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Multicriteria Modelling And Decision Engineering Of A Chemical Extrusion Process

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Abstract

In the optimization of industrial processes, decision-makers are often confronted with multi-objective problems. For example, in manufacturing it is often necessary to optimize many conflicting objectives, subject to market constraints.

Using the concept of Pareto dominance, we present a multi-objective optimization procedure which yields an optimal zone containing the solution. Pair-wise points are compared, and non-dominated points are collected in the Pareto region. Then a ranking is established and the decision-maker selects the first-best solution. Finally we give an application of our procedure to the chemical engineering process of manufacturing cattle feed.

INTRODUCTION

In formulating a new product it is often desirable to meet many different objectives. The extrusion process for cattle feed is a good illustration of this problem. For example, it may be necessary to obtain a desired texture and flavour, as well as a certain degree of dampness and pulverization, while attempting to optimize yield or minimize production costs. This may be considered as an optimization problem with a scalar objective which is a linear combination of a number of criteria [Courcoux *et al.* 1995]. Another approach is to optimize one criterion, while treating all others as effective constraints. The use of one approach over the other depends on the user's preference.

Process engineers have recently turned to a third approach, called the *Dominance Principle* , based on the well-known Pareto criterion [Pareto, 1927] ; see also [Dominique, 1987]. In this approach two vectors: $X_{[N]}$ and $Y_{[N]}$ are considered. The vector $X^*_{[N]}$ is Pareto preferred, or, equivalently $X^*_{[N]}$ is said to dominate $Y_{[N]}$, if :

a) for $\mathbf{X}_{[N]} = \{ x_1, x_2, ... x_N \}$, $\mathbf{Y}_{[N]} = \{ y_1, y_2, ... y_N \}$

$\exists\, x_i \succ y_i \quad \forall i \in N$, and

b) $\exists\, x_j \geq y_j$, $j \neq i, \forall i,\ j \in N-1$, and where x_i , y_i are desired objectives.

Since a Pareto vector $\mathbf{X}^*$ is not known *a priori*, due to the fact that there is no prior knowledge about the combination of the elements of $\hat{\mathbf{X}}^*_{[N]}$, it is desirable to generate a number of vectors $\mathbf{X}_{[N]}$ that can be compared. All the $\mathbf{X}^*$'s generated form the Pareto set, from which $\hat{\mathbf{X}}^*$ is selected according to a) and b) above.

Many researchers, including Schaffer [1985], Horn *et al.* [1994] and Fonseca *et al.* [1993], have developed algorithms for selecting $\mathbf{X}^*$. However, their results find only the Pareto set containing $\hat{\mathbf{X}}^*$. The direct intervention of the decision-maker is still necessary in order to subjectively select $\hat{\mathbf{X}}^*$. For example, the result of Courcoux *et al.* [1995], shown in Figure 1, points to a border line $\hat{\mathbf{X}}^*$, which is not robust under small uncontrollable changes in the problem's environment, such as process temperatures and/or extruder diameters.

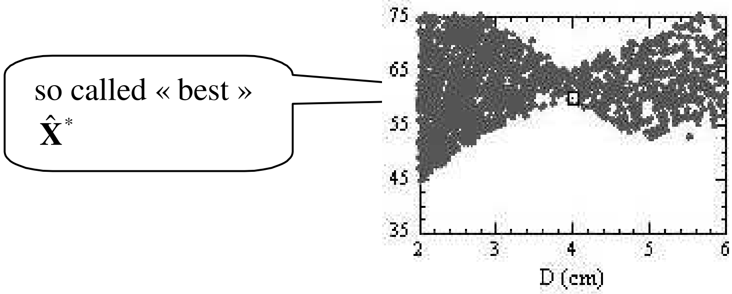

Figure 1 Representation of the Pareto zone and a supposed " best point " by Courcoux *et al.* [1995]

This paper develops an alternative approach, which we call Outranking Synthesis (OS), according to the ELECTRE principle [Roy & Bouyssou 1993]. The philosophy behind the ELECTRE algorithm is to first invite the decision-maker to define his thresholds as indifference, preference and veto, vis-à-vis each of the objectives. Next, indices of concordance and discordance are calculated. The difference between these indices yields a number of net flows equal to the cardinality of the Pareto set, from which a first-best $\hat{\mathbf{X}}^*$ and a worst-best (that is, a nadir) $\hat{\mathbf{X}}_*$ are identified.

MULTICRITERION ANALYSIS FRAMEWORK

To apply OS to the granulation process, we first consider the input parameter set, containing 2 elements , the discrete temperature points T (35 and 75 ^{0}C) and the extruder's diameter profile D (2 and 6 cm). Next, we consider the output parameter set, containing 3 elements, energy consumption and the indices of pulverization and dampness. In the application considered here, the elements of the output parameter set had to be minimized, regardless of the initial conditions.

Algorithmic design

More generally, our multicriterion analysis algorithm is essentially a comparison process, where the decision-maker's preferences are expressed by means of some constraint-like parameters: indifference, preference, veto thresholds and the weights of the criteria. In order to run the Multi-criterion Analysis Process (MAP) operationally, it is necessary to introduce the MAP input set Ψ, with the following synoptic structure [Derot, Gareau, Kiss, Martel, 1997]:

$$\Psi = \Gamma_{\S}\ [m, n, M_{[m \times n]}, K_{[n]}, \Phi_{[n]}, R_{[n]}, Q_{[n]}, P_{[n]}, V_{[n]}, W_{[n]}],$$

where

$\Gamma_{\S}$ = the MAP algorithm;

m = the number of alternatives;

n = the number of criteria;

$M_{[m \times n]}$ = a real m x n cardinal matrix containing alternative evaluations in relation to the criteria and $\overline{m}_{ij} = f_j(x_i) \in M_{[m \times n]}$; i = 1,…,m; j = 1,…,n;

$K_{[n]}$ = a binary n cardinal vector indicating the nature of the criteria evaluation, with

$$\kappa_j = \begin{cases} 0 & \text{if the evaluation in relation to the } j^{th} \text{ criterion is an ordinal type} \\ 1 & \text{if the evaluation in relation to the } j^{th} \text{ criterion is a cardinal type} \end{cases}$$

and $\kappa_j \in K_{[n]}$; j = 1,…,n (in this paper all criteria are of cardinal type);

$\Phi_{[n]}$ = a binary n cardinal vector indicating the nature of the orientation of the criteria, with

$$\phi_j = \begin{cases} 0 & \text{if the } j^{th} \text{ criterion is to be minimized} \\ 1 & \text{if the } j^{th} \text{ criterion is to be maximized} \end{cases}$$

and $\phi_j \in \Phi_{[n]}$; j = 1,…,n (0 for all criteria in this paper);

$R_{[n]}$ = a real n cardinal vector containing the range for every criteria and $\rho_j \in R_{[n]}$; j = 1,…,n;

$Q_{[n]}$ = a real n cardinal vector containing the indifference thresholds and $q_j \in Q_{[n]}$; j = 1,…,n;

$P_{[n]}$ = a real n cardinal vector containing the preference thresholds and $p_j \in P_{[n]}$; j = 1,…,n;

$V_{[n]}$ = a real n cardinal vector containing the veto thresholds and $v_j \in V_{[n]}$; j = 1,…,n;

$W_{[n]}$ = a real n cardinal vector containing the criteria's relative importance coefficients w_j and $w_j \in W_{[n]}$; j = 1,…,n.

The following relations must be verified between the thresholds:

$$0 \le q_j \le p_j \le v_j \le \rho_j ; \; j = 1,\ldots,n.$$

The relative importance coefficients are normalized, that is, $\sum_{j=1}^{n} w_j = 1$.

For the application considered in this paper, the decision-maker's indifference, preference and veto thresholds were given (defined by a chemical engineer's appraisal). These thresholds are presented in Table 1.

	q_i	p_i	v_i
Friability index	0.2	0.5	0.8
Moisture	0.5	1.5	3
Energetic consumption	1	3	6

Table 1: Indifference, preference and veto thresholds for each criterion

This input set Ψ is processed using the MAP algorithm $\Gamma_{\S}$ to determine the ranking from the best to the worst (that is, the "nadir") alternative with possible *ex aequo*.

For each pair of alternatives [i, $\bar{i}$] ; i = 1,…,m; $\bar{i}$ =1,…,m; i$\neq \bar{i}$ a global concordance index $C\|[i;\bar{i}]$ is calculated as follows:

$$C\|[i;\bar{i}] = \sum_{j=1}^{n} w_j \cdot \delta_j\|[i,\bar{i}] \text{ ; } i = 1,\ldots,m;\ \bar{i} = 1,\ldots,m;\ i \neq \bar{i}\text{ ; } j = 1,\ldots,n.$$

where $\delta_j\|[i,\bar{i}] = \begin{cases} 1 & \text{if } -\Delta_j \leq q_j \\ \dfrac{(\Delta_j + p_j)}{p_j - q_j} & \text{if } q_j < -\Delta_j \leq p_j \\ 0 & \text{if } p_j < -\Delta_j \end{cases}$

and $\Delta_j = \overline{m}_{ji} - \overline{m}_{j\bar{i}} \in M_{[m \times n]}$; j = 1,…,n; i = 1,…,m; $\bar{i}$ =1,…,m; i$\neq \bar{i}$. If some criteria call for maximization while others require minimization, we introduce two temporary *sentry* (that is, guard) variables k and $\bar{k}$ in our calculations of Δ_j.

If ϕ_j = 0, meaning that this criterion is to be minimized, then k = $\bar{i}$ and $\bar{k}$ = i ; if ϕ_j = 1 (the criterion to be maximized), k = i and $\bar{k}$ = $\bar{i}$; this way $\Delta_j\|[i,\bar{i}] \equiv \Delta_j = \overline{m}_{jk} - \overline{m}_{j\bar{k}}$; $\forall\ \phi_j$; j = 1,…,n; k = 1,…,m; $\bar{k}$ = 1,…,m; k $\neq \bar{k}$.

A discordance index $\mathbf{D}_j\|[i;\bar{i}]$ is also calculated for every pair of alternatives at each criterion level with ELECTRE III :

$$\mathbf{D}_j\|[i;\bar{i}] = \begin{cases} 0 & \text{if } -\Delta_j \leq p_j \\ \dfrac{-(\Delta_j + p_j)}{v_j - p_j} & \text{if } q_j < -\Delta_j \leq v_j \\ 1 & \text{if } v_j < -\Delta_j \end{cases} \text{ ; } j=1,\ldots,n, i=1,\ldots,m, =1,\ldots,m,\ \bar{i}=1,\ldots,m; i \neq \bar{i}\ .$$

Using the concordance and discordance indices, we generate outranking degrees $\sigma\|[i;\bar{i}]$ for every pair of alternatives. These outranking degrees are obtained using the following Rousseau-Martel formula: $\sigma\|[i;\bar{i}] = C\|[i;\bar{i}] \prod_{j=1}^{n} \left[1 - \left(\mathbf{D}_j\|[i;\bar{i}]\right)^3\right]$ [see Rousseau & Martel, 1994].

The resulting outranking relation sets may be represented as an outranking matrix. Working with outranking relations implies synthetizing that matrix in such a way as to provide a process control recommendation in the form of an alternative ranking.

We make that synthesis following the notion of outgoing flow and incoming flow from PROMETHEE [Brans *et al.*, 1984].

For calculations we use

$$\sigma_i^+ = \begin{cases} \sum_{\bar{i}=1}^{m} \sigma\left[i;\bar{i}\right] & \text{if } i \neq \bar{i} \\ 1 & \text{if } i = \bar{i} \end{cases} \quad \text{and} \quad \sigma_i^- = \begin{cases} \sum_{\bar{i}=1}^{m} \sigma\left[\bar{i};i\right] & \text{if } i \neq \bar{i} \\ 1 & \text{if } i = \bar{i} \end{cases} ;\ i = 1,\ldots,m;\ \bar{i} = 1,\ldots,m;\ i \neq \bar{i}$$

and we get the total ranking of the alternatives from the net flow as in PROMETHEE II [Brans *et al.*, 1984]:

$$\sigma_i = \sigma_i^+ - \sigma_i^- \quad i = 1,\ldots,m.$$

Results

In order to illustrate the proposed approach, in Table 2 we present the results using the $\Gamma_{\S}$ algorithm described earlier. These results were obtained with the support of a Decision Support System (DSS) shell, based on our multicriterion analysis concept with the thresholds indicated in Table 1.

Best		Nadir	
D (cm)	T (°C)	D (cm)	T (°C)
3.3186	64.909	5.2921	72.677

Table 2: Best and worst parameter alternatives.

This DSS shell produces very appealing results for the problem at hand. To illustrate the robustness of these results, the input parameters of the process are presented quintile by quintile, as well as their first-best and worst-best points in Figure 2. The ranking involves some 2500 points, with 500 in each quintile.

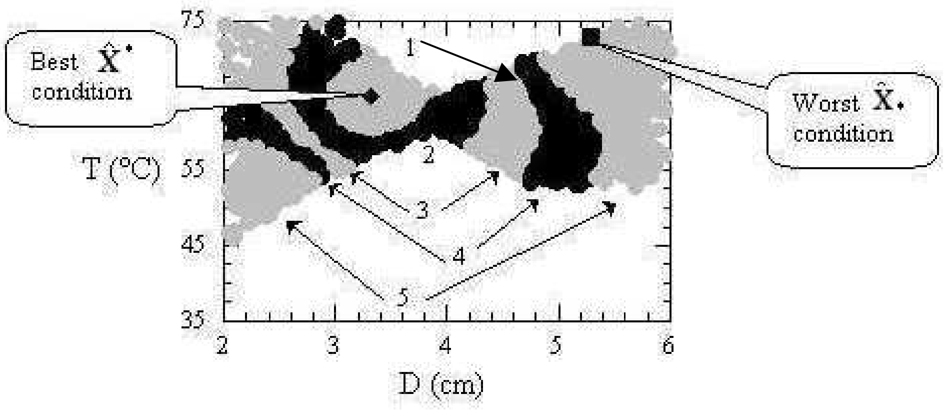

Figure 2: Best (◆) and worst (■) conditions given by DSS shell.

As can be seen in Figure 2, the first-best point is robust in the sense that no small variations in working conditions can take it out of the Pareto region. Likewise, no small variations can bring the worst-best solution into the Pareto region. The quintiles are represented by numbered concentric zones. When Courcoux's result, shown in Figure 1, is compared with ours, it becomes obvious that it is not robust. In addition, it falls in the second quintile.

CONCLUDING REMARKS AND FURTHER CONSIDERATIONS

In this paper we consider a multi-objective decision problem arising in an extrusion process. The main objective was the determination of the optimal operating conditions for the process, in accordance with certain technical requirements and with the decision-maker's preference. The first step was the determination of the Pareto region, and the second was the multi-criterion analysis proper. The best trade-off point was then selected from the region. Clearly, this procedure is also applicable to the design of the process, but in order to take full advantage of its power, the designer must have some familiarity with the technical environment to which the procedure is applied.

The DSS shell gives the complete ranking of the Pareto region. The first-best solution is robust under any perturbation in operating input conditions. The presentation in terms of quintiles gives the decision-maker some latitude to account for his or her own preference.

Our results extend previous decisional procedures, and open new perspectives in the extrapolation of robustness of optimal choice and effective control in the area of chemical processes. An immediate area of application is the polymerization process, which is exceedingly complex to model [Massebeuf *et al.*, 1998]. Another is in the area of engineering decisions in the chemical industry, where decision parameters (w_j, p_j, q_{ij}, v_j, $\forall$ j) are usually defined in terms of intervals, or in terms of fuzzy membership functions. In other more complex processes, the unresolved difficulties center around the inclusion of simple preference and the robustness of the solution. The manner in which our procedure accounts for these is simple and quite natural.

Our analysis concerns, first and foremost, a search for a solution to a multi-objective decision problem arising in the chemical industry. In some ways, it may be seen as a bridge between decision sciences and the chemical industry. Multi-objective optimization is common in the chemical industry, and the decision sciences need concrete areas of application. Both stand to benefit.

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