}$ or $\delta_{ij}^{\mathrm{blw}}$ could be active. Similar to (6), equation (7) ensures that each machine is assigned one distinct orientation. The remaining constraints, equations (8) to (12), establish bounds on the variables. In particular, (8) and (9) restrict machine to be inside the shop floor. Condition (12) enforces a discrete domain for the decision variables, which is necessary for the application of MLP-ACO to solve the model.

This modelling approach is similar to that used by Corry and Kozan (2003). Apart from the addition of the waste removal penalty function, there is one significant extension on the previous models. Machine layout problems previously studied, assume that parts are loaded and unloaded at machine centroids, which is usually justified given the size of machines relative to the floor space (Heragu and Kusiak 1988). The proposed model allows for large machines that have parts loaded onto one side, away from the centroid. It is inefficient to have a pair of large machines close together for high volumes of flow if their part loading points are at opposite ends. Therefore, the centroid assumption is removed increasing the number of allowed orientations from two to four. This extension is also important for satisfying the requirement of access to waste bins.

ACO for Job Shop Layouts

When applying a search algorithm to machine layout problems, the search space is quite irregular because of the geometry constraints. Adapting general-purpose meta-heuristics is not strait-forward as with combinatorial problems like travelling salesman problems, quadratic-assignment and job-shop scheduling. MLP-ACO was proposed in Corry and Kozan (2003) and is extended in this study to cope with additional features required for the proposed model. The modified algorithm is called Job Shop Layout – Ant Colony Optimisation (JSL-ACO).

Ant colony optimisation (Dorigo and Di Caro 1999, Dorigo et al 1999) is an optimisation strategy that shares computational resources amongst a virtual colony of cooperating ants. For an ACO problem, there must be a representation consisting of a finite set of solution components linked in a network by a set of connections. Virtual ants will wander through the network in a controlled manner, searching for paths of lowest cost. Every path completed by an ant corresponds to a solution of the given problem. Each ant deposits pheromone along the connections in its path, whilst pheromone accumulated from previous ant journeys gradually evaporates. The intensity of an ant's pheromone deposit increases with the quality of solution represented by its path. Individual ants use the pheromone trails and a local heuristic to guide their journeys through the network of solution components. By this method the colony shares global information that highlights favourable regions of the search neighbourhood. In the final iterations of the algorithm, the pheromone deposits favour connections that belong to a small number of low-cost solutions. By this stage ant exploration is concentrated in a small neighbourhood of solutions and the algorithm can be terminated.

Graph Representation of MLP-ACO

Every layout constructed in MLP-ACO is based on two kinds of decision, the *order* in which machines are positioned and the *location* of each machine. Based on these decisions, a graphical framework is constructed as a medium for pheromone communication. The graphical framework used in JSL-ACO has not been modified from the original MLP-ACO algorithm.

There are two kinds of node in the layout graph, machine order nodes and grid region nodes. An ant defines its next machine to position by travelling to the order node for that machine. The machine location is defined when the ant selects a set of *grid region* nodes that will represent the area to be occupied by the machine. A grid region in the x'th column and y'th row of the grid is denoted by $[x,y]$, ($x = 1, ..., W$ and $y = 1, ..., H$).

Figure 2 gives an example of a machine layout expressed using graphical framework of MLP-ACO. Three machines have been positioned in the order of machine 1, then machine 2 and finally machine 3. This is represented by the sequence of arcs (1, 2) and (2, 3) between the machine order nodes. Emanating from each machine order node are arcs to the grid region nodes corresponding to grid regions occupied by that machine. In this way a machine is like a stamp, and stamps its position on the grid.

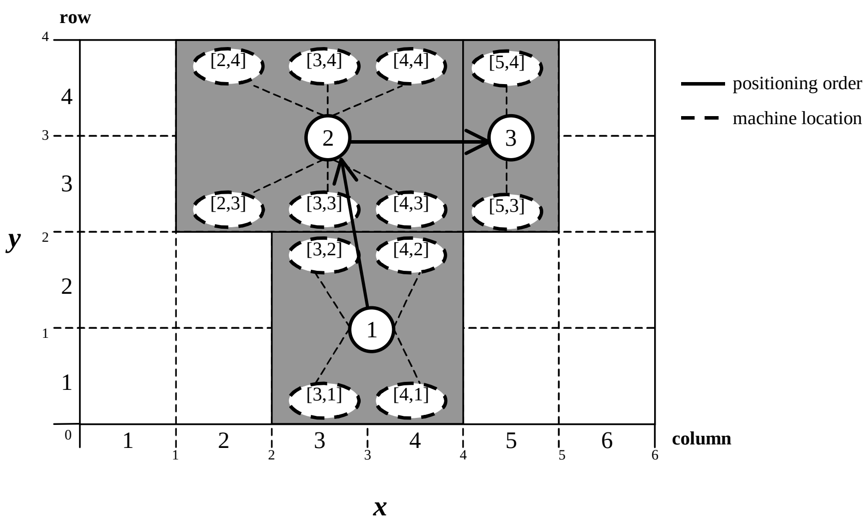


FIG. 2. *Example of an MLP-ACO ant's layout path.*

The non-serial structure of solution paths is more complex than typical ACO applications and does not facilitate building feasible solutions. Feasibility maintenance must be embedded into the search procedure. An innovative procedure to track vacant areas of the floor was developed for the original MLP-ACO algorithm. Readers are referred to Corry and Kozan (2003) for details of this approach.

In total there are $n(n\text{-}1)$ arcs linking machine nodes and $(WH\text{-}1)n$ arcs linking machine nodes to grid region nodes. An accumulated pheromone value must be stored for each arc in the graph. For this reason the graph should be kept as small as possible to gain the most efficiency from the algorithm. The following notation is used to describe the MLP-ACO algorithm.

Action Choice Rules

Virtual ants are controlled by an *action choice rule* that guides their exploration of the graph. It is a stochastic decision process favoring locally attractive decisions as well as those known from experience to be good. Consider a virtual ant that has partially constructed a solution is currently situated at some node *i*. To select the next node(s) to visit, the ant's decision is made using the action choice rule. Since there are two types of decision to be made in layout construction, (order and position) there are two action choice rules used in MLP-ACO.

The local gain of moving to any given node is determined using some function based on a heuristic. In MLP-ACO, two heuristic functions are used, $H_{\text{ord}}(\cdot)$ and $H_{\text{pos}}(\cdot)$ for machine order and position respectively. These functions are defined in a later section. The amount of pheromone accumulated on a give arc is denoted by $trail(\cdot)$ which is used with the heuristic functions in the action choice rules. The relative importance of pheromone and heuristic functions is controlled through setting of the parameters δ and β respectively.

Equation (13) is the rule used by ants to decide the order in which they position machines. It refers to an ant *q* currently at node *i* after machine *i* has already been positioned. Ant *q* is about to decide which machine node to visit next. Let Fre_i^q be the set of machine nodes yet to be visited by ant *q*. Also let *R* be a random variable uniformly distributed over [0,1]. The next machine *j* visited by ant *q* is determined by.

$$j = \begin{cases} \arg\max\limits_{m \in Fre_i^q} \left\{ [trail(i,m)]^{\delta} [H_{\text{ord}}(i,m)]^{\beta} \right\} & \text{if } R \le R_0^{\text{ord}} \\ J & \text{otherwise} \end{cases} \tag{13}$$

where *J* is a machine node from Fre_i^q selected according to the probability below:

$$p_q(i,J) = \begin{cases} \dfrac{[trail(i,j)]^{\delta}[H_{\text{ord}}(i,j)]^{\beta}}{\sum\limits_{m \in Fre_i^q} [trail(i,m)]^{\delta}[H_{\text{ord}}(i,m)]^{\beta}} & \text{if } j \in Fre_i^q \\ 0 & \text{otherwise} \end{cases} \tag{13a}$$

Ant *q* generates a random variable *R* to decide whether to make a deterministic or a biased random choice. The deterministic rule chooses the machine with maximum trail-heuristic score. The stochastic rule chooses a machine randomly, giving high trail-heuristic scoring machines a better chance of selection. Tuning the parameter R_0^{ord} controls the degree of exploration by ants. Recall that δ and β are also control parameters used to control the relative influence of pheromone and heuristic information. For JSL-ACO the action choice rule (13) of machine order requires no modification from the original MLP-ACO algorithm.

After applying (13), node j will be the next node visited in the path of ant q. Machine j is then allocated a position using a rule similar to (13). Machine position, however, is defined by an ant by visiting several grid region nodes simultaneously. Therefore, the decision rule for machine position considers the pheromone accumulated on all arcs leading to nodes for the grid regions to be occupied. MLP-ACO uses the average pheromone over the arcs.

Recall that the original MLP-ACO only considered two possible machine orientations but for this study there are four. Because of this the original machine location rule must be modified for JSL-ACO. Several issues arose in applying MLP-ACO to four-orientation problems. Pheromone accumulated over an area gives an indication of good location but there is no data to assist in choosing the best orientation. Additionally, the number of possibilities considered by ants is doubled from the original two-orientation MLP-ACO. This should be avoided since pheromone has little effect when ants have many possible moves to consider (Dorigo et al 1999).

These issues have been addressed in JSL-ACO whilst keeping modifications to the original MLP-ACO to a minimum. For a potential location there are two possible orientations that would occupy the same area, for example 0° and 180°, or 90° and 270°. Ants in JSL-ACO are programmed to select the most attractive of two possible orientation as measured by the heuristic function H_{pos}. If both orientations are equally attractive, a random choice is made. This keeps the number of possible locations to consider at the same level as a two-orientation problem.

Let $Lcn_i = (\mathbf{x}_i, \mathbf{z}_i)$ denote a possible position for machine i defined by its origin $\mathbf{x}_i = (x_i^0, y_i^0)$ and orientation $\mathbf{z}_i = (z_i^0, z_i^{90}, z_i^{180}, z_i^{270})$. Given a location Lcn_i, the identical location with opposite orientation is denoted $\overline{Lcn_i}$ and called the *complement* of Lcn_i. For example, if $Lcn_i = (\mathbf{x}_i^0, [1,0,0,0])$ then $\overline{Lcn_i} = (\mathbf{x}_i^0, [0,0,1,0])$. Similarly if $Lcn_i = (\mathbf{x}_i^0, [0,1,0,0])$ then $\overline{Lcn_i} = (\mathbf{x}_i^0, [0,0,0,1])$.

Let $M(Lcn_i)$ denote the set of grid regions occupied by machine i in position Lcn_i, and let Vct_i^q denote the set of vacant grid regions in the partial layout of ant q. For some machine i, the set of positions considered by an ant will be denoted by A_i^q. This contains every location in the vacant area that is determined to be more attractive than its complement.

$$A_i^q = \left\{ Lcn_i \mid M(Lcn_i) \subseteq Vct_i^q \wedge H_{\text{pos}}(i, Lcn_i) \geq H_{\text{pos}}(i, \overline{Lcn_i}) \wedge \overline{Lcn_i} \notin A_i^q \right\}$$

Equation (14) is the action choice rule used by ant q to determine a location for machine i. A new random variable $R \sim Uniform[0,1]$ is generated to decide between a deterministic or a biased random choice.

$$Lcn_i = \begin{cases} \arg\max\limits_{Lcn'_i \in A_i^q} \left\{ \left[trail_{\text{avg}}(i, Lcn'_i) \right]^{\delta} \left[H_{\text{pos}}(i, Lcn'_i) \right]^{\beta} \right\} & \text{if } R \leq R_0^{\text{pos}} \\ LCN_i & \text{otherwise} \end{cases} \tag{14}$$

where $trail_{\text{avg}}(i, Lcn_i) = \dfrac{1}{v_i w_i} \sum\limits_{[x,y] \in M(Lcn_i)} trail(i, [x,y])$

and LCN_i is a position such that $M(LCN_i) \subseteq Vct_i^q$ selected by with probability:

$$p_q(i, LCN_i) = \begin{cases} \dfrac{[trail_{\text{avg}}(i, LCN_i)]^\delta [H_{\text{pos}}(i, LCN_i))]^\beta}{\sum_{Lcn_i' \in A_i^q} [trail_{\text{avg}}(i, Lcn_i')]^\delta [H_{\text{pos}}(i, Lcn_i')]^\beta} & \text{if } LCN_i \cup A_i^q \\ 0 & \text{otherwise} \end{cases} \quad (14a)$$

The expression *trail*(i, [x,y]) denotes the pheromone accumulated between machine node i and grid region node [x, y]. After determining a position for machine i, ant q will visit the nodes to corresponding grid regions and immediately return to the node for machine i. Ant q will then reapply equation (13) to determine the next machine to position. This process continues until all machines have been positioned.

Pheromone Updating Rules

The pheromone updating rules used in MLP-ACO are a combination of ant system (Dorigo et al 1999) with an elitist strategy and the max-min ant system (Stutzle and Hoos 2000). Initially all pheromone levels are set to the value of $\tau_{\max}$. This value also sets an upper limit that pheromone levels cannot exceed. Once the ants have completed their layouts, every arc in $G(V, E)$ experiences pheromone evaporation. Any arc present in the layout path of an ant is reinforced with a pheromone deposit. The intensity of each deposit is inversely proportional to the objective evaluation of the corresponding layout. An elitist strategy is also used to reinforce the best-known solution from all previous iterations. Equation (15) is used to calculate the pheromone deposit for a given arc. Equation (16) is used to apply pheromone evaporation and the pheromone deposit calculated in (15).

$$\Delta trail(i, j) = \sum_{q \in K} \frac{U}{C_q} + e_{ij}^{\text{in}} \frac{U}{C_{\text{best}}} \quad (15)$$

$$\text{where } K = \{q \mid (i, j) \text{ is in ant } q\text{'s path on } G(V, E), q = 1, \ldots, N_{\text{ant}}\}$$

$$\text{and } e_{ij}^{\text{in}} = \begin{cases} e & (i, j) \text{ part of path in best known solution} \\ 0 & \text{otherwise} \end{cases}$$

$$trail'(i, j) = \min\left(\tau_{\max}, (1-\alpha) trail(i, j) + \alpha \Delta trail(i, j)\right) \qquad (i, j) \in A \quad (16)$$

Note that for arcs denoted (i, j), node j can represent either a machine order node or a grid region node. C_q is the objective cost of agent q's layout path. Initially set to ∞, C_{best} is the objective cost of the best layout obtained in all previous iterations. U is a parameter that controls the intensity of pheromone deposits. The parameter e represents the number of elitist ants reinforcing the best solution. α controls the amount of pheromone evaporation. Together, the tunable parameters N_{ant}, $\tau_{\max}$, U, e and α shape the evolution of the pheromone distribution during the algorithm. Some experimentation is necessary to find a good combination for these parameters. Note that the pheromone updating rules of MLP- ACO have not been modified for JSL-ACO.

Constructive Heuristic

Virtual ants use pheromone information and a local heuristic to guide their journey through the problem states. For MLP-ACO a constructive heuristic was developed to guide the ants. The first decision an ant faces is which machine to position next. For some ant q, assume that s machines have already been positioned and machine i was the most recent. Let $L_q^{(s)}$ be the set of machine already positioned by ant q after s machines have been positioned (note that $\left|L_q^{(s)}\right| = s$). Equation (17) defines the function used to evaluate the benefit of selecting a machine j as the next to be positioned.

$$H_{\text{ord}}(i,j) = \sum\nolimits_{m \in L_q^{(s)}} \varepsilon_{mj} F_{mj} \tag{17}$$

Equation (17) determines the total product flow between machine j and those machines already in the partial layout. This strategy is intended to place machines of high interaction in good locations before less appropriate machines occupy these areas. No modification has been made to H_{ord} from the original MLP-ACO algorithm.

Once machine j has been selected for positioning a second function is invoked to evaluate the benefit of potential locations. Equation (18) defines this function.

$$H_{\text{pos}}(j, Lcn_j) = \frac{1}{s_j S(x_j^S, y_j^S) + \sum\nolimits_{m \in L_q^{(s)}} F_{mj}\left[\left|x_m - x_j\right| + \left|y_m - y_j\right|\right]} \tag{18}$$

The denominator of (18) is the incremental cost of assigning machine j to the location represented by Lcn_j. Taking the sum of the incremental costs over all machines in a layout will give the total distance-flow cost as calculated by (1). The function H_{pos} has been modified for JSL-ACO because of the addition of the waste removal penalty function.

Machine Layout Ant Algorithm

Below is an ant algorithm for the job shop layout problem. One iteration of the algorithm is of complexity $O(N_{\text{ant}} N^2 WH)$.

ALGORITHM: *JSL-ACO*
** *INITIALISE* **
$trail(i,j) = \tau_{\max}\ , \quad \forall (i,j) \in E$
repeat
  ** *GENERATE SOLUTIONS* **
  start each ant on a different machine (if possible)
  for $s = 1$ **to** N **do**
    for $q = 1$ **to** N_{ant} **do**
      assign position Lcn_i to current machine i by Eq. (14)
      if $s < n$ **then** choose next machine j by Eq. (13)
      update Blk_s^q
    end for
  end for
  ** *UPDATE TRAIL LEVELS* **
  calculate $\Delta trail(i,j), \forall (i,j) \in E$ by Eq. (15)
  evaporate and reinforce $trail(i,j), \forall (i,j) \in E$ by Eq.(16)
until termination requirement met

An Application

The problem under consideration was motivated by the modernization of a large iron foundry in Toowoomba, Australia. The foundry is establishing a 26.3m × 30.55m cell for machining a variety of medium to low volume products. The cell will consist of fifteen machines. One of these machines has a fixed position and cannot be moved. Five products *A* (axle box bodies), *B* (axle box back covers), *C* (motor-pump shaft clamps), *D* (large pump volutes) and *E* (helical rotor pumps) of varying sizes, weights and production volumes, will make up the majority of work in the cell. Aisles and product arrival and departure points have been predetermined. The layout must also accommodate for all obstacles including aisles, roof supports and fixed items of infrastructure.

Movable machines must be surrounded by an area large enough to allow operator space, access to waste bins and pallets used as in-process storage between machines. Before positioning any movable machines, the obstacles must be positioned one at a time whilst maintaining a record of the vacant grid regions that remain. In addition to the physical obstacles there are two practical requirements relating to accessibility issues.

- Waste bins must be accessible from the aisles for forklift removal. The total distance travelled by the swarf removal forklifts through the cell is not considered important.
- Machines receiving or sending batches to the entry/exit points of the cell should be adjacent to an aisle if possible. Total distance travelled from the entry/exit points to the machines is of secondary importance to aisle accessibility.

Total distance from entry/exit points and to waste bines is not considered important since the distance travelled within the cell is small compared within the total distance travelled. Secondly, when this aspect is considered important, the aisle adjacency requirement is weakened.

Product flow between machines was extracted from annual production quantities obtained from the foundry. It was decided to measure the product flow as the annual quantity of units of product being transferred between machines. Other possibilities are total product weight or the number of product batches. The data was gathered into a single flow matrix which is symmetric. All data was converted to readable input for JSL-ACO, which was then applied to the problem. See appendix for problem data.

The objective function equation (1) must be adapted to the considered problem. This is done through the waste removal function $S(x_i^{\mathrm{S}}, y_i^{\mathrm{S}})$. To allow access to waste bins by forklift, the function returns a weighted-distance to the nearest aisle. Aisles are predetermined, and assumed to be either horizontal or vertical forming two sets A_{h} and A_{v} respectively. Aisles are defined by a start-finish range, location in the x direction for vertical aisles and in the y direction for horizontal aisles. A horizontal aisle $a \in A_{\mathrm{h}}$ running between $x = x_{\mathrm{st}}$ and $x = x_{\mathrm{fn}}$ along $y = y_{\mathrm{lv}}$ is defined by $\left([x_{\mathrm{st}}, x_{\mathrm{fn}}], y_{\mathrm{lv}}\right)$. Similarly a vertical aisle $b \in A_{\mathrm{v}}$ running between $y = y_{\mathrm{st}}$ and $y = y_{\mathrm{fn}}$ along $x = x_{\mathrm{lv}}$ is defined by $\left(x_{\mathrm{lv}}, [y_{\mathrm{st}}, y_{\mathrm{fn}}]\right)$. See the appendix for the aisles used in this study to represent the foundry data-set. The distance from a point (x, y) to the nearest point on an aisle a is calculated by equation (19).

$$D(a,(x,y)) = \begin{cases} |y_{\mathrm{lv}} - y| + (x_{\mathrm{st}} - x)^+ + (x - x_{\mathrm{fn}})^+, & a \in A_h \\ |x_{\mathrm{lv}} - x| + (y_{\mathrm{st}} - y)^+ + (y - y_{\mathrm{fn}})^+, & a \in A_v \end{cases} \qquad (19)$$

Equation (20) describes the waste removal function used for this study.

$$S(x_i^{\mathrm{S}}, y_i^{\mathrm{S}}) = \sigma_i \min_{a \in A_{\mathrm{h}} \cup A_{\mathrm{v}}} D(a,(x,y)) \qquad i = 1,\ldots,N \tag{20}$$

where σ_i is a scaling parameter to reflect the importance of the waste removal point of machine i being adjacent to an aisle. Experimentation revealed that $\sigma_i = 50000$ was a suitable value for ensuring that the waste removal accessibility constraint was satisfied.

At Toowoomba Foundry, it is necessary for machines that are receiving parts directly from the cell entry or sending parts to the cell exit, to be as close as possible to the aisles. For this reason a new penalty function is introduced, $IO(x_i, y_i)$ within the objective function to meet the arriving and departing parts requirement. For any given loading point, these functions measure the weighted distance to the nearest aisle. The weighting is there to reflect the importance of satisfying the constraint. Since this is a soft constraint the weighting will be small compared with that used for the swarf removal hard constraint. Equation (22) defines the input/output penalty function used in the Toowoomba Foundry objective function. This function replaces the expressions $|x_0 - x_j| + |y_0 - y_j|$ and $|x_i - x_{N+1}| + |y_i - y_{N+1}|$ in the objective function (1).

$$IO(x_i, y_i) = \mu_i \min_{a \in A_h \cup A_v} D(a,(x,y)) \qquad i = 1,\ldots,N \tag{22}$$

The parameter μ_i is the weighting used to reflect the importance of satisfying the soft constraint. For this study the value $\mu_i = 1.0$ was used. The objective function used in conjunction with the foundry data is given below.

$$Z = \sum_{(i,j)\in\Omega} \varepsilon_{ij} F_{ij} \left[|x_i - x_j| + |y_i - y_j| \right] + \sum_{i=1}^{N} \left[s_i S(x_i^{\mathrm{S}}, y_i^{\mathrm{S}}) + IO(x_i, y_i) \right] \tag{23}$$

where $\Omega = \{(i,j) \mid i = 1,\ldots,N-1 \wedge j = i+1,\ldots,N \wedge i \neq j\}$

Control Parameter Settings

This section investigates the impact of two control parameters of JSL-ACO. There are nine tunable parameters to be considered, which are $[\delta, \beta, R_0^{\mathrm{ord}}, R_0^{\mathrm{pos}}, U, \alpha, N_{\mathrm{ant}}, e, \tau_{\mathrm{max}}]$. Because JSL-ACO is computationally intensive, experimental optimisation of these parameters would be a time consuming process. For this study, most of the parameter values were determined using experience and informal experimentation. Values for the remaining two parameters, e (number of elitist ants) and $\tau_{\max}$ (maximum pheromone limit) were chosen based on organized experimentation. The experiment tested all combinations from 5 values of e and 5 values of $\tau_{\max}$. For each of 25 parameter combinations, JSL-ACO was run 5 times each starting with a different random seed. Each run is terminated after 750 iterations. Table 1 gives the averages obtained and Table 2 gives the best solutions found for each parameter setting. Table entries in bold type are equal to the best result obtained. For each parameter setting in Table 2, the number of times a layout of lowest known cost was obtained is given in parentheses.

The parameter settings achieving the lowest average cost were $[\delta, \beta, R_0^{\mathrm{ord}}, R_0^{\mathrm{pos}}, U, \alpha, N_{\mathrm{ant}}, e, \tau_{\mathrm{max}}]$ = [1, 3, 0.5, 0.8, 24000, 0.025, 50, **25**, **6**]. Most of the parameter settings tested obtained a best solution (Z = 218032) at least once in the five trials. One of these solutions is displayed in figure 3. Other solutions with Z = 218032 only differ by a few machines that can move slightly without affecting the overall objective cost. Table 2 also indicates that the above parameter settings achieved the best results. They were the only settings that obtained an equal best solution in every trial.

TABLE 1. *Averages of five JSL-ACO runs for each combination of parameter settings.*

	e				
τ_{max}	0	13	25	38	50
0.5	228190.4	225723.6	224152.4	224155.6	222446.6
2	224293.4	224287.8	220140.6	222265.6	222109.6
4	223429.4	220362.2	220362.2	221334.2	219530.0
6	224275.0	229874.8	***218032.0***	219833.0	222632.6
8	239224.4	226837.6	230421.2	224529.0	219533.2

TABLE 2. *Best of five JSL-ACO runs for each combination of parameter settings. Numbers in parentheses indicate the number of times a layout of cost 218032 was obtained.*

	e				
τ_{max}	0	13	25	38	50
0.5	224647 (0)	220678 (0)	220881 (0)	220678 (0)	220678 (0)
2	218032 (1)	218032 (1)	218032 (3)	218032 (3)	218032 (1)
4	218032 (1)	218032 (3)	218032 (3)	218032 (3)	218032 (3)
6	220678 (0)	218032 (2)	***218032 (5)***	218032 (4)	218032 (3)
8	218032 (1)	218032 (3)	218032 (1)	218032 (2)	218032 (4)

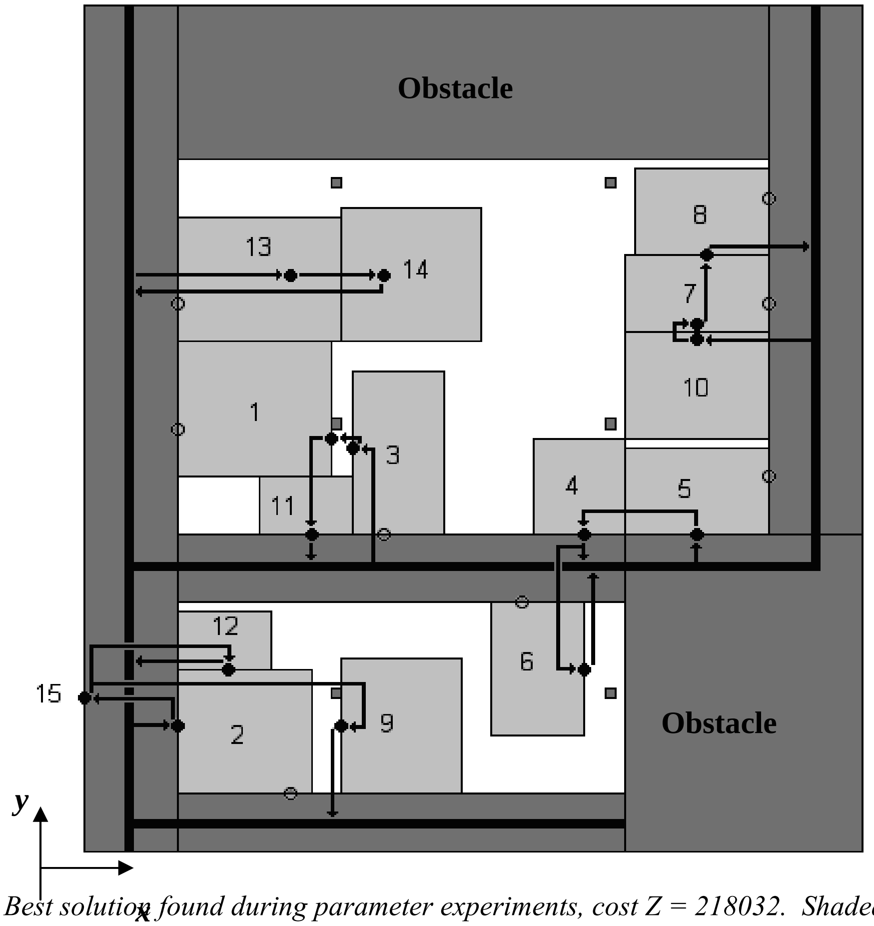


FIG. 3. *Best solution found during parameter experiments, cost Z = 218032. Shaded regions with numbers indicate the location of corresponding machines. Solid circles represent part loading points and hollow circles are swarf removal points. Shaded regions without numbers represent obstacles. Bold lines represent aisles and directed lines indicate product flows. White areas represent vacant space.*

Performance Comparisons

To assess the performance of JSL-ACO, variations of the algorithm were compared. Firstly, the algorithm was run five times with pheromone reading switched off, that is $\delta = 0$. Secondly, a greedy algorithm based on the JSL-ACO heuristic function was tested. The greedy heuristic constructs fourteen layouts each beginning from a different machine. During the construction of each layout, the next available machine to be positioned is the highest scoring H_{ord} evaluation. The location chosen for the selected machine scores the highest H_{pos} evaluation out of the available locations. These steps are repeated for each layout until all machines have been positioned. This algorithm can be achieved by one iteration of JSL-ACO with the parameter settings $[\delta, \beta, R_0^{\text{ord}}, R_0^{\text{pos}}, U, \alpha, N_{\text{ant}}, e, \tau_{\max}]$ = [0, 1, 1, 1, n/a, n/a, 14, n/a, n/a], where "n/a" means "not applicable". A comparison of these approaches is given in Table 3 below.

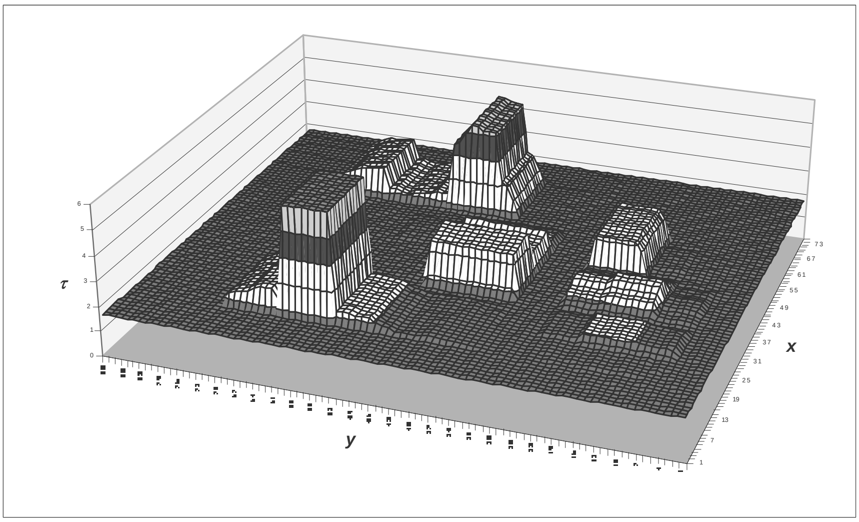
TABLE 3. *JSL-ACO performance comparisons (%>Best = 100*(Min-218032)/218032).*

	Avg	Min	%>Best
$\delta = 0$	227324.8	226132	3.7
Greedy	505356.3	260501	19.5

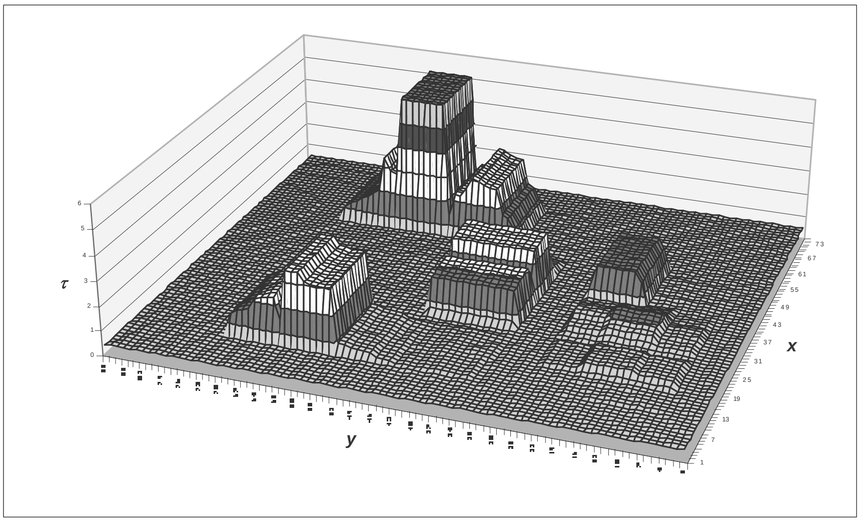
The values quoted for the greedy heuristic are the average and minimum costs obtained from 14 layouts, each starting with a different machine. One of these layouts was infeasible and is not included in the calculation of the average result. The minimum cost layout was started with machine 4. Neither of the two JSL-ACO variations obtained an objective cost as low as the best obtained in the parameter experiments. This result confirms that pheromone communication had a positive effect on the quality of solution found.

Pheromone Distributions

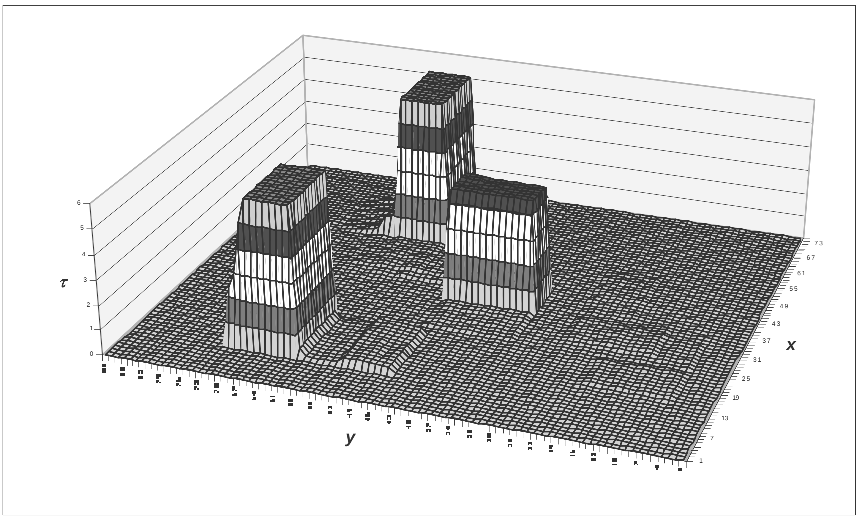
This section illustrates the progression of JSL-ACO by graphically displaying the pheromone distribution of machine 7 as the algorithm progresses. Figure 4 is a series of 3-dimensional graphs representing the shop floor in the horizontal plane and pheromone level in the vertical direction. These results were recorded at various stages during the JSL-ACO run that generated Figure 3.



(a)



(b)



(c)

FIG. 4. *Graphs (a),(b) and (c) represent the pheromone distributions of machine 7 recorded at 50, 100 and 750 iterations respectively.*

As the JSL-ACO run progressed Figures 4(a)-(c) show the ants narrowing their search to a few favoured locations. At 50 iterations, Figure 4(a) shows four main areas of exploration each covering an area of two or three machines. The only major difference between Figures 4 (a) and (b) is that ants had changed their favoured location from an area at the bottom of the graph to an area at the top. From iterations 100 to 750, evaporation erodes the edges of favoured regions until they cover only a slightly bigger area than machine 7. After 750 iterations, Figure 4 (c) shows tight variations around three distinct locations being the focus of ant exploration. There is also evidence of limited exploration in other areas. Another observation from Figure 4 is that ants are only investigating locations adjacent to aisles because machine 7 requires waste removal. This is because the waste removal penalty function $S(x_i^S, y_i^S)$ makes interior locations highly unattractive.

<u>CPU Times</u>

The JSL-ACO algorithm was implemented in the C language using a *gcc* compiler on a SGI Origin 3000 Supercomputer. The CPU time required to complete a run of 750 iterations with 50 agents was about 105 minutes. On average, each run had achieved its best solution by 228 iterations, however some runs required about 700 iterations to achieve their best solution. Although the computational burden of JSL-ACO is significant, it is quite acceptable in the context of machine layout problems. In real life scenarios, planning layouts is a long-term project so that CPU time is not a critical factor, unlike applications such as production scheduling.

Conclusions

This paper demonstrates the application of ant colony optimization to a real machine layout problem. An ACO algorithm from a previous study was modified to deal with the added complexities of a real production environment. Experimental results determined suitable values for some of the control parameters and demonstrated the benefit of pheromone communication in achieving better solutions.

An important enhancement to previous machine layout models from the literature was to allow four possible machine orientations instead of two. This enables more realistic problems to be formulated such as the problem addressed in this study. Overall, this paper has shown that ACO can be successfully adapted to realistic machine layout problems.

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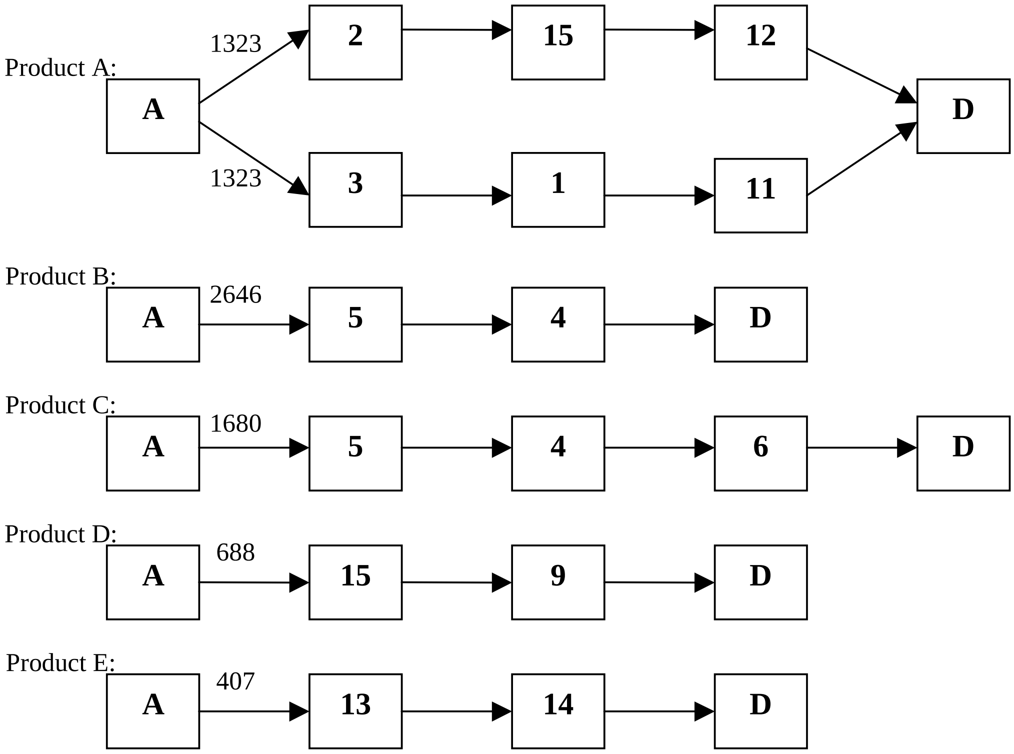
Appendix: Problem Data

Machine data.

Index	Dimensions		Origin	Load Pt	Waste Pt	Description
i	w_i	v_i	(x_i, y_i)	(x_{il}, y_{il})	(x_{is}, y_{is})	
1	15	14	-	(15, 4)	(0, 5)	movable machine
2	13	13	-	(6, 0)	(13, 11)	movable machine
3	17	9	-	(8, 0)	(17, 3)	movable machine
4	9	10	-	(5, 0)	-	movable machine
5	14	9	-	(7, 0)	(14, 6)	movable machine
6	14	9	-	(7, 0)	(14, 6)	movable machine
7	14	8	-	(7, 0)	(14, 3)	movable machine
8	13	9	-	(7, 0)	(13, 6)	movable machine
9	14	11	-	(7, 0)	-	movable machine
10	14	11	-	(7, 0)	-	movable machine
11	9	6	-	(5, 0)	-	movable machine
12	9	6	-	(5, 0)	-	movable machine
13	16	13	-	(5, 6)	(16, 9)	movable machine
14	14	13	-	(7, 4)	-	movable machine
15	0	0	(0, 16)	(0, 0)	-	fixed machine
16	57	16	(9, 72)	-	-	storage area
17	23	33	(52, 0)	-	-	aisles and occupied area
18	9	55	(66, 33)	-	-	aisle: (66, [33,72])
19	9	88	(0, 0)	-	-	aisle: (9, [6,72])
20	43	7	(9, 26)	-	-	aisle: ([9,66], 26) & ([9,66], 33)
21	43	6	(9, 0)	-	-	aisle: ([9,52], 6)
22	1	1	(24, 16)	-	-	roof support
23	1	1	(50, 26)	-	-	roof support
24	1	1	(24, 44)	-	-	roof support
25	1	1	(50, 44)	-	-	roof support
26	1	1	(24, 69)	-	-	roof support
27	1	1	(50, 69)	-	-	roof support
28	15	14	(9, 33)	(15, 4)	(0, 5)	fixed machine

Product flow chart



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Negotiation with Customer Priority and Dynamic Aspiration Level for Order Acceptance Decision

Sujan Piya, Katsuhiko Takahashi and Katsumi Morikawa
Hiroshima University, Higashi-Hiroshima, JAPAN
(Email: sujanpiya@hiroshima-u.ac.jp ; takahasi@hiroshima-u.ac.jp;
mkatsumi@hiroshima-u.ac.jp)

Abstract

Order acceptance (OA) decision in a make-to-order (MTO) system is usually the result of negotiations between the customer and the manufacturer. Extending the model of Piya et al. 2009, this paper proposes a new method to negotiate with the customer on the contested issues. In the proposed method, the aspiration level is defined as a function of time and the distance between the manufacturer's offer and the customer's counter-offer. The method utilizes the geometry between the offer and the counter-offer to determine the expected slope of the customer, thus assisting the manufacturer to understand the customer's priorities regarding different issues. In addition, three different strategies are introduced for generating new offers during each negotiation round. Numerical analysis is presented to illustrate the working mechanism of the proposed method and the effectiveness of each strategy under various conditions.

Key words: Order acceptance, Negotiation, Dynamic aspiration level, Customer's expected slope

Introduction

Order Acceptance (OA) is where the manufacturer has to decide whether to accept or reject an incoming order. One of the main operational issues of the MTO system (Chetan et al. 2004), it involves co-ordination between an organisation's marketing and production sections. It includes such activities as customer enquiries, interaction with the customer to fix the due date and price, production planning and physical distribution. Over the last decade the significance of the OA decision in the MTO system has been widely recognized in academia as well as in practice.

According to Hendry and Kingsman (1993), the MTO system can be characterized as follows:

- Less standard products.
- A high degree of variability in both demand quantity and product mix.
- Production lead time is vital for customer satisfaction and is agreed with customers.
- Price is agreed with customers before production commences.

In addition to the above characteristics, the arrival of the customer in an MTO system is stochastic over time and the system usually works on a trade off between capacity and profit. This MTO characteristic complicates the decision of whether to accept or reject a new order. Accepting all incoming orders may jeopardize the shop due to overload and may incur heavy penalties in terms of money, good will or lost customers; accepting fewer orders though will reduce the capacity utilization rate and adversely affect on company profit. Therefore, this decision is crucial and has large influence on the performance of a company.

Traditionally, this problem is solved by always accepting orders as long as sufficient capacity is available. We can find many papers that have contributed to the research on the OA decision where the due date is assumed to be fixed exogenously by the customer. In these papers, the manufacturer, by checking the available capacity against the exogenously fixed due date, decides whether to accept or reject the incoming order. This shows a company-centric approach where the OA decision is taken solely by the manufacturer. However, the due dates are often result of a negotiation between the customer and the manufacturer with a trade-off between price and promised delivery date (Moodie and Bobrowski, 1999). From this statement, it is obvious that the OA decision is affected by the method by which the manufacturer negotiates with the customer over the conflicting issues.

The purpose of this paper is to examine the problem of OA decision-making by considering negotiation as a tool for reaching an agreement between the manufacturer and the customer over the conflicting issues. Such an approach allows both manufacturer and customer involvement in the decision-making process allowing them to harness their relationship by reducing the distance on the conflicting issues and reaching an agreement through negotiation. This reflects the customer cum company centric approach to the OA decision.

The rest of this paper is structured as follows: the section **Literature Review** discusses the past research carried out on this area; the section **Problem Description** describes the problem that will be addressed in this research; and the section **Proposed Method** explains the proposed OA method in detail; the section **Numerical Analysis** discusses and presents the results; while the concluding remarks and future research directions are highlighted in the last section **Conclusion**.

Literature Review

Even though the OA decision is one of the main operational issues of the MTO system, there is little literature on the subject. Some of the previous literature considered the static arrival of the customer i.e., the customer arrivals are known in advance. On the other hand, others have considered dynamic arrival i.e., customer arrival is irregular over time and is not known to the manufacturer beforehand.

Few researchers have contributed to the research on OA decision by assuming static arrival of customers. Guerrero and Kern (1988) considered the problem in a demand management situation for an Assembly-to-Order system. The authors proposed a MILP model with an objective of minimizing lateness penalties or penalties for not satisfying demand. Pourbabi (1992) presented a model of job selection using net profit in the context of just-in-time manufacturing. The paper develops a mixed binary linear programming model for making the decision. Slotnick and Morton (1996) studied the situation in a periodic decision setting where the problem is to select a subset of orders from among a set while aiming to maximize revenue. This paper compares a branch-and-bound approach with two heuristics in a single machine setting and show that the heuristics work well for the given setting. Ghosh (1997), in the same setting, showed that the job selection problem considered by Slotnick and Morton (1996) is NP-hard. Lewis and Slotnick (2002) extended the work of Slotnick and Morton (1996) to the multi-period case by considering that the rejection of an order would result in no future jobs from the same customer. Slotnick and Morton (2007) in turn extended this work by formulating the problem as an integer program which jointly performs both sequencing and order acceptance.

Most of the literature in the field of OA has assumed a dynamic arrival of customers. Wester et al. (1992) proposed three different OA approaches under a single machine production environment. Among the three approaches, two are based on workload and the other is based on making a detailed schedule after an arrival of a new order. Hans (1994) compared two different approaches in a single machine case. In the hierarchical approach, the decision is based on the aggregate characteristics of already accepted orders. In the second approach, the order acceptance and production scheduling are integrated. To solve the problem, Wang et al. (1994)

used a neural network approach in a periodic decision setting, where the acceptance decision is based on multiple criteria such as profit, customer credit and available capacity. Wu and Chen (1996) studied the effect of accepting rush orders on a production schedule, proposing an MILP model for calculating the cost of accepting rush orders. This paper is further expanded by Wu and Chen (1997) to simultaneously accommodate four different criteria for decision making. They used a multiple objective programming technique to estimate the cost of producing a rush order.

To determine the performance of workload rules for OA, Raaymakers et al. (2000) analysed the results of a case study of a batch chemical manufacturing plant. For each work centre, the maximum workload was specified as a percentage of the available capacity of that work centre, and orders were accepted if the workload remained below a pre-specified level. Ivanescu et al. (2002) used available capacity as an acceptance rule and compared three order acceptance policies in a setting with Erlang distributed processing times. They showed that a regression policy clearly outperforms a workload policy. In general however, a scheduling policy performs better with respect to both the service level and job lateness. Combining the strengths of the scheduling and regression policies, Ivanescu et al. (2006) proposed a hybrid policy by using simulated annealing technique in a setting with stochastic processing time. Nandi and Rogers (2003) pointed out that the control policy to release an order from the pre-shop pool affects OA decision. Their paper considers the capacity of the machines on the candidate order's route as a criterion for the OA decision. Later, Nandi and Rogers (2004) proposed a simulation based approach to the OA decision under two different classes of customers. Ebban (2005) analysed different workload rules for the OA decision that varied from rules based on aggregate information to a method that considers precedence relationships, release date and due dates of orders. From the result of numerical analysis their paper concluded that earliest due date based scheduling policy generates the optimal result.

Classifying the customers into two different categories Piya et al. (2008) analysed the problem of the OA decision for both static and dynamic arrival of customers in the system and used a layering concept to solve the problem.

However, a limited number of papers have contributed to the combined research on the OA decision and negotiation. Calosso et al*.* (2003, 2004) considered price and due date as negotiation issues in a multi-tiered supply chain. They presented three MILP models to evaluate an order from the customer, evaluate a bid submitted by the supplier, and select the best supplier among the group of bidding suppliers. On the other hand, Ebadian et al*.* (2008) considered only delivery time as a negotiable issue where the trade-off between price and due date is utilized to negotiate with the customer. The major drawbacks of these papers are that they either lack a comprehensive structure on negotiation (Calosso et al*.* 2003; Ebadian et al*.* 2008) or on the OA decision (Calosso et al*.* 2003; 2004). Furthermore, these papers do not define any methods for finding the best offer that can be submitted to the opponent during the negotiation round.

Recently, Piya et al. (2009) incorporated a quotation and negotiation mechanism into an OA decision model and provided a constructive structure on these three issues. The paper considered static step size while calculating aspiration level during the negotiation process. With such an approach, the aspiration level does not react to the counter-offer received from the customer and at the same time the step size of the aspiration level for all the negotiation rounds are fixed for each specific customer. To overcome this shortcoming, this paper proposes a new method for the OA decision whereby the aspiration level will be dynamic in nature. Basically, we expand the existing model of Piya et al. (2009) to accommodate the reactive aspiration level. Such aspiration level allows the manufacturer to negotiate with the customer according to the counter-offer received and the number of negotiation rounds. Accordingly, depending on the situation, the manufacturer can accelerate or decelerate the pace of negotiation. Also in Piya et al. (2009), while generating a new offer, the degree of customer priority towards different issues is defined similar to the slope of the manufacturer. This approach will generate a new offer which may be more favourable to the manufacturer, but may result more rounds of negotiation or in an inability to reach an agreement with the customer on the conflicting issues. Therefore, utilizing the geometry between the manufacturer's offer and the customer's counter-offer, this paper will discuss a method by which the manufacturer can determine the expected priorities of the customer regarding different issues.

Problem Description

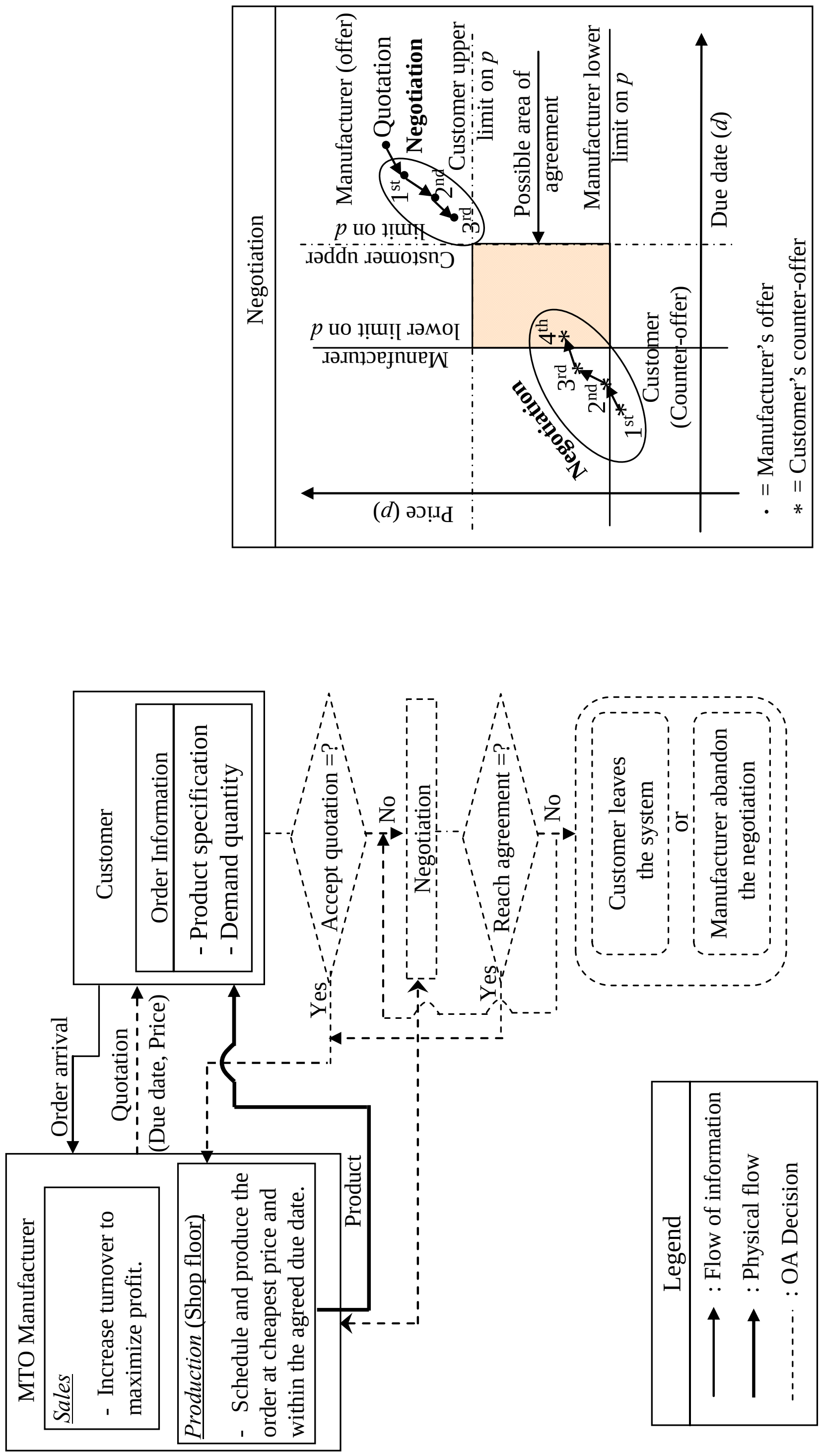
The OA framework considered in this paper consists of a customer and a manufacturer as shown in Figure 1. In the MTO system, negotiations between manufacturer and customer are integrated with production planning, with the due date and the price of the order being the main issues under negotiation (Calosso et al., 2004). We consider these two issues in our paper. The customer arrives stochastically to the sales department for the initial enquiry as to the due date and price for which the company can deliver their order. At this time, the customer will provide certain information such as product specifications and quantity demanded. The manufacturer, based on its strategy and production plan, will submit a quotation (initial offer) to the customer. If the customer is not satisfied with this initial offer, they will then negotiate with the manufacturer. Negotiations may continue for several rounds and end with either an agreement or the rejection of the order. During each round, the manufacturer proposes a new offer and the customer makes a counter-offer with another due date and price, within their limit level as shown in Figure 1(b). The manufacturer will start the offer from its upper limit which indicates the quotation; the customer in contrast will submit their counter-offer from the lower limit. In an attempt to reach an agreement they each move in the opposite direction which reduces the distance between them on the conflicting issues. Once an offer or counter-offer is accepted, the order will be handed to the floor shop for production and then finally the product will be delivered to the customer. On the other hand, if the manufacturer and customer cannot reach an agreement even after rounds of negotiation, the customer may leave the system to search for another manufacturer or the manufacturer may themselves abandon the negotiation process. Thus, in our OA framework both the manufacturer and the customer take part in the decision process. Under such circumstances, the negotiation strategy adopted by the manufacturer plays a crucial role in reaching an agreement on the conflicting issues.

The objective of this paper is to improve the structure of OA decision framework by implementing negotiation strategies that increase the benefits to the manufacturer and allow the reaching of an early agreement on the conflicting issues, thus, satisfying both the manufacturer and the customer engaged in the negotiation. The following assumptions are made in proposing this new method:

i) Only one single negotiation is considered at a time.
ii) The maximum number of rounds within which negotiation should terminate is fixed.

Proposed Method

As two different issues are considered in each negotiation, to understand the proposed OA decision, it is necessary to understand the acceptance/rejection geometry followed in this research. The geometry shows different regions and explains the situations when the manufacturer can accept the order, reject it, or negotiate with the customer. The regions are distinguished on the basis of the maximum, the minimum and the limit level on due date and price as shown in Figure 2.



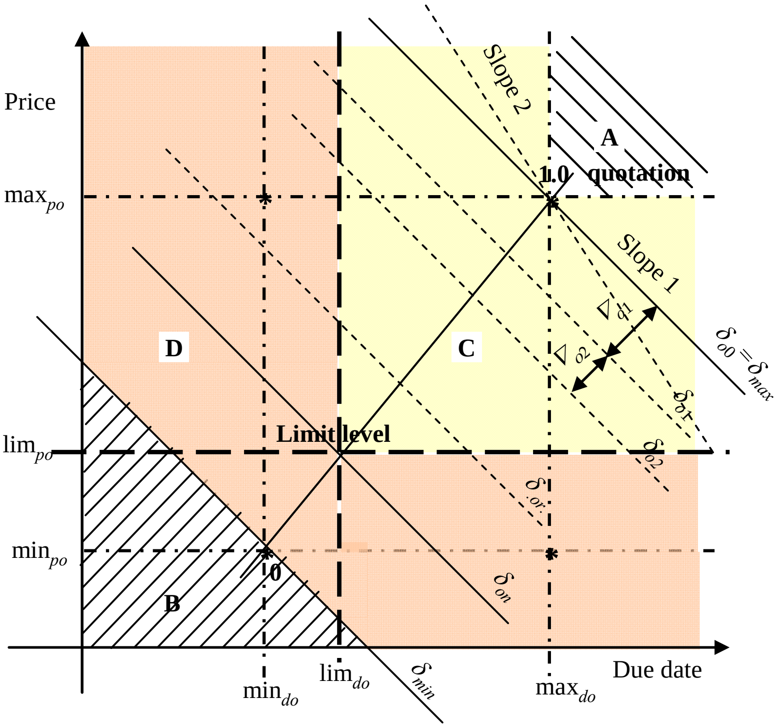
a) OA decision with quotation and negotiation structure

b) Negotiation between the manufacturer and the customer

Figure 1: Framework of negotiation based approach to OA decision

The maximum level for the due date and price is the quotation (initial offer) submitted by the manufacturer to the customer. When contesting this quotation the customer will never propose a counter-offer at region A, because any counter-offer here shows that the quotation can be accepted without negotiation. If a counter-offer on both the issues lies within region B, the order will be rejected immediately by the manufacturer. A counter-offer that falls between the limit level and the maximum level (region C) shows that the order will be accepted after negotiation. Moreover, a counter offer in region D shows that the OA decision will be based on the outcome of negotiation.

In Figure 2, the slope of a line represents the weight given by the manufacturer to different issues. By assigning a higher weight to one issue compared to the other, the slope of the line can be varied. The intercept of the line indicates the aspiration level (δ_{or}) desired by the manufacturer at a particular negotiation round *r*. As shown in the figure, different step sizes $(\nabla_{o1}, \nabla_{o2})$ are used to calculate the aspiration level for different rounds of negotiation. The limit level is the lower limit on due date and price below which the manufacturer is not willing to accept the order. The manufacturer will use this geometry to generate any new offers during the negotiation process.



A: Order accepted without negotiation
B: Order rejected without negotiation
C: Order accepted with negotiation
D: OA decision will be based on the outcome of negotiation

Figure 2: Different regions in an OA geometry

With the above geometry, the proposed OA decision can be explained by the flow chart as shown in Figure 3. Once a new order arrives in the system, the manufacturer has to quote the due date and price at which they are willing to provide said order (a). If the quoted due date and price is acceptable to the customer, the order will be handed to the shop floor for production (b).

But, usually in an MTO system, the agreed due date and price is the outcome of negotiation. Therefore, the manufacturer can expect a counter-offer from the customer in regard to the conflicting issues. After receiving this counter-offer, calculate its score (c). If the score falls below the manufacturer's minimum aspiration level, the new order will be rejected (d). Otherwise, the distance between the previous offer and the counter-offer will be measured (e). Next, based on the distance the aspiration level for the next round of negotiations is defined (f) and compared with the score of the customer (g). If the score is greater than the aspiration level of the next round, the order will be accepted (h) and handed to the shop floor (b). Otherwise, the current round of negotiation is checked (i).

If the current round of negotiation is equal to the maximum number of rounds, *n,* then the new order will be rejected (d). Otherwise, the customer's expected slope is calculated (j). Next, based on this slope and different negotiation strategies, the new offer for the next negotiation round will be found (k) and submitted to the customer. In our proposed method we consider three different strategies to find the new offer.

- Highest probability of acceptance (HPA) strategy
- Perpendicular distance from customer's expected slope (PDC) strategy
- Shortest distance from customer's expected slope (SDCS) strategy

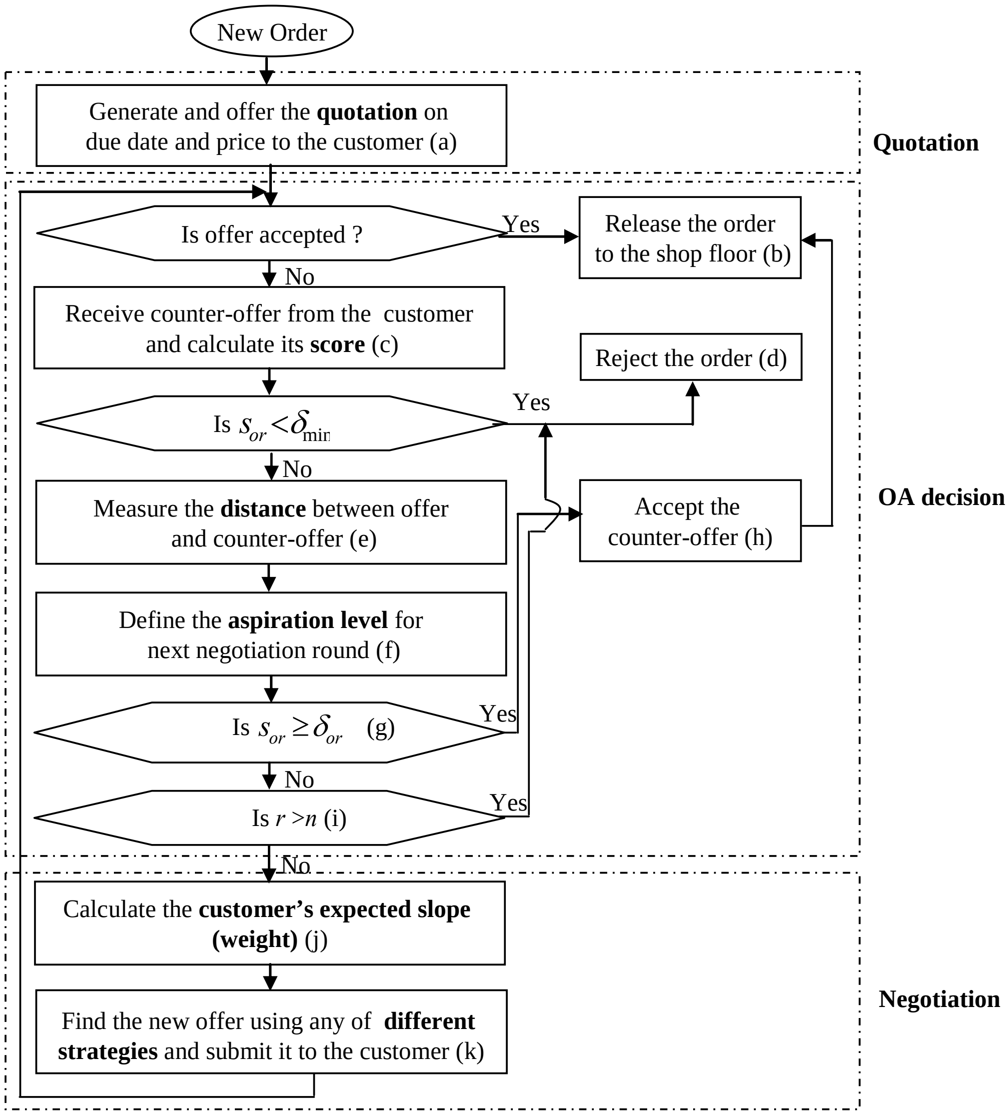


Figure 3: Flow chart for OA decision

The above process is continued until any one of the following occurs:
- The offer or counter-offer is accepted by the customer or the manufacturer respectively.
- The counter-offer on both the issues is below the manufacturer's minimum aspiration level.
- The current round of negotiation exceeds the maximum number of rounds.

Thus, the proposed method can be divided into three major phases:
1) Quotation 2) OA decision 3) Negotiation

Table 1: Mathematical Notation

ζ = Coefficient of processing time uncertainty
ψ = Coefficient of negotiation margin for due date
τ = Coefficient of profit margin
π = Coefficient of negotiation margin for price
CT_{of} = Completion time for operation f of order o
ER_{of} = Earliest release time for operation f of order o
C_o^{pes} = Production cost of order o with pessimistic processing time
$\underline{\tau}$ = Lower limit on profit margin coefficient
S_{or} = Score on counter-offer at r round of order o
j = Negotiation issues {j= price (p) or due date (d)}
w_j= Weight of the manufacturer for issue j
$\max_{jo}$= Maximum level on issue j of order o
$\min_{jo}$ = Minimum level on issue j of order o
$\lim_{jo}$= Limit level on issue j of order o
r = Negotiation round (r= 1,….., n)
δ_{or} = Aspiration level at negotiation round r of order o
δ_{min}= Aspiration level at minimum value on both the issues of manufacturer
d_{or} = Distance between the offer and counter-offer of order o at round r
∇_{or} = Step size for round r of order o
q= Manufacturer's coefficient
BL= Bisection line
HL = Horizontal line
LOD = Line of deflection
θ = Angle of deflection
β = Angle between BL and HL
γ = Angle between HL and LOD
ρ = Customer's expected slope
α = Angle between LOD and aspiration level
i = Alternate option (i =1, 2,….., I)

Quotation

We use a naive approach, similar to that used in Piya et al. (2009), to prepare the quotation. As shown in equation (1), the quotation of a due date is based on the anticipated completion time for the new order, the coefficient of uncertainty margin (ζ) and the negotiation margin (ψ). The coefficient of the uncertainty margin helps overcome the uncertainty in processing time that exists in an MTO production system.

$$\max_{do} = (1+\zeta+\psi)(CT_{of} - ER_{of}) + ER_{of} \qquad (1)$$

If the value of ζ and ψ equals 0, the due date in equation (1) will be equal to the completion time. In relation to Figure 2, this due date represents the minimum level ($\min_{do}$) on due date. When only the value ψ is equal to 0 shows the due date with uncertainty margin, but without negotiation margin. This due date represents the limit level for the due date ($\lim_{do}$). Therefore, during the quotation phase, the value of ζ and ψ should be more than 1 to compensate for uncertainty in processing time and negotiation margin. The quoted due date is considered as the maximum level on due date ($\max_{do}$).

Similarly, the price to be quoted consists of production cost, profit margin coefficient and coefficient of negotiation margin as shown in equation (2). The profit margin coefficient will add the manufacturer's desired profit into the quoted price.

$$\max_{po} = C_o^{pes}(1+\tau+\pi) \qquad (2)$$

If the values of τ and π are equal to 0 in equation (2), the price will be equal to the production cost. In relation to Figure 2, this price represents a minimum level ($\min_{po}$) on price. While, the value of only π equals 0 and τ equals $\underline{\tau}$ represents a price without negotiation margin but with a lower limit on profit margin. This represents a limit level on price ($\lim_{po}$). The quoted price is considered to be the maximum level on price ($\max_{po}$).

In the equations (1) and (2) the coefficient of negotiation margin provides an allowance for the manufacturer to negotiate with the customer on due date and price respectively. Therefore, if the customer asks to reduce the value on the quoted due date and price the manufacturer can do it without risking the chance of the order becoming tardy and without reducing the desired profit margin.

(For details on quotation refer to Piya et al., 2009)

OA Decision

In the OA decision phase the manufacturer has to decide whether to accept or reject the customer's counter-offer or further negotiate with them. In this phase, the manufacturer will calculate the score for the counter-offer received, measure the distance and then based on distance will define the aspiration level for the next negotiation round. The calculated score and aspiration level will be used in the decision process. The method to calculate score, distance and aspiration level will be discussed in the following sub-sections.

Score

Score is the value calculated for the customer's counter-offer. For this purpose, we use a linear scoring function similar to that used by Faratin et al. (1998).

$$S_{or} = \sum_{j \in d,p} w_j \frac{\left(x_{jor}^{c \to m} - \min_{jo}\right)}{\left(\max_{jo} - \min_{jo}\right)} \quad \text{where, } w_d + w_p = 1.0 \tag{3}$$

As shown in equation (3), the score is calculated by aggregating the independent scores of due date and price counter-offered by the customer. The weighted average method is used to aggregate the independent score which will fall between (0-1).

Distance

The distance d_{or} shows the closeness between the manufacturer's offer and the customer's counter-offer. It is calculated by considering the aspiration level of the latest round and the score of the counter-offer.

$$d_{or} = \delta_{o(r-1)} - S_{or} \tag{4}$$

The value of $\delta_{o(r-1)}$ can be calculated by replacing $x_{jor}^{c \to m}$ with $x_{jo(r-1)}^{m \to c}$ in equation (3). The calculated distance is utilized to define the aspiration level of manufacturer for the next round of negotiation.

Aspiration Level

Aspiration level is the level of benefit sought at any particular time. In the paper by Cakravastia and Nakamura (2002), aspiration level is utilized for the arrival time of materials in a buyer-supplier relationship. This paper lacks clarity on how to relate the aspiration level as a function of time.

In Piya et al. (2009), aspiration level is defined as a decreasing function of time. With the approach used in this paper, the aspiration level for any negotiation round can be determined in advance if the maximum number of rounds, *n,* is fixed. The drawback of this approach is that it reduces the aspiration level between rounds by an equal step size and does not take the information from the counter-offer into consideration in defining the aspiration level for the next round.

In our proposed model, aspiration level is adaptive in relation to the information in the counter-offer. As shown in Figure 2, the maximum level on due date and price is mapped to 1.0 while the minimum level is mapped to 0. The value 1.0 represents the maximum aspiration level (δ_{o0}) and indicates that the aspiration level desired by the manufacturer at the beginning (quotation phase) will be at its highest value. As the negotiation progresses, this aspiration level decreases, as shown by the dotted slopes (δ_{o1}, δ_{o2},…, δ_{on}) in the figure. This decrease in the aspiration level will increase the chances of reaching an agreement with the customer on the conflicting issues. As shown in equation (5), the aspiration level for any given round, *r,* is calculated by reducing the step size from the aspiration level of the latest negotiation round.

$$\delta_{or} = \delta_{o(r-1)} - \nabla_{or} \tag{5}$$

$$\nabla_{or} = \left\{ \left(\frac{d_{or}}{d_{o(r-1)} - \nabla_{o(r-1)}} \right)^{\left(1-\frac{r}{n}\right)} \left(\frac{\delta_{o(r-1)} - Z}{n - r + 1} \right)^{(1-q)} \right\} \tag{6}$$

$$-1 \le q \le 1, \tag{7}$$

$$Z = \delta_{on} \text{ if } S_{or} < \delta_{on} \text{; Otherwise, } Z = S_{or} \tag{8}$$

When r=1, $\nabla_{o(r-1)} = 0$ and $d_{o(r-1)} = 1.0$; Otherwise, $\nabla_{o(r-1)} = \delta_{o(r-2)} - \delta_{o(r-1)}$ (9)

From equation 6, it can be noted that the reduction in step size is based on the distance in the previous round (r-1) and the latest round (r), the step size that is reduced to calculate the aspiration level of the previous round, remaining aspiration level, remaining rounds of negotiation and the manufacturer's coefficient. Aspiration level can be reduced up to the limit value (δ_{on}), when the current round of negotiation equals maximum number of rounds n.

The manufacturer's coefficient (q) indicates assertiveness by the manufacturer during the negotiation process. As shown in equation (7), it lies between -1 to 1.

Equation (8) implies that if the score of the counter-offer is less than the aspiration level for the n round of negotiation, the value of Z will be equal to the aspiration level of n round. Otherwise, it will be equal to the score of the counter-offer.

Equation (9) implies that for the first round of negotiation the step size and the distance of the latest round will be equal to 0 and 1 respectively. Otherwise, the step size will be given by the difference between the aspiration levels of the previous two consecutive rounds.

Therefore, the proposed method defines the aspiration level as a function of negotiation round and distance. The step size of aspiration level will be dynamic in nature and different for different rounds of negotiation. Using this method, the manufacturer can accelerate or decelerate the pace of negotiation by manipulating the value of q.

[Note: δ_{on} can be calculated by replacing $x_{jor}^{c \to m}$ with $\lim_{jo}$ in the equation (3)]

Negotiation

Negotiation is defined as a process by which a joint decision is made by two or more parties through concession making (Pruitt, 1981). Raifa (1982) presented a two-party and multi-issue negotiation structure using a value scoring system. This structure was later expanded to accommodate multi-lateral negotiation (Faratine et al. 1998). Elhafsi and Roland (1999) developed a negotiation model by considering the probability of machine failure. Moodi and Bobrowski (1999) proposed a negotiation strategy that considered the trade-off relationship between price and due date. Opera (2002) introduced neural network techniques to develop a model which can learn the opponent's negotiation strategy. The major drawback of these papers is that they are all based on using same negotiation strategy for all customers. After classifying customers based on their sensitivity towards due date and price, Piya et al. (2009) proposed two different strategies of negotiation. While generating a new offer, during the negotiation process, only the slope of the manufacturer is considered and the customer's slope is treated in a similar manner to that of the manufacturer. This type of approach will generate a new offer which may be more favourable to the manufacturer. It may also result in more rounds of negotiation or an inability to reach an agreement between the manufacturer and the customer on the conflicting issues. To overcome this drawback, in this paper we introduced the term "customer's expected slope" which represents the expected degree of customer priorities in relation to the different issues arising during negotiation. By using this, the manufacturer can generate a new offer equally favourable to both parties engaged in the negotiations.

The method to calculate customer's expected slope and the generation of a new offer will be explained in following sub-sections.

Customer's Expected Slope

The customer's expected slope, ρ, is calculated by considering the angle of deflection, θ, between the customer's counter-offer received and the manufacturer's latest offer. As shown in Figure 4, the steps taken to calculate the customer's expected slope are as follows:
Step 1: Connecting points C and E, calculate the angle of deflection θ.
The angle of deflection, θ, will be different for different counter-offers, and can be explained in relation to the following three cases:
Case (i) When the counter-offer on both the issues is less than or equal to the latest offer (Area 1), the angle of deflection is calculated as shown in equation (10).

$$\theta = \beta - \gamma \tag{10}$$

Case (ii) When the counter-offer on price is more than the latest offered price (Area 2), the angle of deflection is calculated as shown in equation (11).

$$\theta = \beta + \gamma \tag{11}$$

Case (iii) When the counter-offer on due date is more than the latest offer (Area 3), the angle of deflection is calculated as shown in equation (12).

$$\theta = 360 - (\beta + \gamma) \tag{12}$$

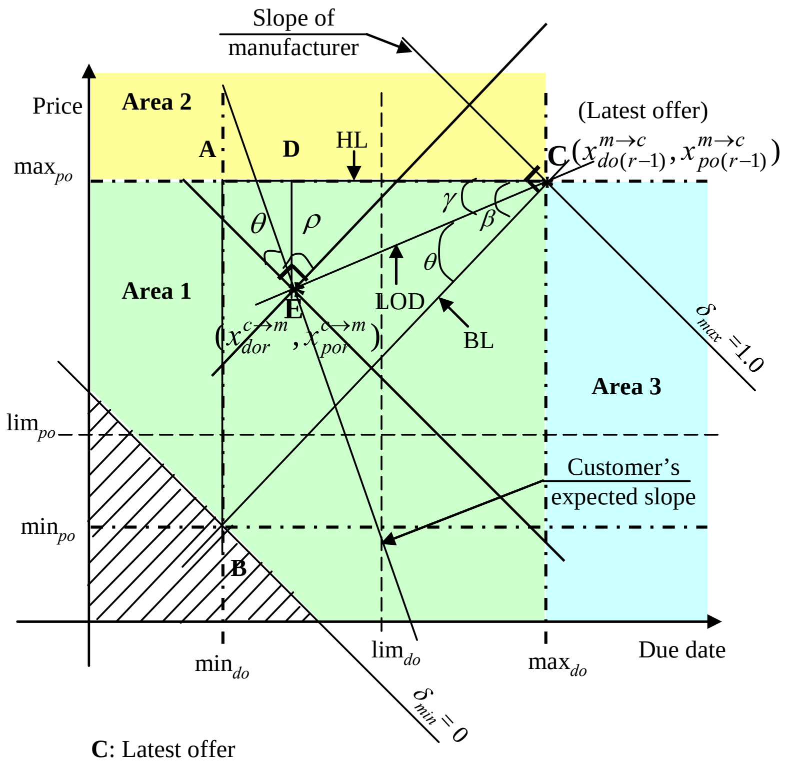
Here, β and γ are calculated from the latest offer and the counter-offer as follows.
From Δ ACB,

$$\beta = \tan^{-}\left\{\frac{\left(x_{po(r-1)}^{m\to c} - \min_{po}\right)\left(\max_{do} - \min_{do}\right)}{\left(x_{do(r-1)}^{m\to c} - \min_{do}\right)\left(\max_{po} - \min_{po}\right)}\right\} \tag{13}$$

From Δ DCE,

$$\gamma = \tan^{-}\left\{\frac{\left(x_{po(r-1)}^{m\to c} - x_{por}^{c\to m}\right)\left(\max_{do} - \min_{do}\right)}{\left(x_{do(r-1)}^{m\to c} - x_{dor}^{c\to m}\right)\left(\max_{po} - \min_{po}\right)}\right\} * (u) \tag{14}$$

Where, u = -1 if $x_{po(r-1)}^{m\to c} < x_{por}^{c\to m}$; otherwise, +1.



C: Latest offer

E: Counter-offer

Figure 4: Defining customer's expected slope

Step 2: Draw a slope on point E similar to the slope of manufacturer.
Step 3: Rotate slope on point E by angle θ.
Step 4: Calculate customer's expected slope ρ.

$$\rho = (x)(90-\theta) \tag{15}$$

Where, x = +1 for case (i) and case (ii); otherwise, -1.

New Offer

We propose three different strategies to find a new offer for each negotiation round. These offers can be generated at the aspiration level defined by equation (5). To generate a new offer, all the strategies utilize the customer's expected slope. The three strategies are as set out below.

i) *HPA Strategy*: As a new offer, the HPA strategy selects the alternate option, from among many options, that will have the highest probability of acceptance. Since all the options at the same aspiration level are of equal priority to the manufacturer, by using this strategy the manufacturer can increase the probability of reaching an agreement with the customer. The steps to find the new offer using this strategy are described below.
Step 1: Find the number of alternate options within the acceptable range i.e., (lim_{do}, lim_{po}) of manufacturer.

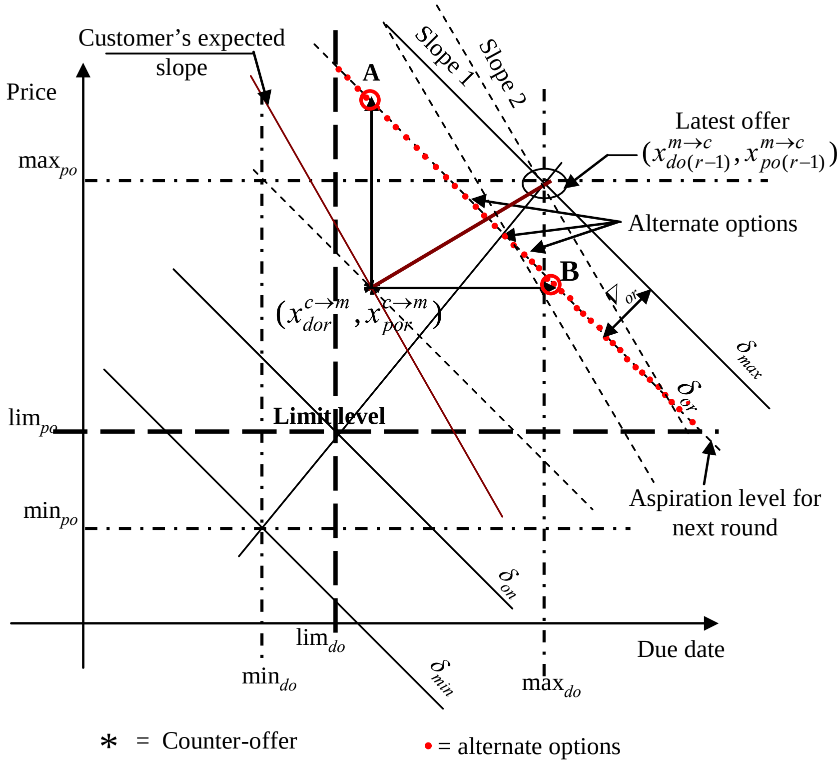


Figure 5: Generating a new offer by the HPA strategy

As shown in Figure 5, numbers of alternate options are generated at the same aspiration level by manipulating the due date and price as compared to the latest offer. We know that

$$\delta_{or} = w_d\left(\frac{(x^{m\to c}_{dor})_i - \min_{do}}{\max_{do} - \min_{do}}\right) + w_p\left(\frac{(x^{m\to c}_{por})_i - \min_{po}}{\max_{po} - \min_{po}}\right) \tag{16}$$

For i = 1; $(x^{m\to c}_{dor})_i = \lim_{do}$ (17)

i = 2, 3, ……….., I-1; $(x^{m\to c}_{dor})_i = (x^{m\to c}_{dor})_{(i-1)} + 1$ (18)

$i = I$; $(x^{m\to c}_{dor})_i = (x^{m\to c}_{dor})_{\lim_{po}}$ (19)

The value of $(x^{m\to c}_{dor})_{\lim_{po}}$ in equation (19) can be obtained by substituting $(x^{m\to c}_{por})_i$ with $\lim_{po}$ in equation (16). Therefore, by increasing the value of $x^{m\to c}_{dor}$ from $\lim_{do}$ to $(x^{m\to c}_{dor})_{\lim_{po}}$ in equation (16); we can obtain the value of $(x^{m\to c}_{por})_i$ for each value of $(x^{m\to c}_{dor})_i$. The alternate option is represented by $(x^{m\to c}_{dor}, x^{m\to c}_{por})_i$.

Step 2: Calculate the probability of acceptance for all the alternate options generated in step 1.
In the paper by Easton and Moodie (1999) an S-shaped Logit model is used to calculate the probability of the customer accepting the quotation submitted by the manufacturer. This model is

constructed without utilizing any information received from the customer. In the negotiation process, it is possible to get the information on the negotiated issues from the customer. Our model utilizes this information to calculate the probability of their acceptance of each alternate option generated in step 1. This will increase the authenticity of the calculated probability of acceptance.

$$(pr_r)_i = \left\{ \exp\left[w_j^c \left\{ \left[v_j \frac{\min_{jo}\{(x_{jor}^{m\to c})_i - x_{jor}^{c\to m})\}}{\lim_{jo}(x_{jor}^{c\to m})} \right]^{\left[1-\frac{r}{n+1}\right]} \right\} \right] \right\}^{-1} \quad \begin{array}{l} ,\forall i=(1,2,...,I) \\ ,\forall j \in (d,p) \end{array} \tag{20}$$

Where, $w_p^c = \dfrac{\rho}{180}$ and $w_d^c = 1 - w_p^c$

v_j=0 when $x_{jor}^{c\to m} > x_{jor}^{m\to c}$; otherwise 1.

From equation (20), it can be noted that the calculated probability of acceptance considers the difference between the offer and the counter-offer on the given issue, the status about the round of negotiation and the expected slope of customer.

Step 3: Select the alternate option that has the highest probability of acceptance as a new offer.

New offer= $(x_{dor}^{m\to c}, x_{por}^{m\to c})_{\max_i (pr_r)_i}$ (21)

From these steps, as shown in Figure 5, the new offer can be generated near either point A or point B. This is because, among many options, option near point A or point B will have the highest probability of acceptance with respect to the customer's expected slope. If the customer's expected slope is more inclined towards the due date (Figure 5), the new offer will be the coordinate of a point that lies near point A. Otherwise, it will be the coordinate of a point that lies near point B.

ii) *PDC Strategy*: The PDC strategy obeys the preferences of the customer in regards to different issues while generating a new offer. With this strategy, the offer on the desired aspiration level will be generated such that it will lie at the perpendicular distance from the customer's expected slope that passes through the counter-offer point. Thus, the reduction ratio on due date and price of new offer will be equal to the sensitivity ratio of the customer.

The sensitivity ratio (Z_{or}) shows the degree of customer sensitivity towards due date and price. It can be calculated by considering the latest offer and the counter-offer.

$$Z_{or} = \frac{(x_{do(r-1)}^{m\to c} - x_{dor}^{c\to m})x_{po(r-1)}^{m\to c}}{x_{do(r-1)}^{m\to c}(x_{po(r-1)}^{m\to c} - x_{por}^{c\to m})} \tag{22}$$

On the other hand, reduction ratio (R_{or}) shows the rate of reduction or increment on each issue considered in the new offer. It can be calculated by utilizing the latest offer and the new offer.

$$R_{or} = \frac{(x_{do(r-1)}^{m\to c} - x_{dor}^{m\to c})x_{po(r-1)}^{m\to c}}{x_{do(r-1)}^{m\to c}(x_{po(r-1)}^{m\to c} - x_{por}^{m\to c})} \tag{23}$$

The new offer therefore will be the coordinate of a point at the desired aspiration level that satisfy the following equation (in the figure $Z_{or} = a:b$ and $R_{or} = a':b'$).

$Z_{or} = R_{or}$ (24)

In Figure 6, the new offer using the PDC strategy will lie at point B. The coordinate of point B can be obtained as shown below.

Step 1: Generate the first equation by considering points A and B.

From distance formulae,

$$q' = \sqrt{\{x^{m\to c}_{do(r-1)} - x^{m\to c}_{dor}\}^2 + \{x^{m\to c}_{po(r-1)} - x^{m\to c}_{por}\}^2} \tag{25}$$

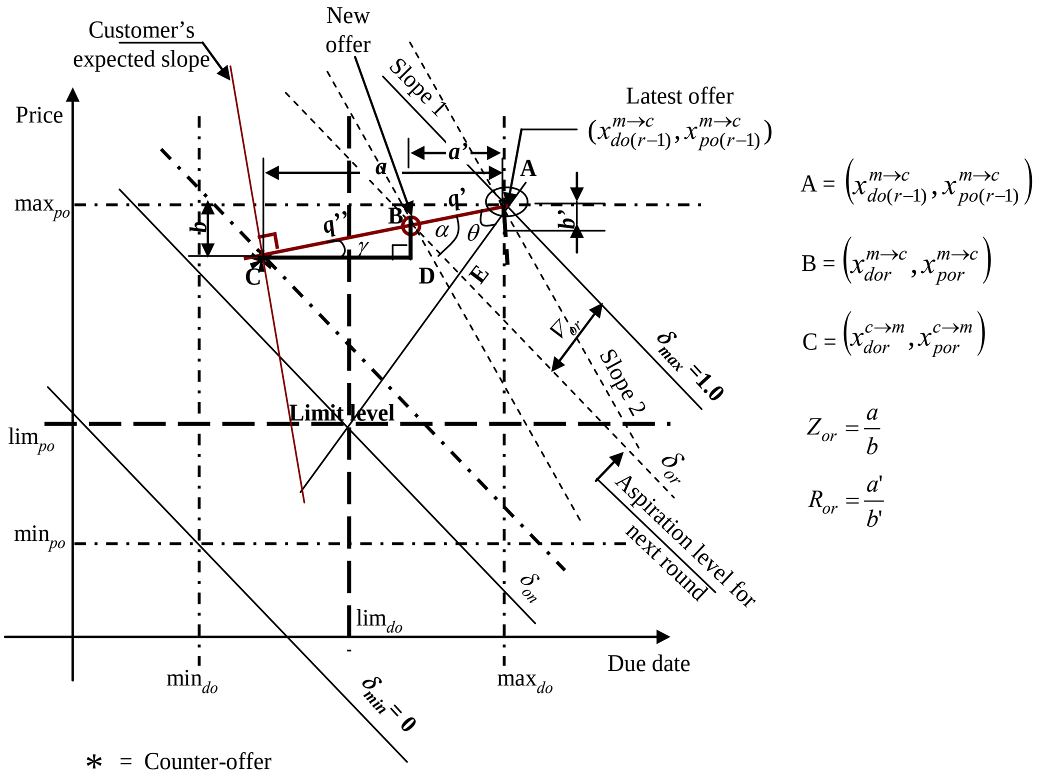


Figure 6: Generating a new offer by the PDC strategy

From ΔABE,

$$q' = \frac{\nabla_{or}}{\sin\alpha} \text{ where } \alpha = 90 - \theta \tag{26}$$

Simplifying equations (25) and (26) we can obtain the first equation as below.

$Fx^{m\to c}_{dor} + Gx^{m\to c}_{por} = H$ where F, G, H are constants for the given latest offer. (27)

Step 2: Similarly, generate the second equation by considering points B and C.

From distance formulae,

$$q'' = \sqrt{\left(x^{m\to c}_{dor} - x^{c\to m}_{dor}\right)^2 + \left(x^{m\to c}_{por} - x^{c\to m}_{por}\right)^2} \tag{28}$$

From ΔBCD,

$$q'' = \frac{(x_{por}^{m\to c} - x_{por}^{c\to m})}{\sin\gamma} \tag{29}$$

Simplifying equations (28) and (29) we can get the second equation as below.

$Lx_{dor}^{m\to c} + Mx_{por}^{m\to c} = N$ where L, M, N are constants for the given counter-offer. (30)

Step 3: Calculate the new offer $(x_{dor}^{m\to c}, x_{por}^{m\to c})$ by solving equations (27) and (30).

In equations (27) and (28), the only two unknowns are $x_{dor}^{m\to c}$ and $x_{por}^{m\to c}$. Therefore, solving these two equations by general mathematics the value of $(x_{dor}^{m\to c}, x_{por}^{m\to c})$ can be obtained.

(Note: The value of θ and γ are obtained from Sub Section "*Customer's Expected Slope*")

iii) ***SDCS Strategy*****:** The SDCS strategy generates a new offer, on the desired aspiration level, such that it lies at the shortest distance from the customer's expected slope. Like the manufacturer, if the customer treats all the options available at their expected slope equally, this strategy will be better in terms of reaching an agreement. In this strategy, depending on different cases, the shortest distance can lie at two different points on the aspiration level.

Case (i): When the point of intersection between the customer's expected slope and the aspiration level lies below the limit level on due date (Figure 7a).

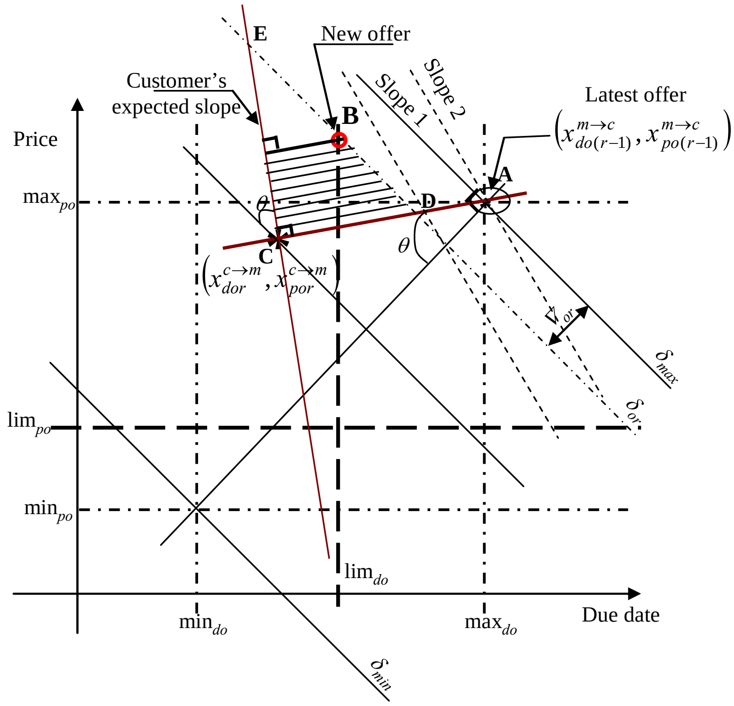
In this case, the shortest distance will be the point of intersection between the limit level on due date and the aspiration level i.e., point B in the figure. Therefore the new offer is calculated as follows:

$(x_{dor}^{m\to c}, x_{por}^{m\to c}) = (\lim_{do}, x_{por}^{m\to c})$ where, (31)

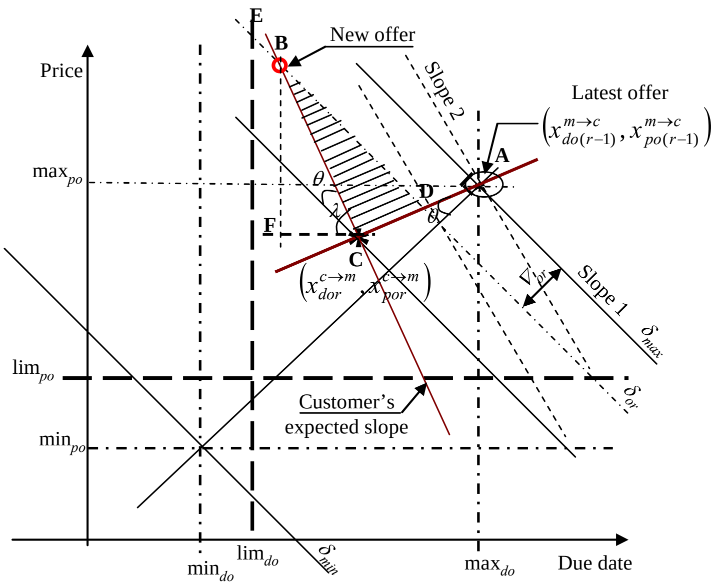
$$x_{por}^{m\to c} = \frac{1}{w_p}\left[\delta_{or}(\max_{po} - \min_{po}) - \frac{w_d(\lim_{do} - \min_{do})(\max_{po} - \min_{po})}{(\max_{do} - \min_{do})}\right] + \min_{po} \tag{32}$$

As shown in equation (31), the new offer on due date will be equal to the limit level on due date. The new offer on price can be calculated by using equation (32). This equation is obtained by simplifying and substituting $x_{dor}^{m\to c}$ with $\lim_{do}$ in equation (16).

Case (ii): When the point of intersection between the customer's expected slope and the aspiration level lies above the limit level on due date (Figure 7b).



a) New offer for case (i)



b) New offer for case (ii)

$*$ = Counter-offer

Figure 7: Generating a new offer by the SDCS strategy

In this case, the shortest distance will lies at the point of intersection between the customer's expected slope and the aspiration level i.e., point B in the figure. Therefore, the coordinate of the new offer can be calculated as follows:

Step 1: Define the coordinate of point D.

The coordinate of point D can be calculated by using the PDC strategy. Let it be (x_4, y_4).

Step2: Define the coordinate of point E.

The coordinate of point E can be calculated by using case (i) of the SDCS strategy. Let it be (x_5, y_5).

Step 3: Generate the equation of line ED.

$$\left(x_{por}^{m\to c} - y_5\right)\left(\max_{do} - \min_{do}\right) = m\left(x_{dor}^{m\to c} - x_5\right)\left(\max_{po} - \min_{po}\right). \tag{33}$$

Here, the slope m can be calculated by the following equation.

$$m = \frac{\left(y_4 - y_5\right)\left(\max_{do} - \min_{do}\right)}{\left(x_4 - x_5\right)\left(\max_{po} - \min_{po}\right)} \tag{34}$$

From equations (33) and (34),

$$\left(x_{por}^{m\to c} - y_5\right)\left(x_4 - x_5\right) = \left(y_4 - y_5\right)\left(x_{dor}^{m\to c} - x_5\right). \tag{35}$$

Step 4: Generate the slope of line BC.

$$\left(x_{por}^{m\to c} - x_{por}^{c\to m}\right) = \tan\lambda\left\{\frac{\left(x_{dor}^{c\to m} - x_{dor}^{m\to c}\right)\left(\max_{po} - \min_{po}\right)}{\left(\max_{do} - \min_{do}\right)}\right\} \quad \text{where, } \lambda = (\theta + 45) \tag{36}$$

Step5: Calculate the new offer $(x_{dor}^{m\to c}, x_{por}^{m\to c})$ by solving equations (35) and (36).

In equations (35) and (36), the only two unknowns are $x_{dor}^{m\to c}$ and $x_{por}^{m\to c}$. Therefore, solving these two equations by general mathematics the value of $(x_{dor}^{m\to c}, x_{por}^{m\to c})$ can be obtained.

The above calculation for case (i) and case (ii) shows the situation when the customer's expected slope is inclined more towards the due date. Similarly, we can generate the new offer for both the cases when the customer's expected slope is inclined more towards price.

Numerical Analysis

Experimental Set up

Numerical analysis is conducted to show the working mechanism and superiority of the proposed strategies on negotiation. Three different categories of customers are considered for the analysis. In the first category (category A) are those who give equal priority to both the issues under negotiation. In contrast, those in the second category (B) have no reservation on price but are highly sensitive towards the due date. Those in the third category (C) are a mixture of the first and the second categories. This category of customer changes their priority on due date and price at each round of negotiation.

Table 2: Generation of counter-offer of the customer for the analysis

Round	Aspiration level		
	Category A	Category B	Category C
1	0.23; (45, $550)	0.115; (30, $550)	0.115; (30, $550)
2	0.3; (50, $650)	0.25; (35, $650)	0.23; (45, $550)
3	0.37; (55, $660)	0.44; (45, $820)	0.44; (45, $820)
4	0.45; (60, $680)	0.55; (48, $960)	0.45; (60, $680)
5	0.57; (65, $720)	0.68; (50, $1090)	0.61; (55, $950)
6	0.61; (73, $770)	0.75; (55, $1150)	0.72; (65, $990)
7	0.68; (80, $790)	(45, $.......)	0.74; (65, $1020)

As shown in Table 2, in this analysis, in order to generate the counter-offer of the customer on the due date and the price, for each category of customer the aspiration level is fixed for all rounds of negotiation. Next, by selecting the due date randomly, the price for the fixed aspiration level is calculated. For example, for the customer with category A, for the 1st round of negotiation, the aspiration level is fixed at 0.23. Then, the due date 45 is picked randomly. Based on this due date and aspiration level the price is calculated as $550. Similarly, the counter-offer for all rounds of negotiation and for all the customer categories is calculated.

During the analysis the maximum, the minimum and the limit levels on due date and price are fixed (see Table 3). Therefore, the quotation on due date and price is (100, $1000). The weight of the manufacturer in relation to both the issues is fixed at 0.5, total expected rounds of negotiation n at 6 and the manufacturer's coefficient q at 0.

Table 3: Manufacturer's data considered for the analysis

($\max_{do}$, $\max_{po}$)	($\min_{do}$, $\min_{po}$)	($\lim_{do}$, $\lim_{po}$)	w_j	n	q
(100, $1000)	(35, $350)	(45, $650)	0.5	6	0

We compare the result of the proposed strategies in terms of the number of negotiation rounds to reach an agreement and the score at which agreement has been reached. Here, the score represents the weighted sum of the agreed due date and price.

The proposed negotiation strategies are affected by various parameters. We also investigate the effect of the manufacturer's coefficient, q, and weight of the manufacturer, w_j, on the negotiation process. For these purposes, we varied q from -0.8 to 0.8 and w_d from 0.1 to 0.9.

Results and Discussion

For all categories of customer, we use three proposed negotiation strategies and analysed the results. Figure 8 shows the process of negotiation between the manufacturer and each category of customer for all the strategies. The numbers 1, 2,…, 6 and 7 in the Figure indicate the rounds of negotiation. From this Figure, for each given strategy the offer and the counter-offer on due date and price in each round of negotiation can be seen.

Results of agreement obtained from Figure 8 are summarized in Table 4. The values inside the parentheses indicate the agreed due date and price while the subscript indicates the round of negotiation at which the agreement was reached.

From the table it can be seen that for customers with category A, the HPA and PDC strategies outperform the SDCS strategy in terms of the least number of negotiation rounds and the score.

For this category of customers, the highest score is received from the PDC strategy. For customers with category B, the HPA and the SDCS strategies outperform the PDC strategy. For this category of customers, the highest score is received from the HPA strategy. On the other hand, we see mixed results for customers with category C. In terms of least number of negotiation rounds the HPA and the PDC strategies outperform the SDCS strategy, but the highest score is obtained from the SDCS strategy.

From these results it can be concluded that the HPA strategy requires the least number of rounds of negotiation to reach an agreement for any category of customer. This is because the HPA strategy generates a new offer in such a way that for any one issue of high priority to the customer, it be close to the customer's counter-offer. It means that the HPA strategy quickly tries to agree on one issue, if the counter-offer on that issue is above limit level and reach an agreement on the other issue through negotiation.

For customers with category A, the PDC strategy seems better as it exhibits a liberal nature while decreasing/increasing the value on due date and price during the negotiations. The new offer is generated such that the sensitivity ratio of the customer is respected. Therefore, this strategy will be better for customers who give equal priority to both the issues or who want reductions on both the issues when compared to the manufacturer's offer.

For customers with category B, even though the SDCS strategy is better in terms of agreed price than the PDC strategy, the score is lower than the HPA strategy. This is because the PDC strategy may generate a new offer where the value on any one issue is lower than the value of the customer counter-offer while the value on the other issue is too high. This shows the aggressive nature of negotiation. Such a strategy is effective when the customer has no problem with paying the higher value on one issue in order to obtain a benefit on the other.

From the result obtained for customers with category C, we can conclude that in a negotiation process a trade-off may exists between the number of negotiation rounds and the score. Here, even though the score is high for the SDCS strategy, the rounds of negotiation taken to reach an agreement is also high. More negotiation rounds to reach an agreement mean more time taken. If the negotiation time is lower, the order may be rejected.

From this result it can be noted that when the counter-offer is less than or equal to the limit level or when the customer's slope doesn't intersect the aspiration level above the limit level on due date/price, the HPA and the SDCS strategies work in a similar manner (first and second rounds of negotiation in the Figures 8(b) and 8(c)). As shown in Figure 8, for most of the rounds and any category of customer, in the case of HPA or SDCS strategy, there will be an abrupt change in one issue as compared to other while generating the new offer. But, in the case of the PDC strategy the generated new offer will be in accordance with the rate of reduction/increment demanded by the customer. Due to this there will be no abrupt change in one issue compared to other, for each category of customer, when using the PDC strategy.

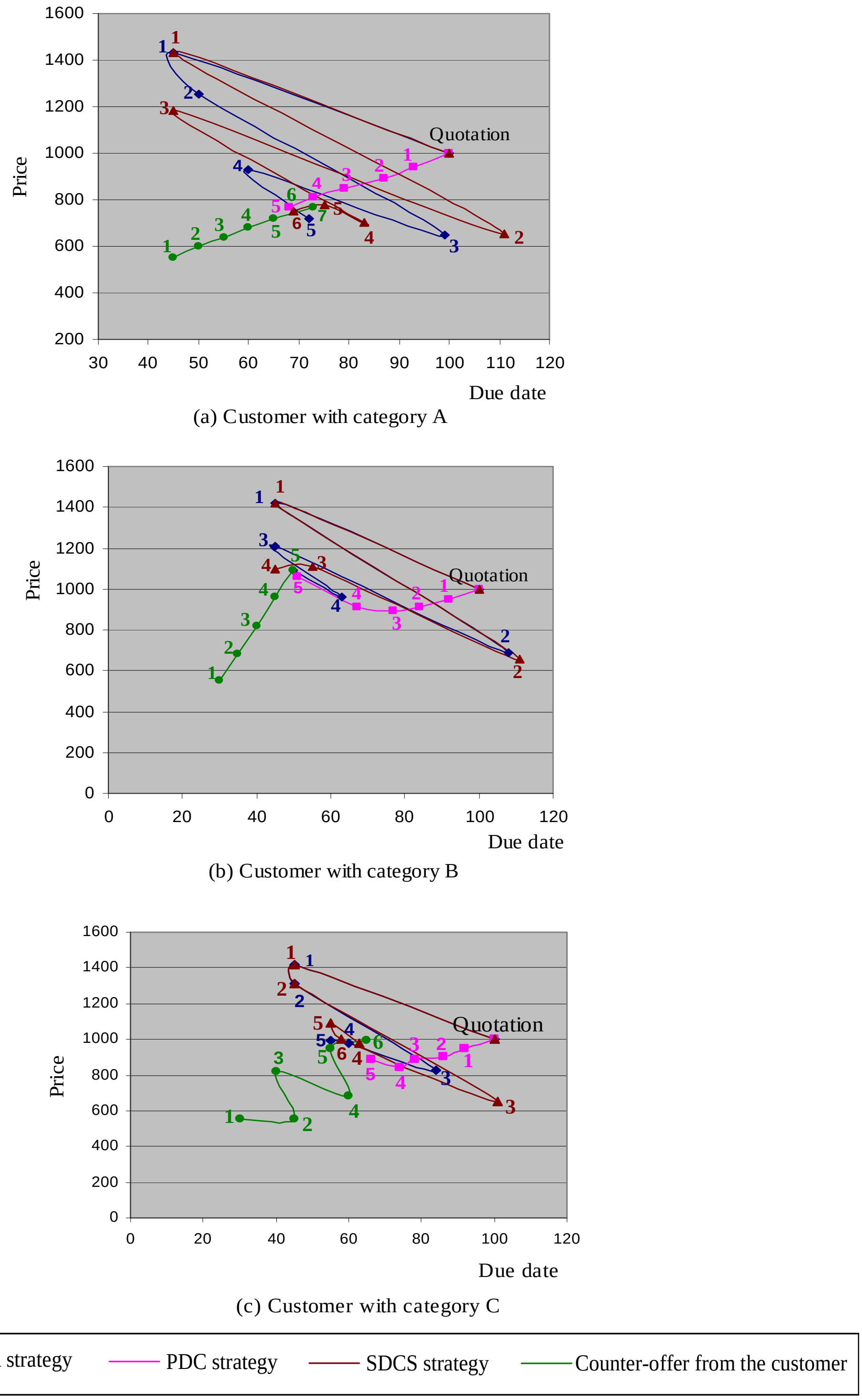


Figure 8: The offers and the counter-offers for different categories of customer

Table 4: Result obtained for the three different strategies

Strategy	Agreement					
	Category A		Category B		Category C	
	$(d, p)_r$	Score	$(d, p)_r$	Score	$(d, p)_r$	Score
HPA	$(72,\$721)_5$	0.57	$(56,\$1080)_4$	0.72	$(55,\$995)_5$	0.65
PDC	$(71,\$772)_5$	0.61	$(51,\$1061)_5$	0.67	$(66,\$885)_5$	0.65
SDCS	$(67,\$750)_6$	0.55	$(48,\$1133)_4$	0.70	$(63,\$980)_6$	0.70

Next, to check the effect of manufacturer's coefficient, q, and weight, w_j, on the negotiation process, we consider the data on counter-offers for customer with category A.

Table 5 shows the effect of the manufacturer's coefficient on the proposed negotiation strategies. The values in the last column of the table, for example $(80, \$767)_{6C}$ indicate that an agreement has been reached in the 6th round and is accepted by the customer. The accepted due date is 80 and price is $767. The subscript M instead of C represents that the acceptance is made by the manufacturer.

Table 5: Result obtained for different value of manufacturer's coefficient

q	Strategy	Offer						OA decision
		1st round	2nd round	3rd round	4th round	5th round	6th round	
-0.8	HPA	(45, 1532)	(50, 1448)	(126, 649)	(60, 1270)	(65, 1155)	(93, 771)	Rejected
	PDC	(99, 992)	(97, 986)	(91, 957)	(87, 950)	(84, 939)	(80, 930)	Rejected
	SDSC	(45, 1532)	(130, 655)	(45, 1570)	(119, 680)	(51, 1295)	(93, 771)	Rejected
-0.4	HPA	(45, 1505)	(50, 1397)	(120, 640)	(106, 680)	(65, 1012)	(80, 764)	$(80,\$767)_{6C}$
	PDC	(98, 976)	(95, 947)	(92, 920)	(86, 892)	(80, 879)	(72, 869)	Rejected
	SDSC	(45, 1505)	(124, 657)	(45, 1390)	(105, 693)	(58, 1095)	(89, 764)	Rejected
-0.2	HPA	(45, 1485)	(50, 1344)	(110, 640)	(60, 1049)	(83, 728)		$(73,\$770)_{5M}$
	PDC	(97, 965)	(92, 924)	(86, 880)	(81, 839)	(76, 798)	(74, 779)	$(74,\$779)_{6C}$
	SDSC	(45, 1485)	(119, 654)	(45, 1290)	(96, 689)	(64, 918)	(75, 770)	$(75,\$770)_{6C}$
0	HPA	(45, 1433)	(50, 1253)	(99, 646)	(60, 932)	(72, 721)		$(72,\$721)_{5C}$
	PDC	(93, 940)	(87, 893)	(79, 848)	(73, 809)	(68, 767)		$(68,\$767)_{5C}$
	SDSC	(45, 1433)	(111, 653)	(45, 1186)	(83, 702)	(75, 779)	(69, 751)	$(69,\$751)_{6C}$
0.2	HPA	(45, 1368)	(50, 1136)	(55, 969)	(69, 686)			$(65,\$720)_{4M}$
	PDC	(91, 918)	(81, 834)	(74, 779)	(64, 743)	(67, 680)		$(67,\$680)_{5C}$
	SDSC	(45, 1368)	(99, 654)	(45, 1069)	(67, 706)	(74, 610)		$(74,\$610)_{5C}$
0.4	HPA	(45, 1277)	(50, 963)	(55, 735)				$(60,\$680)_{3M}$
	PDC	(85, 877)	(70, 763)	(60, 695)				$(60,\$680)_{3M}$
	SDSC	(45, 1277)	(81, 653)	(45, 835)	(51, 710)			$(51,\$710)_{4M}$

From these results it is evident that the rounds of negotiations taken to reach an agreement increases with the decreasing value of q for all the strategies. When the value of q is very low then the chances of the order being rejected are high. This is because the step size of aspiration level considered by the manufacturer will be small at the low value of q. This in turn will increase the distance between the counter-offer and the offer. Therefore, more rounds of negotiations are needed to reach an agreement at this lower value of q. The opposite will be the case when the value of q is high. At $q = 0$, the step size will be neither high nor low. With this value the speed of negotiation will take a normal pace.

Figure 9 shows the effect of the weight of manufacturer, w_d, on the new offer on due date and price generated by the three strategies. From Figure 9(a), it can be seen that, in the HPA strategy with the increase in the weight for due date there will be an increase in the value of price while retaining the same due date. This is because, as shown in Figure 5 where the customer's expected slope is inclined towards due date, with the increasing weight placed by the manufacturer on due date (aspiration level with slope 2), point A will move up - retaining the same due date and increasing the price. This situation continues for a certain value of w_d. When the value of w_d is very high, then the new offer will shift near point B which shows almost a similar price to that of counter-offer price but with a high due date. This is because at a very high value of w_d, the probability of acceptance of coordinate near point B will be higher than the coordinate near point A.

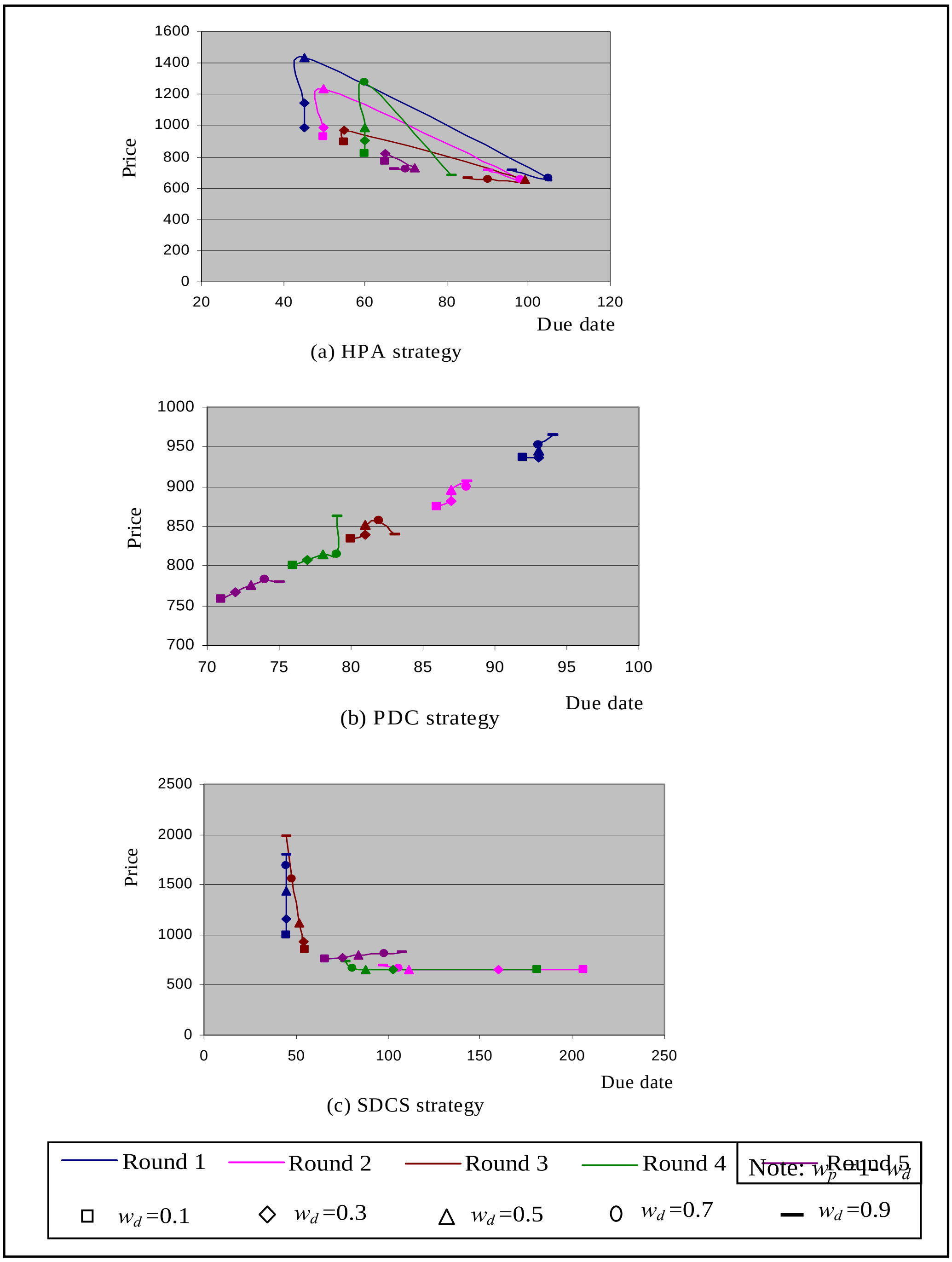


Figure 9: Effect of weight of manufacturer (w_d) on the proposed strategies

From Figure 9(b) it can be seen that the effect of w_d is not very significant in the PDC strategy. Here, with increasing w_d, the value on both due date and price only increases by a small amount. This result is also evident in Figure 6 where point C increases by a very small upwards distance when there is a significant increase in the value of w_d (aspiration level with slope 2).

From Figure 9(c), we can conclude that in the SDCS strategy the effect of w_d will be different with different customer expected slopes. If the customer's expected slope and the aspiration level do not converge at the same point above the limit level on due date and/or price then the effect is similar to that of the HPA strategy i.e., retain the limit level on due date and increase the price or vice-versa. This is decided based on whether the customer's expected slope is inclined towards due date or price. If the customer's expected slope and the aspiration level converge at the same point above the limit level then the effect of w_d depends on whether the customer's expected slope is positive or negative. The slop will be positive when the angle of deflection θ is very large, otherwise it will be negative. If the slope is positive, with the increasing $w_{d,}$ the value on both the issues will increase as shown by round 5 in Figure 9(c). On the other hand, if the slope is negative, with the increasing $w_{d,}$ the due date will decrease and the price increase as shown by round 3 in Figure 9(c).

Conclusion

In an MTO system, when the new customer arrives the manufacturer is asked to submit a quotation for their supply of the product. If the quotation is not acceptable to them the customer will negotiate with the manufacturer. Therefore, in such a system, the quotation and the mechanism by which the manufacturer negotiates with the customer play a vital role in the decision process. Inclusion of a negotiation margin while preparing a quotation helps the manufacturer to realize their expected value on the negotiated issues. This margin should be realistic and not be so high that the manufacturer and customer cannot reach an agreement within the available negotiation time.

In the negotiation process, the exchange of information between the negotiating parties is a key to reaching an agreement. This helps one party to understand the intention or desire of the other so that they can formulate their strategies for further negotiation. The length or the number of rounds of negotiation also plays a crucial role in reaching an agreement. More rounds of negotiation may increase the probability of reaching an agreement but require more time. According to Mehmet (2001) there are many factors that affect the length of negotiation. Basically, the negotiating parties may fix the deadline or number of rounds for the negotiations before negotiation begins.

In this paper, implementing the concept of dynamic aspiration level, we proposed a customer cum company centric approach to the OA decision where negotiation acts as a tool for reaching an agreement on the due date and the price. The aspiration level depends on the counter-offer received from the customer. With this approach, the manufacturer can set the appropriate step size to be reduced for defining the aspiration level for the next negotiation round. This helps them to avoid reducing the aspiration level by a larger step than necessary, and will increase their benefits in terms of agreed due date and/or price. The length of negotiation is defined in terms of maximum rounds of negotiation while calculating the aspiration level. Three different strategies are proposed to generate new offers during the negotiation process. Each strategy utilizes the expected slope of the customer while generating the new offer. Expected slope helps the manufacturer to understand the customer priorities for due date and price. In our numerical analysis we show the working mechanism of each strategy and discuss the situation where a particular strategy outperforms other strategies. From this analysis, it was found that when the customer gives equal priority to both the issues with respect to the offer of manufacturer, the PDC strategy performs better. On the other hand, if the customer gives priority to only one issue and is willing to pay more on the other, the HPA strategy is better. But, if the customer changes their priorities at regular intervals the SDCS strategies performs better. In terms of the least number of negotiation rounds taken to reach an agreement, for any category of customer, the HPA strategy is better. We also checked the effect of various parameters on these strategies and gave reasons for the effect.

The proposed method is based on the assumption that the manufacturer will consider only a single negotiation at any given time. If a new customer arrives in the system prior to the completion of negotiations with a previous customer, the newly arrived customer has to wait until the ongoing negotiation is terminated. This may compel the new customer to search for another manufacturer if the waiting time is too long. The research could be extended to accommodate multiple negotiations at a time. This requires the manufacturer to consider the contingency effect of other orders while negotiating with the customer. In addition, in this paper the preferences of the customer in regard to due date and price have been measured with respect to the customer's expected slope which is based on the angle of deflection between the offer and the counter-offer. Along with the angle of deflection, the inclusion of behavioural theories of customer actions in negotiations may generate more robust ideas for defining the preferences of customers in the negotiation issues.

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Comparison Support Method for Analytic Hierarchy Process

Yumi Tadano[1], Hidenori Kawamura[1], Keiji Suzuki[1] and Azuma Ohuchi[2]
[1] Grad School of Information Science and Technology, Hokkaido University, Japan
[2] Hokkai School of Commerce, Hokkaido, Japan
(Email: yumi-hr@complex.eng.hokudai.ac.jp; kawamura@complex.eng.hokuadi.ac.jp;
suzuki@complex.eng.hokudai.ac.jp; ohuchi@hokkai.ac.jp)

Abstract
The analytic hierarchy process (AHP) proposed by T. L. Saaty is a method for decision making that considers uncertain situations or multiple evaluation criteria. In the AHP, a decision maker compares two elements between evaluation criteria and alternatives. Therefore, comparing all pairs is difficult when evaluating many alternatives. In this study, we present the "comparison support method" for evaluating many alternatives when a decision maker needs to decide the highest priority alternative. The comparison support method stops pairwise comparisons when the best solution, i.e., the highest priority alternative, is found, even if all pairs have not been compared. We experimentally verified the effectiveness of the comparison support method by searching for the necessary amount of data and the number of comparisons that is required to find the best solution. Moreover, for each input order and input value of a pairwise comparison matrix, we searched for the number of input patterns where the best solution is found. The results when there were four alternatives clearly showed that searching for the highest priority alternative requires at least three inputs. Therefore, unnecessary pairwise comparisons can be omitted, which reduces the decision maker's burden. A large difference in the number of patterns where the best solution was found according to the input order was also clearly shown. Hence, if an input order that has high probability for finding the best solution can be suggested, the decision maker can find the solution more quickly.

Keywords: Decision Modelling and Theory, Information Technology, Analytic Hierarchy Process.

Introduction

The analytic hierarchy process (AHP) [1-3] proposed by T. L. Saaty is a decision making support method for expressing human subjective judgments numerically. The AHP calculates overall evaluations by structuring a hierarchy of the problem and comparing pairs of elements at each level. Several decision making problems are solved by using the AHP [4-6] because human subjective judgments including preference or guess can be expressed numerically by the pairwise comparisons. In addition, the AHP is straightforward.

In the AHP, a decision maker inputs values of pairwise comparisons between evaluation criteria and alternatives. Comparing all pairs becomes difficult when evaluating many alternatives. A decision maker has to re-compare all pairs after new alternatives are added.

We present the "comparison support method" to address these difficulties by stopping pairwise comparisons when the best solution is found even if all comparisons have not been finished. We experimentally verified the effectiveness of the comparison support method by searching for the necessary amount of data and the number of comparisons that is required to find the best solution. Moreover, for each input order and input value of a pairwise comparison matrix, we searched for the number of input patterns where the best solution is found.

AHP

This section outlines the AHP and discusses its many-alternatives problem.

Outline of AHP

The AHP is a method for deciding the priority of several alternatives for which the superiority and inferiority cannot be evaluated immediately because the alternatives have several evaluation criteria. The AHP is shown below. Various priority calculation methods for Step 3 have been investigated: the eigenvector method, geometric mean method, and logarithmic least square method.

- Step 1. Investigate the components of the problem and structure a hierarchy of the components.
- Step 2. Compare two elements that share a common parent based on the fundamental scale (Table 1) at each hierarchy level.
- Step 3. Calculate the priorities for the pairwise comparison matrix.
- Step 4. Calculate the priority of each alternative based on the hierarchy.

For example, consider the purchase of a car. The decision maker has three alternatives {Car A, Car B, Car C} and considers three evaluation criteria {Price, Fuel Economy, Displacement}. The hierarchy is shown in Fig. 1. Next, the decision maker performs pairwise comparison of the evaluation criteria and calculates priorities (Table 2) and then performs pairwise comparison of the alternatives for each evaluation criteria and calculates priorities (Tables 3–5). The calculated priority results for each alternative based on the hierarchy are shown in Table 6. Car A has the highest priority

Table 1. Fundamental scale of AHP

Intensity of Importance	*Definition*	*Explanation*
1 (1/1)	Equal importance	Two activities contribute equally to the objective
2 (1/2)	Moderate importance	Experience and judgment strongly favour the row activity over the column
3 (1/3)	Strong importance	Experience and judgment slightly favour the row activity over the column
4 (1/4)	Very strong or demonstrated importance	The row activity is favoured very strongly over the column; its dominance is demonstrated in practice
5 (1/5)	Extreme importance	The evidence favouring the row activity over the column is of the highest possible order of affirmation
Values in parentheses		The column activity is more important than the row

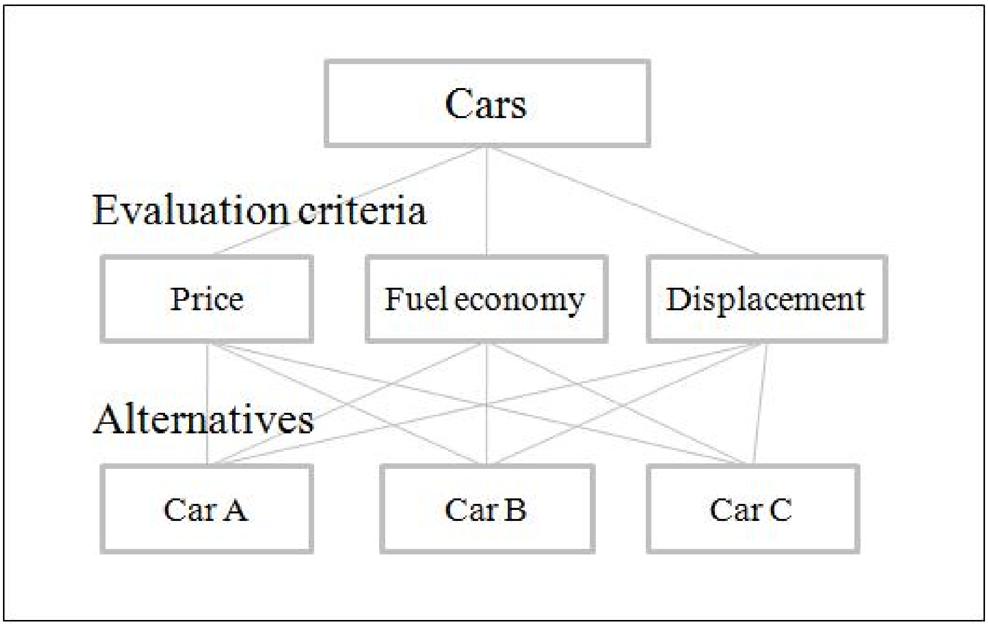

Figure 1. Hierarchy of purchase-of-car problem

Table 2. Pairwise comparison and priorities of evaluation criteria

	Price	*Fuel economy*	*Displacement*	*Priority*
Price	1	3	5	0.64
Fuel economy	1/3	1	3	0.26
Displacement	1/5	1/3	1	0.11

Table 3. Pairwise comparison and priorities of price alternatives

	Car A	*Car B*	*Car C*	*Priority*
Car A	1	3	3	0.6
Car B	1/3	1	1	0.2
Car C	1/3	1	1	0.2

Table 4. Pairwise comparison and priorities of fuel economy alternatives

	Car A	*Car B*	*Car C*	*Priority*
Car A	1	2	3	0.53
Car B	1/2	1	2	0.30
Car C	1/3	1/2	1	0.16

Table 5. Pairwise comparison and priorities of displacement alternatives

	Car A	*Car B*	*Car C*	*Priority*
Car A	1	5	5	0.70
Car B	1/5	1	2	0.18
Car C	1/5	1/2	1	0.11

Table 6. Calculated priority of each alternative

	Calculation	*Priority*
Car A	0.6•0.64+0.53•0.26+0.70•0.11=0.5988	0.60
Car B	0.2•0.64+0.30•0.26+0.18•0.11=0.2258	0.23
Car C	0.2•0.64+0.16•0.26+0.11•0.11=0.1817	0.18

Amount of time for pairwise comparison in AHP

The AHP calculate priorities after decision maker inputs values of all paired comparisons between the evaluation criteria and alternatives. This is called the relative measurement method. In this method, for m evaluation criteria and n elements, the amount of time for pairwise comparison is represented by the following equation.

$$ {}_{m}C_{2} + {}_{n}C_{2} \times m = m(m-1)/2 + nm(n-1)/2 \qquad (1) $$

The first term shows comparisons between evaluation criteria and can be decreased by making a multilevel hierarchy. The second term shows comparisons between alternatives. This term is difficult to decrease because comparisons between alternatives cannot represent a multilevel hierarchy. Therefore, a decision maker's burden increases with n. For example, for 3 evaluation criteria and 10 alternatives, the decision maker has to compare 138 pairs. Furthermore, a decision maker has to re-compare all the pairs after new alternatives are added.

T. L. Saaty proposed another method called the absolute measurement method [3, 7]. This method evaluates each alternative for each criterion by absolute measurement instead of pairwise comparison. Therefore, this method could decrease the amount of time for pairwise comparison between alternatives. However, not every problem with a large number of alternatives can be solved using the absolute measurement method, particularly ones in which setting the priorities of the evaluation criteria independently from the structural effects is undesirable. Consequently, the relative measurement method must be used for these problems.

Comparison Support Method

When a decision maker calculates priorities using the AHP, there are several conditions: the relative priorities of each alternative are needed; the alternative that has the highest priority is needed, etc. The relative measurement method even compares low priority alternatives. The pairwise comparisons that do not influence the decision about the highest priority alternatives are included in the comparisons between low priority alternatives. Therefore, the pairwise comparisons include omissible comparisons when a decision maker needs the alternative that has the highest priority.

Algorithm

We present the comparison support method for solving the many-alternatives problem when a decision maker needs to decide the highest priority alternative and when a two-level hierarchy is assumed (Fig. 2).

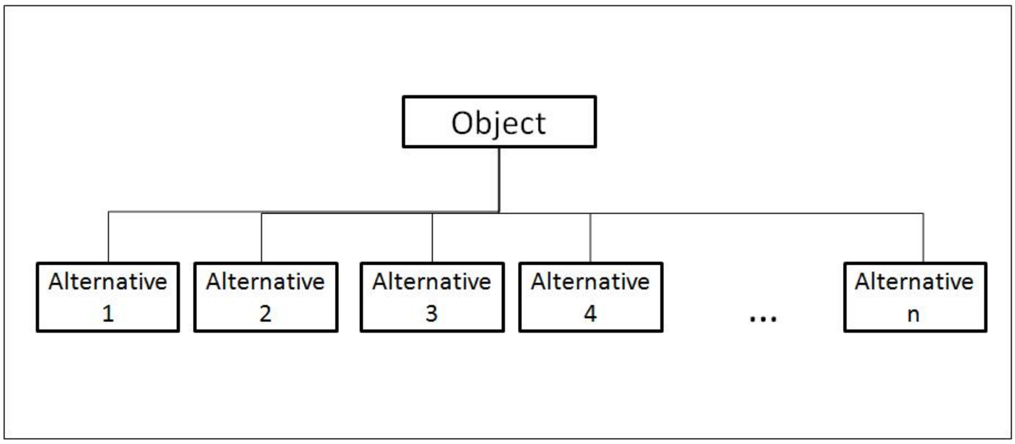

Figure 2. Assumed hierarchy for many-alternatives problem

The comparison support method stops pairwise comparisons when the best solution is found even if all comparisons have not been finished. The algorithm of this method for calculating one alternative that has the highest priority follows. From Step 1 to Step 3, all input patterns are enumerated and the highest priority alternative of each input pattern is found. In Step 4, "*" represents a wild card that is an arbitrary numerical value of the nine numerical values in Table 1. A pairwise comparison matrix X represents the input pattern that is input by a decision maker. The initial values of all elements of the pairwise comparison matrix X are "*". In Steps 5 and 6, whether the best solution is found is checked whenever the decision maker inputs a value in an element of the pairwise comparison matrix. The situation when the best solution is found represents the case of the highest priority alternatives being the same in all the input patterns; an arbitrary numerical value is entered as the wild card.

Step 1. Input the number n of alternatives.
Step 2. Enumerate all input patterns of pairwise comparison matrix X' for which the number of alternative is n.
Step 3. Calculate the priorities of all input patterns of X' using the simplified eigenvector method, and find the highest priority alternative.
Step 4. Input * (wild card) in all elements of another pairwise comparison matrix X.
Step 5. Input a value of the pairwise comparison matrix's element x_{ij} to compare the alternatives i and j.
Step 6. Based on X', check whether the best solution is found, even if elements of * in X are not yet decided.
- If the best solution is found:
  Stop pairwise comparison and output the solution.
- If the best solution is not found:
  Return to Step 5.

The algorithm of the simplified eigenvector method [8] follows.

Step 1. Calculate the summation of the elements of each column in the pairwise comparison matrix.
Step 2. Divide each element of each column in the pairwise comparison matrix by the summation.
Step 3. Calculate the average of the elements calculated in Step 2 for each row. These averages represent the priorities.

A sequence chart of the comparison support method is shown in Fig. 3. Class 1 enumerates the input patterns and the best alternative (the highest priority alternative) of each pattern. Class 2 searches to see if each pattern's highest alternative can be obtained and if all of the patterns are the same. If the alternatives are the same, the best solution is found and the alternative is returned.

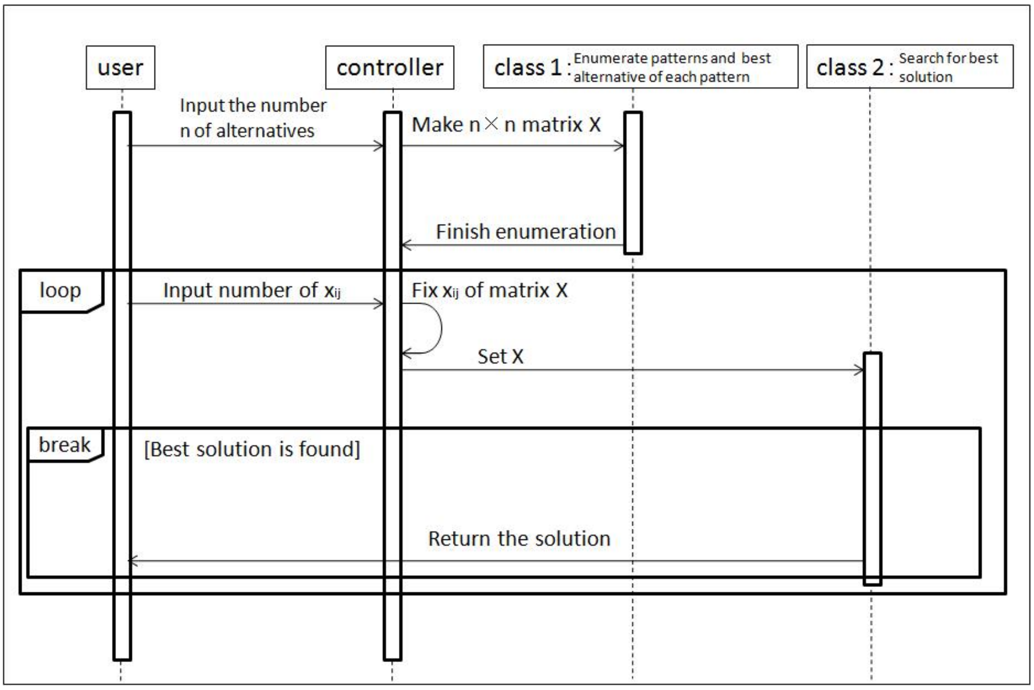

Figure 3. Sequence chart of comparison support method

Time complexity of comparison support method

The time complexity of the comparison support method is exploded. For n alternatives, the pairwise comparison has $n(n-1)/2$ patterns because all diagonal elements of the pairwise comparison matrix are one and element x_{ij} is the inverse of the number of element x_{ji}. For example, a case of four alternatives is shown in Table 7. In this case, we consider the six elements at the upper right of the pairwise comparison matrix. Moreover, nine kinds of numerical values are input in each element of the matrix (Table 1). Therefore, there are $9^{n(n-1)/2}$ input patterns in the pairwise comparison matrix when there are n alternatives. The time complexity for calculating the priority of each pattern is o(n) when using the simplified eigenvector method. Therefore, the time complexity for calculating priorities of all the patterns is o(9^{n*n})*o(n).

Table 7. Pairwise comparison matrix of four alternatives

	Alternative 1	*Alternative 2*	*Alternative 3*	*Alternative 4*
Alternative 1	1	x_{12}	x_{13}	x_{14}
Alternative 2	$1 / x_{12}$	1	x_{23}	x_{24}
Alternative 3	$1 / x_{13}$	$1 / x_{23}$	1	x_{34}
Alternative 4	$1 / x_{14}$	$1 / x_{24}$	$1 / x_{34}$	1

We propose a new method for decreasing the complexity that prepares a database of all input patterns and each pattern's best alternative. Steps 2 and 3 of the comparison support method's algorithm or class 1 of the comparison support method's sequence chart (Fig. 3) are omissible when the decision maker uses the database to evaluate alternatives. The amount of data increases in proportion to $9^{n(n-1)/2}$ when there are n alternatives. However, the amount of data can be decreased by using wild cards and grouping the input patterns that have the same best alternative.

Experiment

We experimentally verified whether the comparison support method was effective by searching for the necessary amount of data and the number of comparisons that is required to find the best solution. We focused on the following three points.

- Comparison of the amount of data and the number of comparisons between two cases: one enumerates all input patterns and the best alternatives, and the other compresses the data using wild cards. In addition, the breakdown of the input patterns of the number of wild cards for the compression data was investigated.
- In each number of pairwise comparisons, the number of patterns where the best solution was found was investigated.
- For each input order and input value of the pairwise comparison matrix, the number of input patterns where the best solution was found was investigated.

<u>Setup</u>

We investigated the case of four alternatives shown in Table 7. We considered the six elements at the upper right of the pairwise comparison matrix. Nine patterns of numbers (Table 1) were input in each matrix's element.

Each input pattern and the best alternatives were brought together in one line and output to a comma separated value (CSV) file when compared with the amount of data. The output form of one line was "x_{12}, x_{13}, x_{14}, x_{23}, x_{24}, x_{34}, Best Alternative number". An example is shown in Fig. 4. In this data file, "0" represents a wild card. This CSV data file is used as the database in the comparison support method.

The values of x_{12} and x_{13} were fixed as 5 when the number of input patterns that find the best solution for each input order and input value of the pairwise comparison matrix was investigated. Therefore, for each input order and input value of four elements {x_{14}, x_{23}, x_{24}, x_{34}}, we searched for the number of input patterns where the best solution was found. There were 24*94=157,464 patterns for the input order and input value.

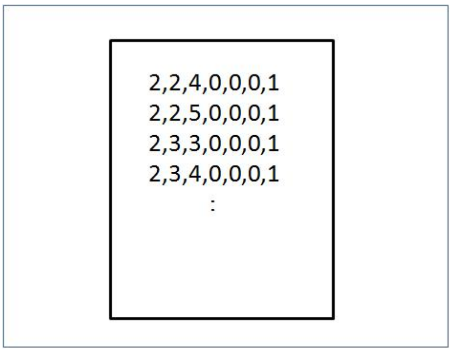

Figure 4. Example of output form

<u>Results</u>

Table 8 shows the results of comparing the amount of data and the number of comparisons and of investigating the breakdown of the input patterns for the number of wild cards for the compression data. In the case of four alternatives, there are 96=531,441 input patterns. The CSV data file was 10,901 KB when all input patterns were enumerated. In contrast, when wild cards were included, the CSV data file was 1935 KB. The data file including wild cards reduces the data by 82% compared with the data file that enumerates all input patterns. In the breakdown of the input patterns for the number of wild cards, the number of patterns where the wild card was not included was the same as the number of patterns that required all pairwise comparisons to find the best solution. Therefore, 61,520 patterns are required for all pairwise comparisons to find the best solution.

Table 8. Comparison of data of all input patterns and data compressed by using wild cards

		Number of input patterns	*CSV file size* (KB)
All input patterns		531,441	10,901
Compressing data using wild cards	Total	97,457	1935
Breakdown of input patterns for number of wild cards	6	0	
	5	0	
	4	0	
	3	240	
	2	2976	
	1	32,721	
	0	61,520	

Table 9 shows the number of input patterns that find the best solution for each number of input pairwise comparisons. Figure 5 shows the percentage of each number of input pairwise comparisons of all input patterns that find the best solution. From Table 9, searching for the best alternative clearly requires at least three inputs. Moreover, from Fig. 5, 33% of all patterns can find the best solution when 3 pairwise comparisons are input, and 88% of all patterns can find the best solution before all pairwise comparisons are input. In contrast, 12% of all patterns require all the pairwise comparisons to be input.

Table 9. Number of input patterns that find best solution for each number of inputs

Number of inputs	*Number of input patterns that find best solution*
1	0
2	0
3	174,960
4	320,760
5	569,921
6	531,441

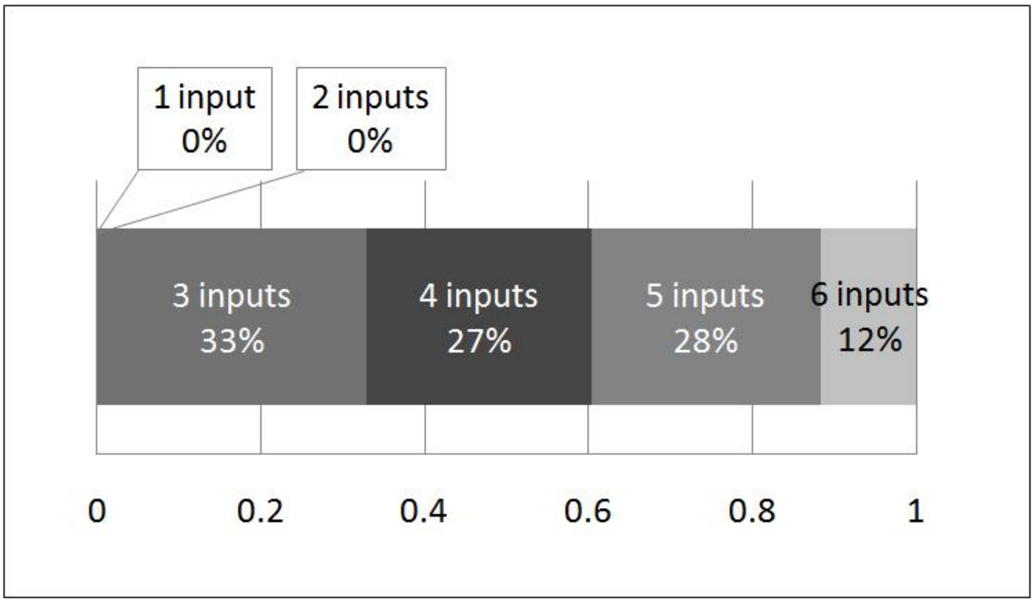

Figure 5. Percentage of each number of input pairwise comparisons of all input patterns that find best solution

Figure 7 shows the tree structure for the input order and number of input patterns when the solution is or is not found. In each node of the tree structure (Fig. 6), the four characters on the left show the order of inputting each element in the pairwise comparison matrix, respectively corresponding to x_{14}, x_{23}, x_{24}, and x_{34}. The "*" symbol represents a wild card. For example, "①③*②" represents the decision maker inputting in the order of x_{14}, x_{34}, and x_{23}, and x_{24} has not been input yet. Therefore, any value could be input into x_{24}. The value at the centre shows the number of input patterns excluding the patterns where the best solution is already found. This centre value is calculated as the product of nine and the number of patterns where solutions are not found until the parent node. The upper right value shows the number of patterns where the best solution is found, and the lower right value shows the number of patterns where the best solution is not found.

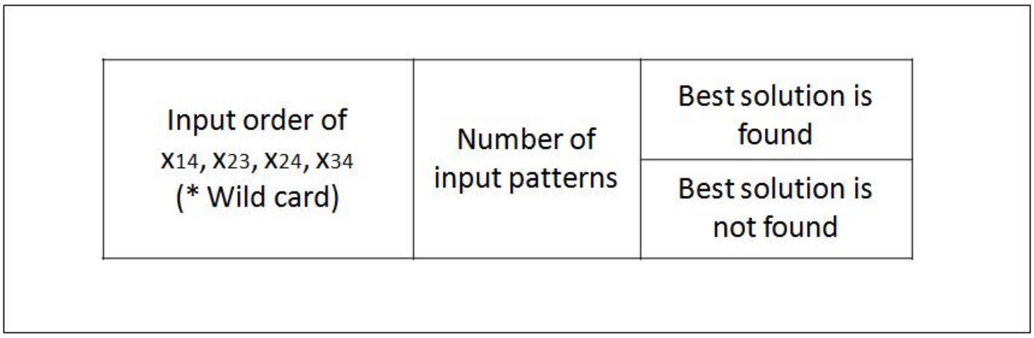

Figure 6. Nodes of tree structure

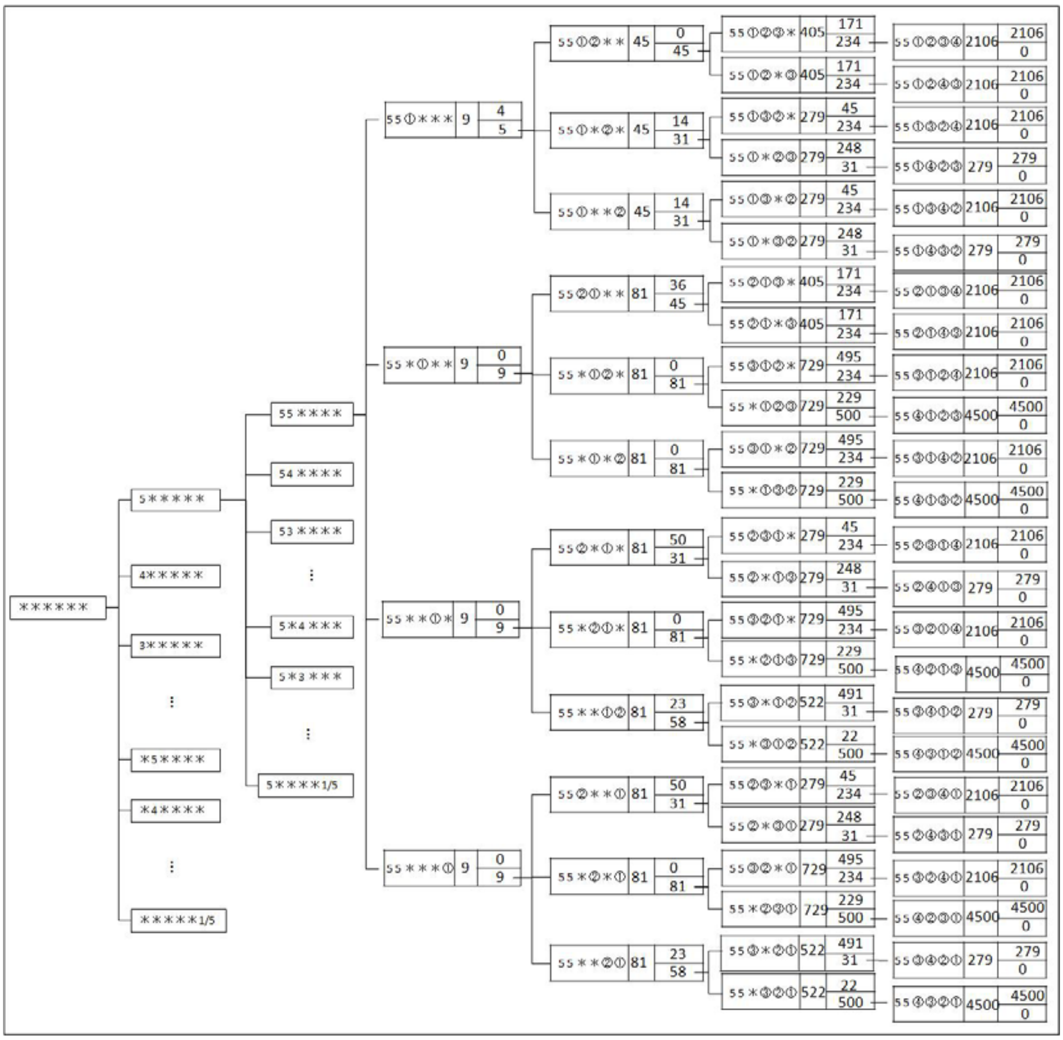

Figure 7. Tree structure of input order and number of input patterns when best solution is or is not found

According to Fig. 7, when x_{14} is input first, about half of all patterns find the best solution. However, when other elements are input first, no patterns find the best solution earlier. The result is a large difference in the number of patterns where the best solution is found according to the input order. The cases of input in the order of x_{14}, x_{24}, x_{34}, x_{23} or x_{14}, x_{34}, x_{24}, x_{23} have the highest probabilities of finding the best solution. For some elements the best solution was not found although all values were input into the elements. Calculating the sum of the number of patterns where the best solution was found in the leaf node clearly showed that there were 53,946 patterns where the best solution was found when all elements were input.

Discussion

The results clearly show that the CSV data file that includes wild cards has an 82% reduction in data compared with the CSV data file that enumerates all input patterns when there are four alternatives. Whenever the number of alternatives is increased, the number of all input patterns becomes 9^n because there are $9^{n(n-1)/2}$ input patterns of the paired comparison matrix when there are n alternatives. In this experiment, the reduced amount of data for each alternative by using wild cards could not be calculated. However, compressing the data by using a wild card could be effective. In this experiment, we made a data file that enumerates all input patterns and compressed the data file by using wild cards. In this case, the time complexity for making a database that includes wild cards is $o(9^{n*n})*o(n)*o(10^{n*n}*9^{n*n})$. $o(9^{n*n})*o(n)$ is the time complexity for making a CSV data file that enumerates all input patterns, and $o(10^{n*n}*9^{n*n})$ is the time complexity for making a CSV data file that includes wild cards based on the all-input-patterns data file. Therefore, it is necessary to develop an algorithm that reduces the time complexity to make a database that corresponds to more alternatives.

In the results in Fig. 7, there are 53,946 patterns where the best solution was found when the all elements were input, considering the input order and number of input patterns. The 157,464 patterns of input order and input value are 34% of all the patterns. Therefore, 66% of all patterns can find the best solution before all elements are input, and a decision maker can stop comparison when the computer finds the best alternative.

In this case, the values of x_{12} and x_{13} are fixed as 5. Therefore, alternative 1 is much more important than alternatives 2 and 3. Alternative 2 could be expected to be as important as alternative 3 without a comparison being conducted. In the experiment results, there are few patterns where the best solution is found when comparing alternatives 2 and 3; there are zero patterns at the second input, and there are a maximum of 45 patterns at the third input. This is because alternative 2 is already expected to be as important as alternative 3. In contrast, there are many patterns where the best solution is found when comparing alternative 4 and other alternatives. This is because alternative 4 had never been compared with other alternatives. In particular, when alternatives 1 and 4 are compared, there are many patterns where the best solution is found. This is because alternative 1 is clearly more important than alternatives 2 and 3. Therefore, if it is clear that alternative 4 is less important than alternative 1, the solution is decided by alternative 1. In the results, the best solution may be found earlier by pairwise comparison between alternatives whose relative importance cannot be expected from past input.

The tree structure in Fig. 7 is a portion of the all-input-patterns tree structure. Therefore, when the values of x_{12} and x_{13} are not fixed as 5, the input order and number of input patterns where the solution is or is not found are similar. Moreover, these experiment results include inconsistency in the comparison between alternatives. Hence, in practice, the best solution may be found earlier.

Conclusion

In this paper, we focused on a case where the decision maker needs the highest priority alternative. We presented a comparison support method for solving the difficulty of comparing all pairs when evaluating many alternatives. The comparison support method stops pairwise comparisons when the best solution is found, even if all pairs have not been compared. An experiment showed that 88% of all patterns can find the best solution before all elements are input, and the decision maker can stop comparison when the computer finds the best of four alternatives. Searching for the best alternative requires at least three inputs. Therefore, unnecessary pairwise comparisons can be omitted, which reduces the decision maker's burden. However, the time complexity of the comparison support method is exploded. We proposed a method for decreasing the complexity that prepares a database of input patterns and each pattern's best alternative and that uses the data when the decision maker evaluates alternatives. The data file size becomes enormous because the number of all input patterns is increased by 9^n whenever the number of alternatives is increased. A data file that includes wild cards has an 82% reduction in data size compared with a data file that enumerates all input patterns when there are four alternatives. Therefore, the method of compressing data by using wild cards is effective. Moreover, a large difference is clear in the number of patterns where the best solution is found according to the input order. Hence, if an input order that has high probability of finding the solution can be suggested, the decision maker can find the solution more quickly.

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Price Stabilization Using Buffer Stock in Duopoly-Like Market Considering Expectation of Stakeholders

Wahyudi Sutopo[1], Senator Nur Bahagia[2],
Andi Cakravastia[2], and T.M.A. Ari Samadhi[2]
[1]Ph. D student of Industrial Engineering & Management, Bandung Institute of Technology
[1]Dept. of Industrial Engineering, Sebelas Maret University, Surakarta, Indonesia
[2]Dept. of Industrial Engineering, Bandung Institute of Technology, Bandung, Indonesia
(Email: sutopo@uns.ac.id; {senator, andi, asamadhi@mail.ti.itb.ac.id})

Abstract

This paper considers a staple food distribution problem in agro-industry. There is a great difference in staple food supplies during the harvest season and planting season while the demand is relatively stable in whole year. Moreover, the domestic supply is lower than the total consumption. These situations will motivate the speculators to take action for their own benefit, so that it will cause price volatility and scarcity. It will bring disadvantages to the stakeholders such as producer, wholesaler, consumer, and government. The government usually takes initiatives to stabilize the price; one of them is market intervention policy by using buffer stocks schemes. The objective of this research is to develop buffer stock model for operating market intervention program. In the previous researches, some models had been developed separately to protect either producer or consumer from loses. Therefore, in this research, we developed a model with considering the expectation of the stakeholders simultaneously on the market distribution comprise with 2 actors, government and wholesaler in such a way that it calls duopoly-like market. A nonlinear programming has been developed to solve the decision variables of quantity and price of buffer stocks. A market model with inventory was applied for solving the market price equilibrium. The study shows that the proposed model can solve the problem of staple food price stabilization.

Keywords: buffer stocks, market intervention, price stabilization, staple food.

Introduction

Supply Chain Management (SCM) is the integration of key business processes from end user through original suppliers and provides products, services, and information for customers and other stakeholders. The concept of SCM is required to achieve suitable economic results together with the desired consumer satisfaction levels (Guilléna *et al*., 2005). The SCM problem may be considered at different levels such as strategic, tactical and operational. The level depends on the planning horizon and the detail of the analysis (Chopra and Meindl, 2004). In this work, a strategic level Supply Chain (SC) design problem is addressed to determine a buffer stock model.

The SC design problem discussed here is a staple food distribution system in agro-industry. There is a great difference in staple food supplies, such as sugar, in the harvest season and planting season while the demand is relatively constant. The period of supply is only six months while the consumption is twelve months in whole year. The quantity of supply during the harvest season could fulfil around 80% of total consumption (Ismail, 2001). This situation will trigger price volatility and lead the food security problems, especially related to the scarcity and price hikes for households. Therefore, the stakeholders such as producer, wholesaler, consumer, and government will get loses. Sutopo *et al.* (2008) analysed the financial losses and market risks for the stakeholders at free market situation. The producer is forced to sell staple food at lowest price during the harvest season. On the other hand, the consumer has to deal with the scarcity of staple food and price hikes during the planting season and the wholesaler is forced to spend a larger procurement cost when the supply of goods is scarce.

Market intervention program should be conducted to reduce losses and market risks for both producer and consumer. The purposes of this program are to protect the producer from selling the staple food at lower price when excess supply; and to keep the consumer from buying the staple food at higher price when shortage supply. In order to keep the expectation of stakeholders, the government can apply the buffer stock schemes to maintain the market-price on certain price-band (William and Wright, 2005). Therefore, the government has several price stabilization policies; one of them is market-intervention program by using buffer stock schemes (Athanasioa *et al.*, 2008). The market-intervention policy is able to improve producer's profit, cut consumer's expenditure, and sustains wholesaler's margin-profit by implementing price-support and price-stabilization program (Sutopo *et al.,* 2009).

Table 1 Summary of previous researches

Author (s),	Criteria		Uncertainty		Stakeholders		BS Policy			Model	
(Published)	TC	TB	Q	P	1S	2S	Q	P	T	D	P
Labys (1980).	V			V	V		V	V		V	
Nguyen (1980)	V		V		V		V	V		V	
Edwards & Hallwood (1980)		V	V		V		V	V		V	
Newbwry & Stiglitz (1982)	V		V		V		V	V		V	
Harker (1986)	V		V		V			V			V
Guder (1988)		V	V		V		V				V
Tersine (1992)	V		V		V		V		**V**		V
Chavas *et al.* (1998)	V		V		V		V	V		V	
Jha & Srinivasan (1999)	V		V		V		V	V		V	
Graves (1999)	V		V		V		V		**V**		V
Coulson *et al.* (2001)	V		V		V		V	V		V	
Véricourt et al. (2002)	V		V		V		V				V
Brennan (2003)	V		V		V		V	V		V	
Rossi-Hansberg (2005)	V		V		V		V				V
Pompermayer *et al.* (2007)	V		V		V		V	V			V
Athanasioa *et al.* (2008)	V		V		V		V	V		V	
Sutopo *et al.* (2008)		V	V	V		V	V	V			V

There were several researches available regarding this issue performed before, results in the making of several models. The previous models can be classified according to relevant features, *i.e.* the performance criteria: total cost (TC) and total benefit (TB); the source of uncertainty: quantity (Q) and price (P); the number of stakeholder: producer or consumer (1S), producer and consumer (2S), and producer, wholesaler and consumer (3S); the buffer stock policy: quantity (Q), price (P), and time (T); and the model types: descriptive (D) and prescriptive (P). The optimization method is used to decide the level of goods availability that market intervention policy consists of time and amount of buffer stocks. On the other hand, the econometrics method is applied to find out the equilibrium price by using the selling price and amount of buffer stocks.

Those previous developed model can be summing up as can be seen on Table 1.

Labys (1980), Nguyen (1980), Edwards and Hallwood (1980), Newbwry and Stiglitz (1982), Underwood and Davis (1997), Jha and Srinivasan (1999), Brennan (2003) and Athanasioa *et al*. (2008) developed buffer stock models based on supply-demand approach. This approach is employed to measure the excess of supply and demand in order to achieve market price stability. The models addressed the reduction of uncertainty of demand side by determining the buffer stock schemes consisting of amount and price of procurement. Harker (1986), Guder (1988), Chavas *et al.* (1998), Coulson *et al.* (2001), Véricourt *et al.* (2002), Rossi-Hansberg (2005) and Pompermayer *et al.* (2007) have developed buffer stock models based on location-allocation approach. This approach is used to identify the condition when the location of supply influences the price instability. The models addressed the reduction of uncertainty of supply side by deciding buffer stock schemes consisting of amount and price of procurement. Tersine (1992) and Graves (1999) have developed buffer stock models based on inventory system approach. The inventory system is utilized to determine stock needed to anticipate the shortage. The models addressed the reduction of uncertainty of supply side only by determining buffer stock schemes consisting of time and amount of buffer’s procurement.

As can be seen on Table 1, this research has addressed the gap that currently exists in the literature available while also come up from the real problem of sugar price stabilization in the supply chain of sugar in Indonesia. The objective of this research is to determine the buffer stock schemes decision required for market intervention program. The decisions concerns quantity and price of buffer stock schemes.

Problem Description

The relevant system of the problem is illustrated in Figure 1 and consists of three main components that are producer, wholesaler and consumer as well as government as regulator. The producer-wholesaler-customer relationships are based on the free market mechanism while the producer-government-customer relationships are based on the intervention of market mechanism. The equilibrium price consists of two level of price including purchasing price to producer and selling price to consumer. The purchasing price is taken from producer and wholesaler, while the selling price is resulted from transaction between wholesaler and consumer. The total production is lower than the total consumption; consequently import of staple food is permitted by the government to anticipate market shortage.

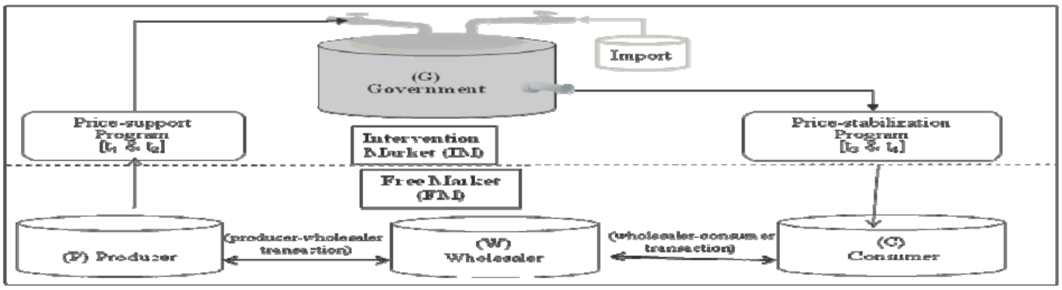

Figure 1 Overview of system relevant.

In a free-market (FM), the theory of supply and demand states that price is determined by supply and demand forces. At the harvest season (period t_1 and t_2), producer sells staple food to the wholesaler, and the wholesaler sells them to the consumer. The purchasing price and selling price are set by an equilibrium process. At the planting season (period t_3 and t_4), only the wholesaler sells staple food to the consumer and it is often that wholesaler with excess inventory will speculate the market by increasing price.

In an interventioned-market (IM), the market-price is determined by supply-demand forces and buffer stock schemes forces. In the harvest season, the government controls the purchasing price when the price falls. Price support program is conducted through a market operation program where the government buying a large amount of staple food from producer. The government purchases the staple food during the boom periods so that the purchasing price goes up. During the planting season, the government controls the selling price when the selling price soars. The price stabilization program is conducted through the market operation using buffer that is stocked by the government. The government releases the staple food in shortage periods so that the selling price goes down. In this case, the distribution system is handled by two actors; government and wholesalers so that it is called duopoly-like market.

It is assumed that the market situation through four mentioned periods can be depicted as shown in Table 2. The planning horizon is divided into four 4 periods as follows: (i) the beginning of harvest season (period t_1); (ii) the end of harvest season (period t_2); (iii) the beginning of planting season (period t_3), and (iv) and the end of planting season (period t_4). It is assumed that staple food cannot be substituted by other products but it is consumed continuously in a year. The market intervention program is run by government in two ways: purchasing and releasing staple foods. The government is able to purchase the staple foods in booming periods (period t_1 and t_2) and releases them in collapse periods (period t_3 and t_4). If the government's stock is lack then the government could purchase it from import. The market intervention program will be conducted by government when indicator of volatility is happening.

Table 2 List of market assumptions

Period	t_1	t_2	t_3	t_4
production	normal/boom	boom	none	none
consumption	Stable	Stable	Stable	Stable
availability	Sufficient	surplus	sufficient /shortage	Shortage
price market	normal/lower	lowest price	normal/higher	highest price
Intervention	Support	support	stabilization	Stabilization

The objectives of this research are to determine buffer stock and its price for both producer and consumer during the market-intervention program in order to maximize benefit to both the producer and the consumer and also minimize cost to both the wholesaler and the government. Total benefit or total cost of market-intervention program can be calculated by doing a comparative analysis of transaction between FM and IM.

Mathematical Formulation

The staple food distribution condition and all relevant data (costs, supply-availability-demand and other factors) were collected using historical data and appropriate forecasting methods. Before presenting mathematical formulation for the price stabilization problem described in Section 2, the notations used in the formulation will be described and all cost parameters and decision variables are measured in Indonesia Domestic Rupiah (IDR).

The parameters are defined as following:

c_p	production cost of producer per unit (IDR/Kgs)
c_d	distribution cost of wholesaler per unit (IDR/Kgs)
c_o	operation cost for staple food per unit from government (IDR/Kgs)
c_h	holding cost for buffer stocks per unit (IDR/Kgs)
c_i	import cost per unit (IDR/Kgs)
q_t^s	supplies of staple food in period t (tons)
q_t^d	demand of staple food in period t (tons)
q_t^a	maximum availability of staple food from wholesaler in period t (tons)
q_0^g	amount of staple food from government in the beginning period t (tons)
p_{pwt}^{p0}	purchasing price in the FM in period t at FM (IDR/Kgs)
p_{pwt}^{p1}	purchasing price in the IM in period t at IM (IDR/Kgs)
p_{wct}^{s0}	selling price in the FM in period t at FM (IDR/Kgs)
p_{wct}^{s1}	selling price in the IM period t at IM (IDR/Kgs)
irr	percentage of internal rate of return
nrr	percentage of normal rate of return
srr	percentage of speculative rate of return
VT_1, VT_2	lower and upper limit of volatility target for producer's purchasing price
VT_3, VT_4	lower and upper limit of volatility target for consumer's selling price
TB^P	producer's total benefit (IDR)
TB^C	consumer's total benefit (IDR)
TC^W	wholesaler's total cost (IDR)
TC^G	government's total intervention cost (IDR)

The decisions variables are defined as following:

P^{Min}	minimum purchasing price limit (IDR/kgs)
P^{Max}	maximum selling price limit (IDR/kgs)
Q_{pt}^{OP}	amount of staple food purchased by the government in period t (tons)
Q_{ct}^{OR}	amount of buffer stocks distributed by the government in period t (tons)
Q_t^{OI}	import quota in period t (tons)
Q_t^{OG}	amount of buffer stocks that is stored by government in period t (tons)

Multi-objectives of stakeholders

A buffers stock model based on recourse model with two chronological stages is proposed in this work to incorporate the expectation of all stakeholders. The first-stage, an analysis of historical transaction is done to elaborate the parameters of each stakeholder and one staple food market model. The second-stage, the decision variables are made subject to restrictions imposed by model formulation. The proposed model is developed by combining both econometrics and optimization method. The econometrics method is applied to analyse the parameters of each stakeholder and one staple food market model. And the optimization method is used to formulate a nonlinear programming.

Let consider the simple one staple food market problem, where it is only governed by the supplies of staple food from producer, the demand of staple food from customer, maximum availability of staple food from wholesaler, and its market-price in time period t. Based on partial market equilibrium theory (a linear model), the simple one staple food market model can be represented as follows:

$$q_t^d = a + bp_t^c, a, b > 0 \tag{1}$$

$$q_t^s = -c + dp_t^p, c, d > 0 \tag{2}$$

$$q_t^a = -e + gp_t^a, e, g > 0 \tag{3}$$

Where (a, c, e) are constants; (b, d, g) are price elasticity point; then (a, c, e) and (b, d, g) should be mutually independent parameters. Furthermore, model of price determination in an isolated market is illustrated in Figure 2.

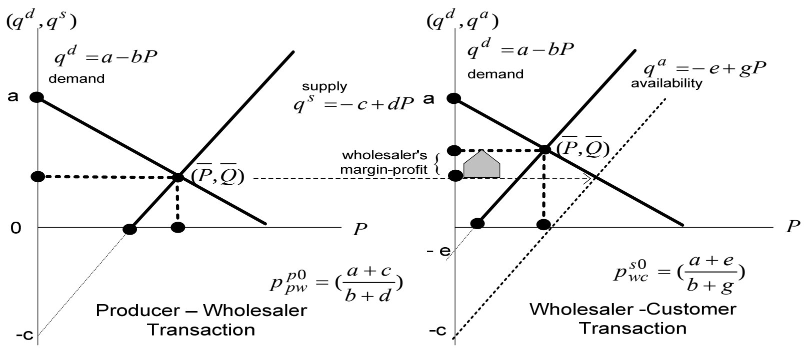

Figure 2. The producer-wholesaler-customer relationship.

In Figure 2., market price is assumed to be set when the supplies of staple food is equal to the demand of staple food for period *t.* When there is inventory available in the market then the approach in Fig. 2 is inappropriate to solve the problem. Therefore, we have to modify the model by applying a market model with inventory. In this model the market price is calculated by subtracting the current market price (without inventory) with multiplying result of stock-induced-price adjustment coefficient and the level of inventory. The stock-induced-price-adjustment coefficient is for describing the price change when the level of inventory changes. In this approach, the mechanism of inventory control can be used as principal to control the market price directly in the price support and stabilization programs by government. In the price support program, the producer gains benefit because the government reduce the inventory level in the market (period t_1 and t_2) so that the purchasing price will go up. On the other hand, when the price stabilization program is conducted the consumer will get advantage because the government increase the inventory level (period t_3 and t_4) then the selling price will go down.

According to the explanation above, market price determination becomes complicated due to conflict of interest. It does not only pertain producer-wholesaler-consumer, but also producer-government-consumer relationship. On the other side, this buffer stock model will not only consider producer and consumer, but also government and wholesaler perspective simultaneously as performance criteria. As consequently, this model considers multi-objectives of stakeholders.

The mathematical formulation for each stakeholder is next described.

i). producer perspective

Total benefit for producer is obtained from the difference between total revenue and the total production cost. Both in FM and IM situations, the total production cost are obtained as the production cost per unit of the staple-food multiplied by its production amount from producer in time period t. Furthermore in FM, total revenue is calculated from multiplication of the production amount and its purchasing-price. On the other hand, total revenue in IM is expected from the amount of staple food bought by the government multiplied by the minimum price-limit and the amount of staple food sold to wholesaler multiplied by the current purchasing-price. Therefore, the total benefit for producer can be expressed as:

$$TB^{P} = \sum_{t=1}^{2} P^{Min} Q_{pt}^{OP} + \sum_{t=1}^{2} p_{pwt}^{p1}(q_t^s - Q_{pt}^{OP}) - \sum_{t=1}^{2} p_{pwt}^{p0} q_t^s \tag{4}$$

ii). consumer perspective

Total benefit for consumer is calculated from the total difference of consumption cost between IM and FM. In FM condition, total consumption cost is expected from the demand of staple food from consumer in time period t multiplied by its selling price. On the contrary, in IM condition, consumer will spend money to fulfil the consumer's total demand at selling-price between wholesaler and consumer during period t_1 and t_2. During period t_3 and t_4, the consumer bought the staple food at the maximum price-limit when market-operation is performed, and the remaining demand will be bought at the selling-price. Therefore, the total benefit for consumer is expressed as:

$$TB^{C} = \sum_{t=1}^{4} p_{wct}^{s0} q_t^d - \sum_{t=1}^{2} p_{wct}^{s1} q_t^d - \sum_{t=1}^{4} (P^{Max} Q_{ct}^{OR} + p_{wct}^{s1}(q_t^d - Q_{ct}^{OR})) \tag{5}$$

iii). wholesaler perspective

Total profit of wholesaler decreases when market operation conducted due to the increase of purchasing price and the decrease of selling price as a result of intervention. The total profit is then calculated from the difference between total revenue and total cost (procurement, distribution and inventory cost) in the IM and FM conditions. The total revenue for wholesaler is a multiplication of total supply amount to consumer at the selling-price in FM condition. Meanwhile in IM condition, the total sales is calculated by total consumer's demand less amount of buffer stocks when market-operation conducted; than the total revenue is obtained as the selling-price multiplied by the total sales. Both in FM and IM conditions, total procurement cost is obtained from sum of the amount of staple food bought from the producer at the purchasing price, total distribution cost that is obtained as the distribution cost per unit of item multiplied by total demand of staple food from the consumer, and total inventory cost that is obtained as a holding cost per unit in stock per unit of time multiplied by total of average inventory in a year. Therefore, the total cost for wholesaler can be written as:

$$TC^{W} = \{ \sum_{t=1}^{4} p_{wct}^{s0} q_t^d - \sum_{t=1}^{2} p_{pwt}^{s0} q_t^s - \sum_{t=1}^{4} c_d q_t^d - \sum_{t=1}^{4} \frac{c_h}{4} q_t^a \} - \{ \sum_{t=1}^{4} p_{wct}^{s1} (q_t^d - Q_{ct}^{OR}) - \sum_{t=1}^{2} p_{pwt}^{p0} (q_t^s - Q_{pt}^{OP}) - \sum_{t=1}^{4} c_d (q_t^d - Q_{ct}^{OR}) - \sum_{t=1}^{4} \frac{c_h}{4} (q_t^a - Q_{pt}^{OP}) \} \quad (6)$$

iv). government perspective

The total intervention cost is obtained as total cost minus total revenue. The total cost consists of procurement cost, distribution cost and inventory cost. The total revenue is obtained from multiplication of amount of buffer stocks that is released to consumer in time period t and the maximum price-limit. Total distribution cost is obtained as cost of market operation by the government multiplied by amount of buffer stocks that should be released to the market. Total inventory cost is obtained as a holding cost per unit in stock per unit of time multiplied by total of average the government's inventory in a year. Total procurement cost is calculated from amount of staple food bought by the government from the producer at the minimum price-limit and amount of staple food bought by the government from import at a purchase cost per unit of the staple food from import. Therefore the total intervention cost for the government is expressed as:

$$TC^{G} = \sum_{t=1}^{2} P^{Min} Q_{pt}^{OP} + \sum_{t=3}^{4} c_i Q_t^{OI} + \sum_{t=3}^{4} c_o Q_{ct}^{OR} + \sum_{t=1}^{4} \frac{c_h}{4} Q_t^{OG} - \sum_{t=3}^{4} P^{Max} Q_{ct}^{OR} \quad (7)$$

<u>The objective function</u>

We develop a buffer stock schemes for stabilizing price of the staple food under volatility target (VT) for fulfilling the expectation of stakeholders. The buffer stock model therefore must attain two targets: (i) maximise the benefit of producer and consumer, and (ii) minimise the total cost of wholesaler and government. The resulting objective function which considers multi-objectives of stakeholders, is finally expressed as follows:

$$Max.\ Z = TB^{P} + TB^{C} - TC^{W} - TC^{G} \quad (8)$$

<u>The constraints</u>

In a free-market, a market model with inventory is used to determine the purchasing price and the selling price and it can be seen on equations (9) and (10), where γ denotes the stock-induced-price adjustment coefficient. The inventory level in determining the purchasing price is calculated from the amount of stock in previous period, added with current stock, and subtracted with the current demand. For the selling price, the inventory level is only the availability of staple food owned by wholesaler.

$$p_{pwt}^{p0} = [(\frac{a+c}{b+d}) - \gamma(q_{t-1}^a + q_t^s - q_t^d)],\ t = t_1, t_2 \quad (9)$$

$$p_{wct}^{s0} = (\frac{a+e}{b+g}) - \gamma q_t^a,\ t = t_1, \ldots, t_4 \quad (10)$$

In an intervention-market, we modify a market model with inventory minus the amount of staple food bought by the government such as in equations (11) and (12).

$$p^{p1}_{pwt} = [(\frac{a+c}{b+d}) - \gamma(q^{a}_{t-1} + q^{s}_{t} - q^{d}_{t} - Q^{OP}_{pt})],\ t = t_1, t_2 \tag{11}$$

$$p^{s1}_{wct} = [(\frac{a+e}{b+g}) - \gamma(q^{a}_{t} - Q^{OP}_{pt})],\ t = t_1, \ldots, t_4 \tag{12}$$

Constraints (13), (14) and (15) are introduced to ensure that the price-equilibrium fulfilled the expectation of the producer and the wholesaler at price-support program. The percentage of rate of return (internal, normal, and speculative) is used to control the lower limit of volatility target for the purchasing price and the upper limit of volatility target for the selling price. The producer's expectation is protected from distortion of selling-price as impact of excess supply; and the wholesaler's expectation is protected from costly buying-price as the impact of price-floor regulated by the government. For ensuring the expectation of the wholesaler and the consumer at price-stabilization program in each period constrains (16), (17) and (18) are employed.

$$VT_1 \leq (p^{p1}_{pwt}, P^{Min}) \leq VT_2,\ t = t_1, t_2 \tag{13}$$

$$VT_1 = c_p\ (1 + irr),\ t = t_1 \tag{14}$$

$$VT_2 = \{(p^{p0}_{pwt} + c_d)(1 + irn) - c_d\},\ t = t_2 \tag{15}$$

$$VT_3 \leq (p^{s1}_{wct}, P^{Max}) \leq VT_4, t = t_3, t_4 \tag{16}$$

$$VT_3 = (p^{p0}_{pwt} + c_p)(1 + irr),\ \ t = t_3 \tag{17}$$

$$VT_4 = (p^{p0}_{pwt} + c_a)(1 + irs), t = t_4 \tag{18}$$

The government has to ensure the market-intervention program could fulfil the demand in each period, constraint (19). We have to ensure that the buffer stock schemes are adequate to hold the market-intervention program in each period by considering constrains (20) and (21).

$$\sum_{t=1}^{4}(q^{a}_{t-1} + q^{s}_{t} - Q^{OP}_{pt} + Q^{OR}_{ct}) \geq \sum_{t=3}^{4} q^{d}_{t}, t = t_1, \ldots, t_4 \tag{19}$$

$$Q^{OG}_{t-1} + Q^{OP}_{pt} > 0, t = t_1, t_2 \tag{20}$$

$$Q^{OG}_{t-1} - Q^{OR}_{ct} + Q^{OI}_{t} > 0,\ t = t_3, t_4 \tag{21}$$

Finally, we have to ensure the supply of staple food are adequate the demand in each period and to ensure that all decision variables cannot be negative by considering constraints (22), (23) and (24).

$$q^{s}_{t} + q^{a}_{t} - Q^{OP}_{pt} \geq q^{d}_{t}, t = t_1, t_2 \tag{22}$$

$$q^{a}_{t} + Q^{OR}_{ct} \geq q^{d}_{t},\ t = t_3, t_4 \tag{23}$$

$$P^{Min}, P^{Max}, Q^{OP}_{pt}, Q_{ct}{}^{OR}, Q_t{}^{OI}, Q^{OG}_{t} \geq 0, t = t_1, \ldots t_4 \tag{24}$$

Solution Methods and Analysis

In this section, we present the solution method and the numerical examples, and analyse them for evaluating expectation of stakeholders. The optimal solution can be obtained by solving the pre-emptive of the non-linear programming above. The procedure to solve the proposed problem is described as follow: (i) forecast all the parameters from the historical data; (ii) set the parameters of the market price function; (iii) predict the market price in the FM; (iv) formulate the objectives function in the IM; (v) formulate all the constraints of the solution model; and (vi) solve the model by using optimization software *i.e.* WinQSB software.

In order to illustrate the capabilities of the proposed-model, a numerical example has been studied. The problem consists of hypothetical data for the Indonesian sugar market. Let a = 32.0, b = 0.17, c = 167.0, d = 4.8, e = 0.1, g = 0.45, γ = 0.1, c_h = 2.0, c_d = 2.0, c_O = 4.0, c_p = 34.0, c_i = 40.0, irr = 5.0, nrr = 10.0, srr = 25.0, in appropriate units. Thus, the supply-demand parameters are shown in Table 3.

Table 3 Hypothetical data (selected)

Period	Stocks	t_1	t_2	t_3	t_4	Total
q_t^d	-	25	25	25	25	100
q_t^s	-	35	55	-	-	90
q_t^a	25	-	-	-	-	-

A non-linear programming associated to the problem described above was formulated in equation (8) to equation (24). The models were formulated in WinQSB software and the solutions are found using available procedure. For instance, by using parameters in Table 3, the six decision variables are solved and the results are shown in Table 4.

Table 4 The result of decision variables calculation

Period	t_1	t_2	t_3	t_4
P^{Min}	35.7	35.7	-	-
P^{Max}	-	-	45.6	45.6
Q_{pt}^{OP}	0.0	21.6	-	-
Q_t^{OI}	-	-	21.6	21.6
Q_{ct}^{OR}	-	-	21.6	21.6
Q_t^{OG}	0	21.6	21.6	21.6

According to the Table 4, the proposed model can be used to decide on the buffer stock scheme for government, including the purchasing amount and price, the amount of market operation and its price, the import quota and the amount of buffer stocks that is stored by government. The price-band schemes can be suggested under 35.7-45.6 staple food per unit. It can be noted that improving or degrading price will influenced by stock-induced-price-adjustment coefficient. With the minimum purchasing price limit set at 35.7, the government should purchase 21.6 the staple food from producer. Furthermore with the maximum selling price limit sets at 45.6, the government should release 43.2 the staple food to consumer. At the planting season (t_3, t_4), the government should purchase the staple food from import respectively 21.6 and 21.6. A comparative analysis of price-equilibrium between FM and IM is depicted in Table 5.

Table 5 Price-equilibrium analysis

	t_1	t_2	t_3	t_4
p_{pwt}^{p0}	36.54	33.54	-	-
p_{pwt}^{p1}	36.84	33.70	-	-
p_{wct}^{s0}	48.27	45.27	47.77	50.27
p_{wct}^{s1}	48.57	47.43	45.61	48.11

Because the focus of this paper are maximizing the benefit of producer and consumer and minimizing the total cost of wholesaler and government, it is assumed that there is no private storage. Under a price-band scheme above, the government does not purchase any of staple food from domestic market unless the purchasing price is 35.7 per unit or lower. The government conducts the price support program at period t_2 because the purchasing price is 33.54 per unit. Moreover, the government can purchase stocks only 21.6 from producer at the maximum selling price limit, so that the purchasing price goes up and expected to 33.70 per unit for period t_2. The government must conduct market-operation when the selling price is 45.6 per unit or higher. At the planting season, the selling price is 45.77 and 50.27 respectively. The government does release its holding up to 43.2 tons at the maximum selling price limit so that the selling price goes down. New selling price is expected 45.61 per unit for period t_3 and 48.11 per unit for period t_4.

Table 6 reports the results proposed-model accounts for the maximization of expected benefit for producer and consumer and for the minimization of expected cost for wholesaler and government. The market-intervention policy can be utilized for improving producer's profit up to 118.79 and for reducing consumer's expenditure up to 108.18. The wholesaler will get disadvantages up to 1,341.51 and the government need to spend budget up to 733.97 as the intervention cost.
Based on a comparative analysis between FM and IM, Government intervention has significantly effect to reduce risks of both producer and consumer side. For a set of hypothetic-parameters given, it can be noted that each of total benefit for the producer and the consumer in loss/benefit ratio are respectively 186.72% and 2.26%. Total cost for the government and the wholesaler in loss/benefit ratio are respectively 15.32% and 96.59%. For price elasticity given, the producer obtains the benefit bigger than consumer. Furthermore, the wholesaler obtains the cost/loss bigger than the government.

Table 6 A comparative analysis between FM and IM

Stakeholder	Earning/Expenditure	FM (Mill. IDR)	IM (Mill. IDR)
Producer	total revenue	3,123.62	3,242.42
	production cost	3,060,00	3,060,00
	profit	63.62	182.42
Consumer	consumtion cost	4,789.92	4,681.74
Wholesaler	total revenue	4,789;92	2,711.39
	procurement cost	3,123.62	2,471.30
	distribution cost	200.00	113.60
	inventory cost	77.50	79.20
	profit	1,388.80	47.29
Government	procurement cost	-	2,499.12
	distribution cost	-	172.80
	inventory cost	-	32.40
	total revenue	-	1,970.35

Conclusion

We have presented a buffer stock model to solve a problem of the staple-food distribution system, incorporating the configuration of one-producer, duo market-buyers, and one-consumer. The buffer stock model has been developed by combining both optimization and econometrics method. To determine the decision variables (quantity and price of buffer stocks), a nonlinear programming model was formulated. The proposed model has a significant effect to enhance the benefit for both the producer and the consumer under the minimum cost/losses for wholesaler and government. The revenue of price stabilization is intended to induce an equivalent reduction in the fluctuations of total market revenue. Moreover, the producer gets bigger benefit than the consumer does, and the wholesaler gets bigger cost/losses than the government does. By considering both the market model with inventory and partial market equilibrium theory (a linear model), it is shown that the proposed model is able to decide the minimum purchasing price limit and maximum selling price limit simultaneously.

There are some extensions from this work that could be derived to elaborate the model formulation such as considering the budget constraint and offering the government's total intervention budget options. In the future, it probably significant to develop a model that is suitable with the indirect intervention system for instance by using the Warehouse Receipt System (WRS).

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Editor-in-Chief
ASOR Bulletin

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[2] Simon, H.A. The New Science of Management Decision. Rev. Ed. Prentice-Hall, N.J. (1977).

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Editor: Address all correspondence and contributions to:

A/Prof. Ruhul A Sarker,
School of Engineering & IT, UNSW@ADFA
Northcott Drive, Canberra ACT 2600
Tel: (02) 6268 8051 Fax: (02) 6268 8581
Email: r.sarker@adfa.edu.au