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Editorial

In this issue, we have accepted two papers. The first paper is a Soft Operations Research contribution on *Water Management in the Australian Capital Territory: Problem Structuring using SODA* by S. El Sawah, A. McLucas and J. Mazanov. The second paper is contributed by S. Kumar, E. Munapo, B. Jones and M. Mehlawat on *Complexity Reduction for Solving a Pure Integer Program by the Branch and Bound Method using the Gomory Constraints*. We are delighted to be publishing these papers here for Bulletin readers. We have also provided a list of Operations Research and related journals, with their ranks, recently proposed by the Australian Research Council (ARC). ARC will use this ranking for judging the quality of publications in the future. To maximize the benefit from our research, ASOR members and the ASOR Bulletin readers are reminded to carefully choose the journals for their future publications.

I am pleased to inform you that the electronic version of ASOR Bulletin is available at the ASOR web site: <http://www.asor.org.au/>. Although the electronic version is prepared as an HTML file, for technical reasons articles posted in PDF format.

ASOR Bulletin is the only national publication of ASOR. I would like to request all ASOR members, ASOR Bulletin readers and OR organizations in the country to contribute to the ASOR Bulletin. The editorial policy is available either from the Bulletin web site or from the inside back cover of the Bulletin. The detailed instructions for preparing the manuscripts is available in the URL: <http://www.asor.org.au/> and <http://www.itee.adfa.edu.au/~ruhul/asor.html>

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Water Management in the Australian Capital Territory: Problem Structuring using SODA

Sondoss El Sawah¹, Alan McLucas², Jason Mazanov³

Abstract: Akin to environmental problems, natural resources management are wicked problems which have no definitive formulation. Each stakeholder structures the situation according to his own valid point of view. Therefore, taking an idiosyncratic view becomes a part and parcel of any effective systemic intervention. This paper reports the results of the early steps of conducting a SODA intervention for structuring the problem of water management in the Australian Capital Territory (ACT). A series of two-session interviews were conducted with 25 residents in order to capture their mental models about the situation. Cognitive Mapping (CM) is used to portray and analyse the elicited data. Our investigations show a lack of consensus among participants in how they conceive the system. Findings from the interviews are used to set up an agenda of the key issues which need to be passed to further stages of analysis. This work belongs to an ongoing research project which aims to design and evaluate a System Dynamics (SD) based learning intervention to foster cognitive and behaviour changes among water users/managers.

Keywords: System Dynamics, SODA, Cognitive Mapping, Water Management

1. Introduction

Messy problems, such as environment management, are subjectively and socially constructed situations whose definition varies widely between the different actors involved. All agents come to the position of creating and managing the problem by reacting to the flux of events and ideas, selectively perceiving and evaluating parts of it, making decisions and initiating actions which become a new part of the flux inviting new perceptions and actions (Checkland & Scholes 1999). Thus, it becomes essential to shift the discourse from a solution-oriented to a problem-oriented domain where the central emphasis is placed on systems thinking, resolving and aligning the multiple views people have to the situation.

In System Thinking literature, Churchman (Churchman 1971) makes a paradigm shift from a view of system boundaries being “given” in the objective world, to a view of boundary as a personal/social constructs defining what relevant to the analysis. At the outset of a modelling process, the modeller has the easy option of walking away from the situation complexity heading directly to quantitative model building. Adopting a single and definite view of reality, the modeller runs the risk of taking her mental model as the legitimate representation of reality, imposing her own bounded rationality to the modelling process/outcomes and depriving the intervention from the requisite variety added by sweeping in other views. A modeller-driven modelling process is more vulnerable to questionable validity as building elegantly sophisticated models are worthless efforts if they find the right solution to the wrong problem. In systemic interventions, it is simplistic to accept the modeller-driven answers to the critical questions of: (1) what is the problem, and (2) what is the system to be modelled. Instead, it becomes essential that the modeller leaves herself to be driven by the problem complexity and richness. In addition, focusing the analysis merely on numbers rather than on ideas, will rarely lead to actual robust strategies and actual organizational learning. This creates the challenge of deliberately collecting and analysing the “bits and pieces” from stakeholder’s mental models with the purpose of reaching a holistic view of the problem.

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Midgley (Midgley 2000) highlights the importance of the critical reflection of agents on choices made between boundaries.

This work-in-progress research project aims to design and evaluate the efficacy of the SD Based Learning environment in promoting learning, improving decision making and fostering behavioural changes regarding water use in the Australian Capital Territory (ACT). This research is expected to provide insights into behaviour and decision making in other contexts where consumption of limited resources is a concern. At the design phase, Strategic Option Development and Analysis (SODA), a soft OR methodology based on the Cognitive Mapping (CM) technique, is used with the prime objective of capturing the belief systems people have about the situation rather than helping them to change their minds. SODA is selected for a couple of reasons. From a methodology-pluralism perspective, it is critically important that methodologies share the same theoretical assumptions (Mingers & Gill 1997). SODA and SD share the same kellyian view for CM. This facilitates linking semantically rich CM, with formal Influence Diagramming (ID) (Coyle 1996) and SD models to ensure that the rich complexity has been copied to the simulation model (Vennix 1996) (Eden 1994). In addition, the design of SODA and SD group model building projects are highly streamlined which facilitates moving back and forth between them (Andersen et al. 2007).

In the present paper, we report the preliminary results of using SODA for problem structuring. At this stage, a series of interviews were conducted with water users in order to identify the central issues proposed for further investigation. As explained by Eden (Eden 1990):

“The interview is conducted so that further questions are built on the 'grounded data' generated during the interview and recorded as a cognitive map. After the interviews, the facilitator(s) constructs a...model of the knowledge and arguments he discovers, merging the various views so that 'synergy' and creativity become possible. The overall map produced by this procedure will highlight the goals, the supporting assumptions, key issues, and the options available to...action-oriented workshops”

The paper is organized as follows: the next section highlights the two methodological threads informing this research: SD and SODA with special emphasis on the pluralism of both methodologies in real life interventions. Section (3) introduces the case study background and context. Section (4) presents the adopted research methodology. Findings are outlined in section (5) followed by a synopsis of the current work in progress.

2. Background

2.1 SD Modelling

Grounded in control theory, nonlinear dynamics and cybernetics, SD is a rigorous feedback oriented methodology for modelling complex systems. In SD, the emphasis is placed on understanding the structure underpinning the problematic behaviour. The structure represents the network of cause-and-effect between the system elements which governs its behaviour.

SD involves more than the construction and analysis of mathematical models. It appreciates qualitative modelling and client involvement through the project lifecycle (Sterman 2007). Forrester (Forrester 1994a) early acknowledged the significance of drawing heavily on the mental model database for effective model building. SD has been concerned to work intimately with clients in the Group Model Building (GMB) process for at least three reasons (Vennix 1999) to: (1) identify the required knowledge, (2) enhance the implementation of model results, and (3) increase client learning through interaction and model ownership.

2.2 Strategic Option Development and Analysis (SODA)

Giving up the mathematical modelling part of hard OR, Soft OR concentrates on defining the situation, resolving conflicting viewpoints, and coming to a consensus about alternative courses of action (Rosenhead & Mingers, J. 2001). SODA is a soft OR methodology developed by Eden and his colleagues (Eden 2004). It provides a systematic way for

collecting, structuring and analysing subjective knowledge using interviews, workshops and CM techniques. This might involve: interviews with key individuals and/or the analysis of background papers; and/or interviews with experts on the topic area (Eden 1990). Figure (1) depicts a typical SODA project as illustrated by Eden (1990). The current research stage is outlined in the diagram.

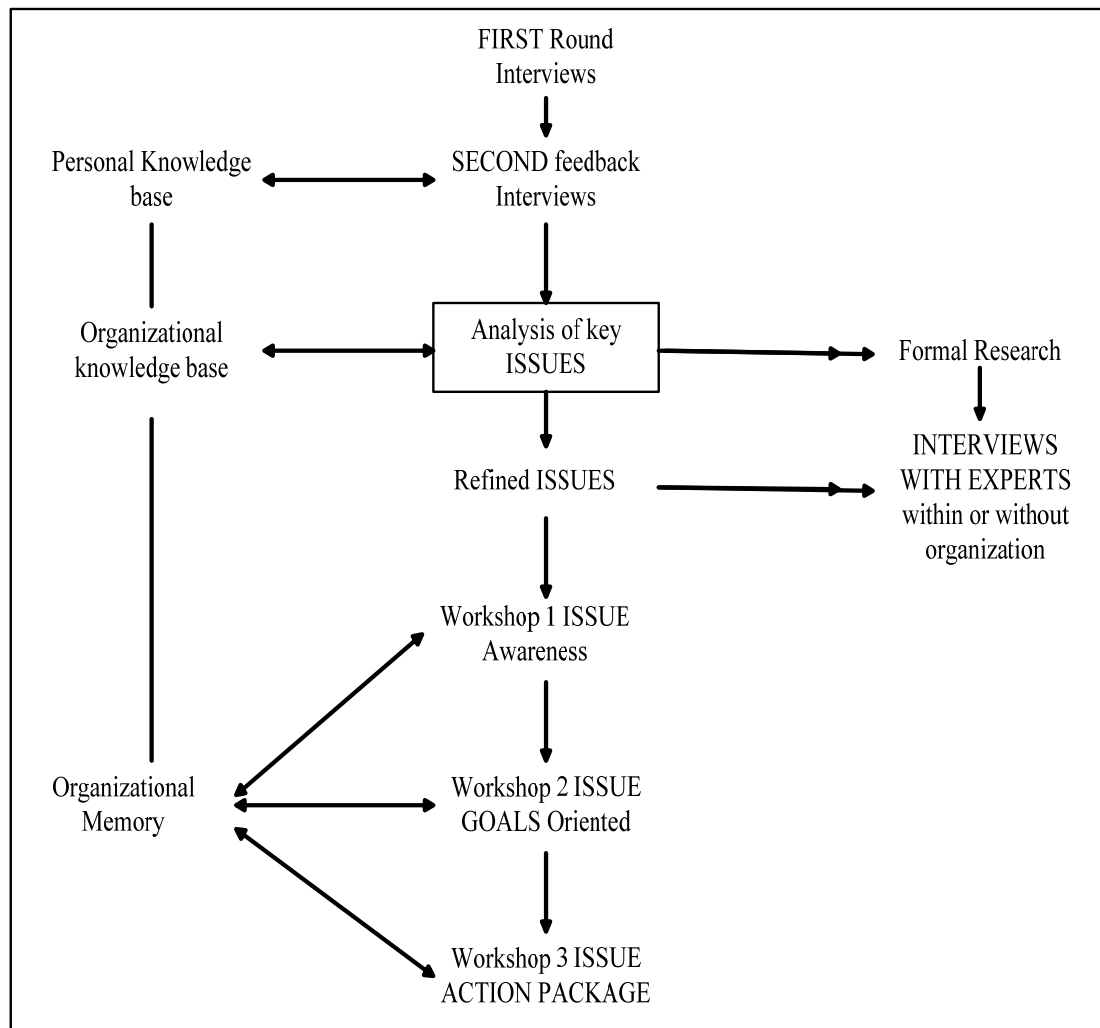


Figure 1: A Typical SODA project as depicted by Eden (1990), where the current stage is marked

At the core of the methodology, the technique of “cognitive mapping” is used to capture different views about the situation. In literature, representing cognition as a causal structure is supported by different theoretical groundings (Huff 1990). The theoretical foundation of CM in SODA is the “personal construct” theory developed by the cognitive psychologist George Kelly (Kelly 1963). In Kelly’s view, people are iteratively involved in developing cognitive constructs of the world as they experience it as a way make sense of their present and anticipate future events. The constructs are bipolar (i.e. they have two ends), or dichotomous and will vary from one person to another dependent on the way he may reason the situation. CM can be seen as a picture or visual aid in comprehending the subject’s understanding of particular, and selective, elements of the thoughts of an individual as a network of means and ends concepts (Eden 1992). Using CM in SODA is used to support the following problem solving activities (McLucas 2003) (Montibeller et al. 2008):

1. Evaluating policy alternatives and strategy development.
2. Negotiating a social order with the aim of communication, conflicts resolution, and consensus building.
3. Learning where people externalize, challenge and revise their mental models about the situation

4. Problem structuring as a basis for model building

2.3 SODA for SD Modelling Intervention

Lane (Lane 1994) (Lane & Oliva 1998) invites the dialogue and synthesis of SD and soft OR with the aim of helping stakeholders generate and articulate a richly divergent set of substantially different views which might then inspire various issues upon which a model building process may centre. Soft OR provides a powerful toolkit of problem structuring techniques which may be effectively support problem identification and model conceptualization (Forrester 1994b).

Integrating SD and SODA, the CM is used to depict the structure of the problem as perceived by the client group (Eden 1994). From a divergent thinking perspective, CM encourages debate among actors and compares the many different views and proposals they have about a topic in an easy self-expressive language (Montibeller et al. 2008). In SD modelling intervention, the modeller is keen to capture the concepts or the “nub of issues” eligible for being usefully treated as variables. The concepts having large number of links combined with feedback are the potential candidates for a SD model (McLucas 2000). Selection of candidates is a convergent thinking-based exercise through which the modeller with the cooperation of stakeholders strives for a legitimate and holistic representation for the situation using tools like influence diagrams (Eden 2004). This double-phase thinking process insures that the richness and complexity of the situation are fairly reflected in the simulation model. Figure (2) illustrates the bi-directional relation between CM, influence diagrams and SD model.

In a case study for modelling complex construction projects, Ackermann (Ackermann et al. 1997) demonstrated how cycling between qualitative analysis of CM and SD simulation analysis richly informed each other, resulting in benefits which are likely unattainable if they were separately implemented. As a result of analysing six case studies in organizational crisis and defence preparedness, McLucas identified the candidates for SD modelling for 3 cases, while other modelling techniques were recommended for the rest of cases (McLucas 2000). Motivated by the desire to add client value to the project, Howick et al. (Howick, Susan et al. 2006) used CM to depict potential scenarios about the renewable energy sources in the U.K. electric power market. Although the causal structure underlying the simulation model was developed by the research team based on pertinent literature review, scenarios are linked to the model in order to visualize the over-time dynamics produced by each scenario. However, the multi-methodological engagement of SD and SODA is still limited in its application (Mingers 2000).

3. Case Study: Water Management in the ACT

Australia is referred to as the driest inhabited continent on Earth. Uncertainty characterizes the supply and demand of water across much of the country where water resources are under pressure on several fronts (Pigram 2006). The research described in this paper focuses on water available in the ACT. The ACT was built early in the 20th Century to house the Australian Federal Government and its many Government departments. According to the Australian Bureau of Statistics in 2005, the ACT is home to just over 326,700 inhabitants, numerous commercial and industrial enterprises and a smaller number of rural and agricultural activities. The ACT is land-locked territory on the southern highlands located within the state of New South Wales. The ACT has a temperate climate with an average rainfall in the range of 450-600 mm. The ACT covers some 65 kilometres from North to South and 35 kilometres from East to West. It is bordered by, and shares many of its resources with, several smaller towns, villages and rural communities.

The ACT is chronically threatened by water shortage due to population growth, increasing usage per capita, the aftermath of the 2003 bushfires in the Cotter catchment and long-term climate change. Despite the water restrictions currently mandated in the city, the current storage levels are less than 48%, and the daily usage limit is often exceeded. Most of the ACT residents are immigrants from other states or countries, who might not have prior

experiences relevant to water problems. This background ultimately affects the way each person views his consumption relative to the whole picture. Traditional water management approach, which used to deal with people as average number with slight attention to the dynamics underlying their behavioural patterns, is no longer beneficial. This highlights the need to spotlight on the individual consumers, as a unit of analysis, to explore their mental models and how they make sense of the situation.

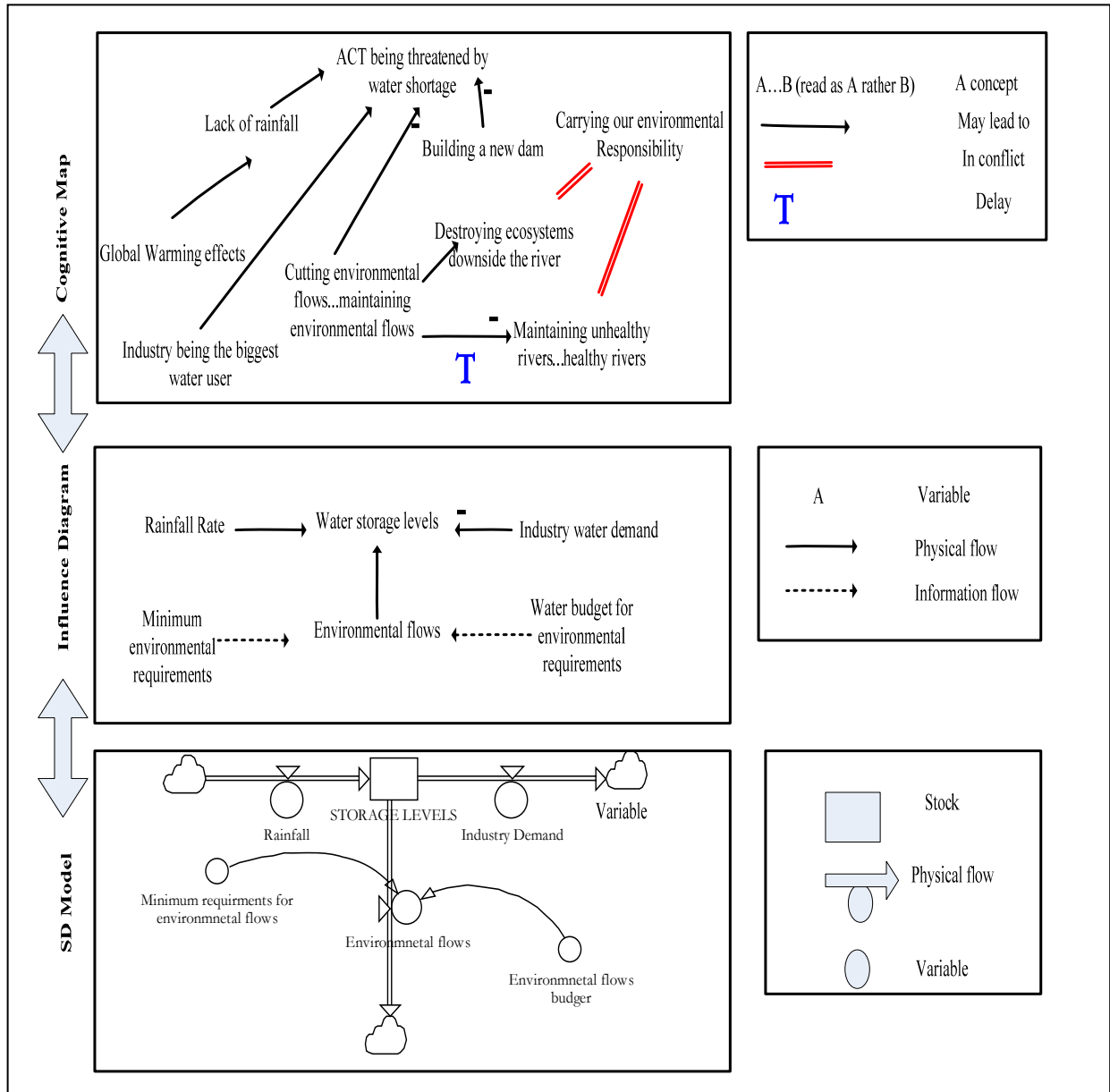


Figure 2: The bi-directional interaction between CM, ID and SD models

4. Research Methodology

A two-session set of interviews were conducted with 25 residents (F=13, M=12). A two step sampling process is followed to select potential interviewees. First, a recruitment advertisement was broadcasted to call for participants. Taking this as an initial sample, participants are organized according to their socio-economic levels and demographic areas. Snowball sampling technique was employed within each cluster to engage more participants. Much of the value of the open ended interviews can be obtained by a relatively small sample where an appropriate sample of 20-30 individuals should reveal most of the beliefs held with any substantial frequency in the population from which they are selected (Morgan et al. 2001).

Moreover, they are effectively used in eliciting mental models when the researcher wishes to gain a very deep understanding of the issues from the standpoint of individuals avoiding early group effects (Eden & Ackermann 1998)(Sterman 2000). Using semi-structured interviews probing around a set of anchor topics is found to be the most efficient approach to elicit rich and valid data (Laukkanen 1998).

In session 1, a semi-structured interview protocol (35-50 min) was used to surface each participant's mental model. In order to avoid pre-contamination, each respondent was told that the interview aims to receive inputs from water users in a study for investigating and modelling the capacity of water utilities. The interview followed a "funnel design", which started generally and proceeds to increasingly focused questions. Initial warm and very general questions work get the participant quickly engaged in the interview. During the interviews, the use of words like "shortage", and "problem" was avoided so the judgment for whether there is a problem and how it is conceptualized is made by the participant. Methodologically, researchers should explicitly consider and the validity, reliability and practicability of their processes in terms of the question deriving their research (Jenkins 1998). While validity questions whether the issues which are salient to the respondent are actually captured, reliability looks whether the researcher has added his own interpretations to the analysis. Like client-centred therapy, a set of follow up questions are targeted to clarify what respondents have in mind, but not to question its legitimacy (Morgan et al. 2001). Some prompting questions were carefully used to round ideas among participants in order to capture the requisite richness of their mental models keeping the attention not to encourage participants reframe their mental models or practise the subject-expectancy bias. Through the mapping process, we kept track of those parts elicited after prompting. At the end of the interview, the researcher set some time for closure where the next step is explained.

After interviews had been transcribed and analysed, the second session interview (15-30 min) was organized to ensure the reliability of the map structure using a question-and-answer technique. For more convergence towards reliability and validity, this process may be repeated subsequently to elicit and test more knowledge. However, this does not seem practical in terms of the participants and researchers time. Moreover, in light of the study research question the purpose of the knowledge elicitation process is capturing a snapshot of the mental models users have about the situation at the present time rather than a longitudinal comparison across maps.

5. Preliminary Analysis and Findings

It is noteworthy that we do not aim to derive any statistically induced results or generalizations about the whole population. This analysis aims to (1) identify the key issues to be further investigated, and (2) derive a working or first-cut holistic view of the situation.

5.1 Identifying Key Issues

The size of elicited maps varies between 10-62 concepts. Broadly, analysis of the maps indicates marked differences and similarities among participants about the problem, underlying causes, anticipated consequences and necessary solutions. The key issues emerged from the discussions are:

- **Water shortage...Poor management**
There is a consensus among participants on the problem existence but not on how it is formulated. Some participants have the belief that the city receives enough amount of rainfall which is poorly managed, while the others perceive a serious shortage problem due to environmental conditions.
- **The drought is a temporary event...The drought is a permanent condition**
There is a strong debate on describing the drought as a temporary or permanent condition. This debate is inherently in the belief of global warming effects. Some of the participants believe that it is just a natural cycles of droughts whereas global warming advocates expect a persistent drought condition.

- **Maintaining environmental flows...Cutting environmental flows**
The release of environmental flows to maintain the health of inland streams is a central conflict point which clearly reflected the very different values participants hold. Some participants argued the legitimacy of releasing such enormous amount of water at the time every drop should be saved to sustain the community. Participants of this view show doubts about the actual reason for maintaining such flows. They believe that they are maintained to support water-expensive crops such as cotton/rice rather than ecosystems downside the river.

Participant A: I don't believe we have the responsibility to look after every plant and animal. I believe that the government has enough trouble looking after the community...I am more concerned about the utility of water to the community because if there is no water, there would be no community.

The second group overweighted the environmental value for keeping healthy rivers and wildlife/aquatic habitants.

Participant B: The choice to kill these ecosystems is based on the simple criteria that we are human who make the decision!

Participant C: We are very lucky that we are the first water users in the Murray basin. This means that we have the responsibility of passing water to others in the queue.

- **2003 bushfire has marginal effect....2003 bushfire has long term effects**
The impact of 2003 bushfires on water quantity and quality is a controversial issue. Some participants think that the effect is marginal in terms of the amount of water used to put off the fires, while other identified the negative impact of vegetation destruction and growth on the amount of runoff.
- **Building a dam...not building a dam**
For most participants, building a dam seems to be the only logical and less risky solution to overcome the situation. Few participants argued about the environmental cost in terms of destroying large ecosystems and doubted if there is enough rainfall to fill in the dam.
- **Residential water demand being the biggest consumption... Non residential water demand being the biggest consumption**

According to the government reports (www.thinkwater.act.gov.au), households are the biggest users (54% of total consumption). However, most participants overestimated the consumption of industry (19% of total consumption) and government (11% of total consumption) and the significance of controlling their usage.

5.2 Developing a working holistic view

Aggregating the beliefs gained from the CMs, ID in figure (3) is developed to present the current understanding of the situation gained from the conducted interviews. This will be passed for further examination during the experts' interviews and the subsequent focus groups to identify the most critical feedback loops to be modelled.

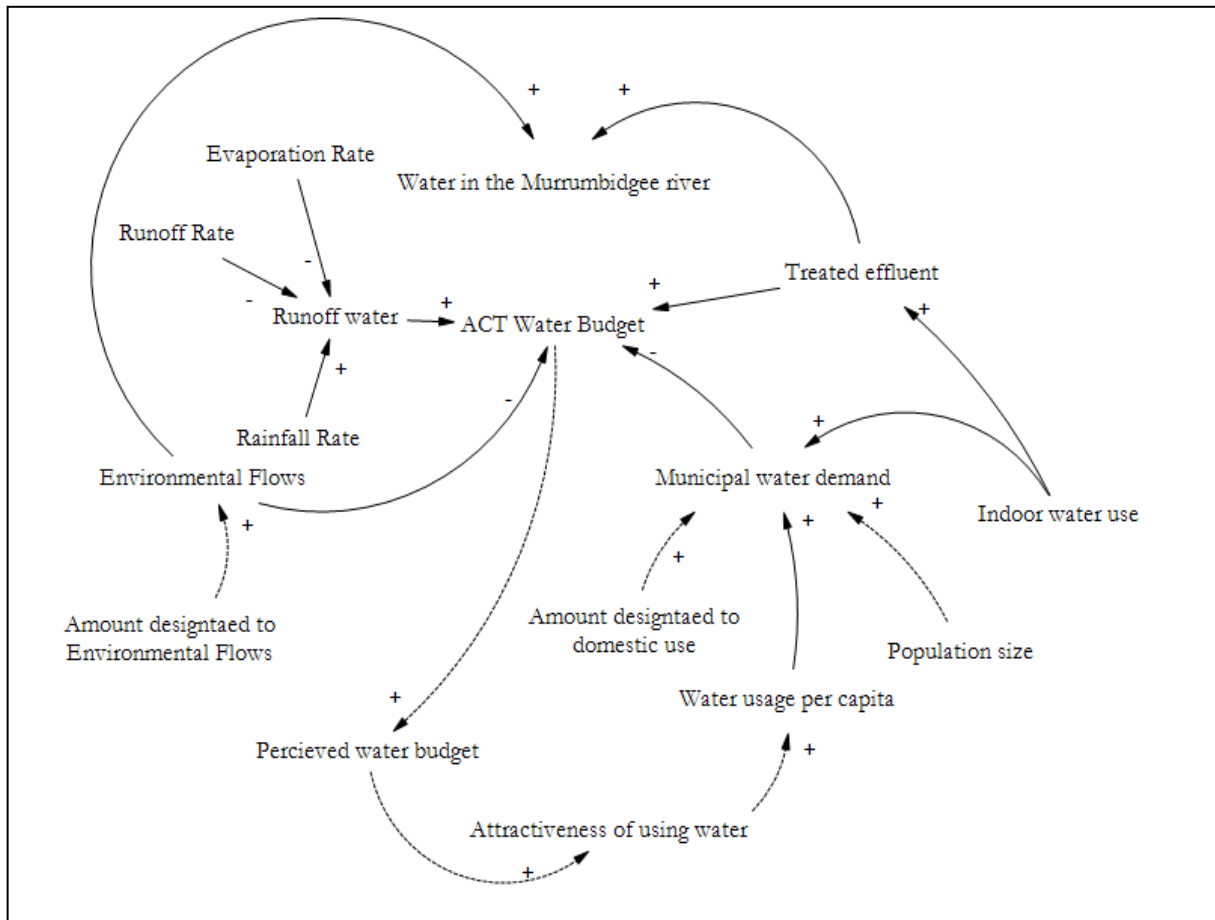


Figure 3: An ID derived from the CMs of participants

6. Conclusion and work in progress

In this paper, we identified the key issues revealed from interviews with 25 residents about the chronic water shortage problem in the territory. This included the doubted effect of global warming on water supplies, the legitimacy of releasing environmental flows and the effectiveness of adding dams. Initial findings indicate that acceptance of public policies is mainly driven by conception of the situation. It was observed that participants who perceive the situation in terms of poor management tend to be more resistant to the employed restrictions. This underlines the need to understand the mental models of water users.

In addition, a holistic ID is developed in an attempt to combine the CMs of all participants. This working tool acts as an aid for understanding the situation. In the next stage, it will be passed for further examination during the experts' interviews and the subsequent focus groups to identify the most critical feedback loops to be modelled. As expected, contrasting the views of laypeople and experts will spotlight on many blind points in the mental models of both groups about how the system behave. While experts base their policies on the rational actor assumption, people interpret and behave in an unanticipated manner. Understanding how people make decision in real life contexts may highlight the impediments for communication, learning and effective decision making. From a SD perspective, we believe that accumulating such understanding is the leverage point for enriching our theoretical base and widening the scope of our applications. This may be more promising way to go rather than looking for applications when best SD may be implemented. In the next phase of the research project, the effectiveness of the intervention is evaluated using a process rather outcome oriented approach, where the emphasis shifts from wether to how/why learning occurred or not. We adopt an integrated view of learning as cognitive and behavioural changes.

In the context of natural resources management, it is expected that engaging participants in the experience of interacting with a simulation game will enable them to better understand the system structure, revise their mental models and change their water using patterns. In this research, we are motivated by our belief that the slowly-but-surely approach to achieve a mature, refutable and universally accepted SD fields involves two interdependent steps:

- (1) Combining SD with soft OR methodologies and process based evaluation approaches will substantially contribute to the effectiveness of our interventions.
- (2) Externalizing and linking the adopted methodologies and presenting well-documented research results.

Figure (4) is a meta-cognitive map for the researchers' understanding of the means and goals for the research.

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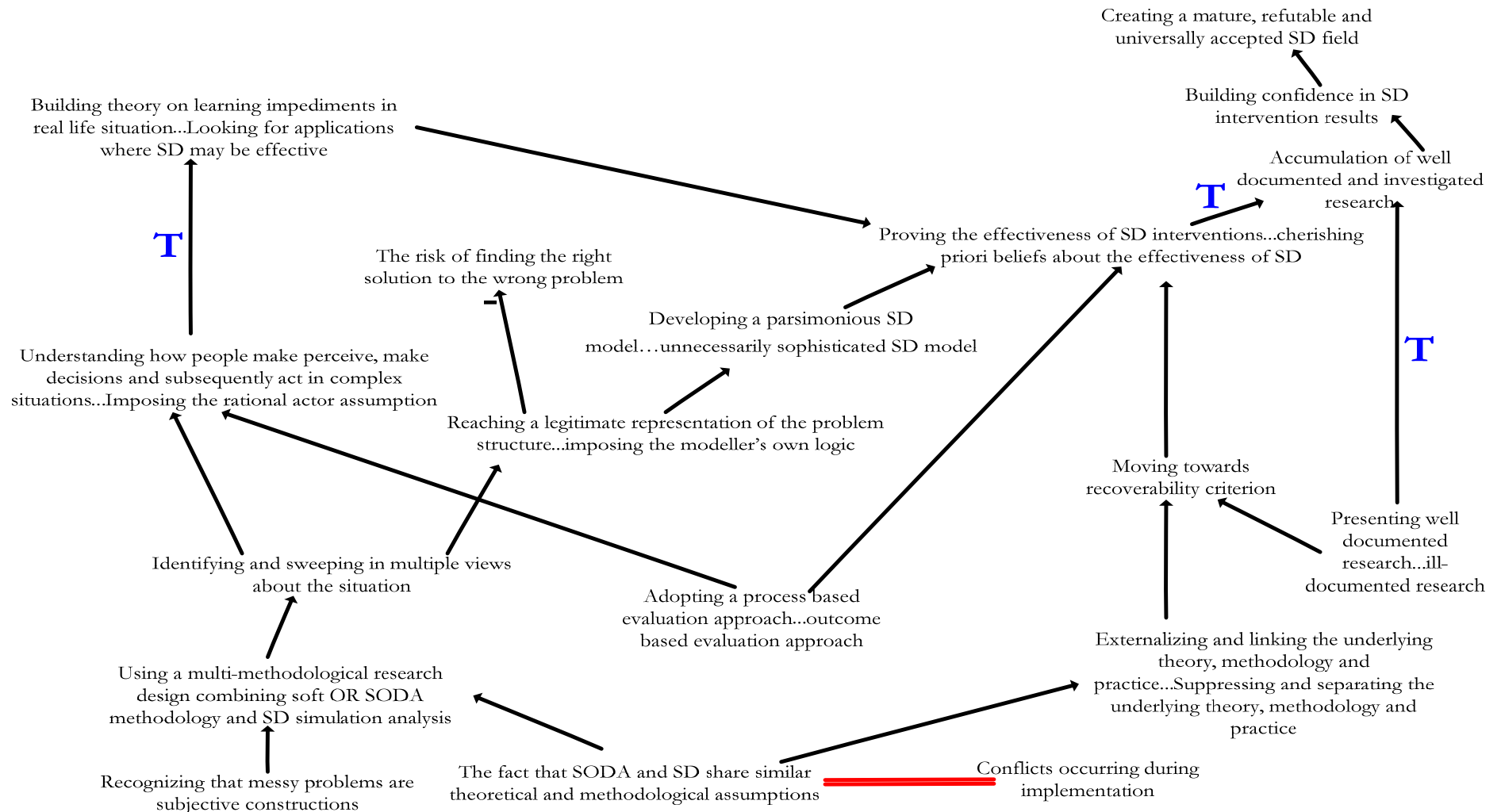


Figure 4: A meta-cognitive map for the researchers' current understanding of the means and goals for the research

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Complexity Reduction for Solving a Pure Integer Program by the Branch and Bound Method using the Gomory Constraints

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Abstract: This paper deals with improving complexity of the branch and bound method for solving a pure integer program. This improvement is achieved by formulating a characteristic pure integer program from all the Gomory constraints arising from the relaxed LP solution of the given problem. The number of sub-problems required in the branch and bound method reduce significantly.

Keywords: Branch and bound method, pure integer programs, characteristic equation, characteristic pure integer program, Gomory constraints, descending hyper-plane method.

1. Introduction

In the case of a pure integer program, if the linear programming (LP) optimal extreme point solution is not an integer point, the required integer optimal point must lie in the interior region of the LP relaxed convex region. Past attempts for solving a pure integer program (PIP) were dealing with modifications of the LP convex region such that the optimal integer point can be reached by using the LP type approaches. The Gomory cutting plane algorithm was the first developed in 1958 as a method for solving a PIP model [4, 5]. Although there is no guarantee that the Gomory procedure will converge to an optimal solution, but when it did the convergence in many cases was slow. For this reason, the ideas of using cuts to solve a PIP were neglected until the 1980s. Later attempts were made to improve the cutting plane approaches, but the end results were still not encouraging, see [1, 9, 15, 17]. Besides the Gomory cut, there are several other types of cuts, these include *Dantzig*, *Chvatal*, *Fenchel*, *knapsack*, *lift-and-project*, etc, see [2, 6, 9]. The resurgence of the interest in the cutting plane algorithm was due to the development of the polyhedral combinatorial approach and consequent implementations of cuts that use facets of the convex hull of the integral feasible points as cut [3, 9, 12]. Cuts are currently used to enhance the effectiveness of other methods. These approaches include: Branch and cut [13]; Branch, cut and price [16]; Interior point cutting plane [10]; heuristics search and other approximating methods [7, 18].

Recently, the authors attempted to reach the required optimal integer point directly by the descending hyper-plane approach [8]. This approach requires solution of a characteristic equation for different values of a parameter. The authors have developed various strategies for efficient solution of the characteristic equation in [14]. Thus an interior integer point can either be

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reached directly, or by modifying the convex region to force the extreme point to be an integer point.

In this paper, a new approach is proposed that can improve effectiveness of the existing branch and bound method significantly by reducing the number of sub-problems created by the method to reach the optimal solution. Also this paper disproves a general belief among researchers in mathematical programming that cuts may not be useful on their own to solve a general *PIP* model. Improved effectiveness is achieved by developing a characteristic PIP from the LP solution and the associated Gomory cuts. The new problem is also a PIP and is called a characteristic PIP, which is a minimizing model. This characteristic model has excellent features and relationships with the original model.

In the descending hyper-plane approach [8, 14], it was established that the required integer optimal point can be determined by increasing values of the non-basic variables from the zero value they have at the LP optimal solution to a non-negative integer value ≥ 0 . This aspect of increasing the values of the non-basic variables to non-negative integer value in the descending hyper-plane approach was achieved through a characteristic equation and in the present paper the same objective is achieved by formulating a characteristic PIP model. Complexity of the characteristic model compared to the original problem is relatively less when it is solved by the branch and bound method. The solution of the characteristic PIP is used to find the required optimal integer solution of the original PIP. Thus this characteristic problem is solved first for an integer solution, which in turn finds the required integer solution of the given PIP.

The paper has been organized in 6 sections. Section 2 deals with the development of the characteristic PIP. In Section 3, the relationship of the given problem with its characteristic PIP model has been discussed. Section 4 outlines the algorithmic steps to solve a PIP using the ideas discussed in this paper. Section 5 presents a numerical example and finally in Sections 6 and 7, computational experiences are discussed with some randomly generated problems and problems selected from the integer programming library.

2. The characteristic PIP model

Let a given PIP model be represented as:

$$\text{Maximize } Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

$$\text{Such that } a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$$

(1)

where a_{ij}, b_i and c_j are discrete constants, Z is the objective function, and $x_j \geq 0$ are the restricted discrete variables.

The LP relaxed solution of (1) be as given in Table 1.

	x_1	x_2	\dots	x_m	s_1	s_2	\dots	s_n	<i>r.h.s.</i>
Z	0	0	\dots	0	ω_1	ω_2	\dots	ω_n	γ
x_1	1	0	\dots	0	α_{11}	α_{12}	\dots	α_{1n}	β_1
x_2	0	1	\dots	0	α_{21}	α_{22}	\dots	α_{2n}	β_2
		.							
		.							
		.							
x_m	0	0	\dots	1	α_{m1}	α_{m2}	\dots	α_{mn}	β_m

Table 1: Continuous optimal tableau

In Table 1, $\omega_1, \omega_2, \dots, \omega_n, \beta_1, \beta_2, \dots, \beta_n$ are non-negative constants, whereas α_{ij} and γ are constants with positive, negative or zero values. For convenience, all basic variables in the final table are represented by x_1, x_2, \dots, x_m and the non-basic by s_1, s_2, \dots, s_n . Thus x_1, x_2, \dots, x_m are not necessarily the same in model (1) and Table 1. Similarly, s_1, s_2, \dots, s_n are not necessarily the slack variables. The basic variables and the objective function can be expressed as a function of the non-basic variables. From Table 1, these relationships between the basic and the non-basic variables are given by (2),

$$\begin{aligned}
 Z + \omega_1 s_1 + \omega_2 s_2 + \dots + \omega_n s_n &= \gamma \\
 x_1 + \alpha_{11} s_1 + \alpha_{12} s_2 + \dots + \alpha_{1n} s_n &= \beta_1 \\
 x_2 + \alpha_{21} s_1 + \alpha_{22} s_2 + \dots + \alpha_{2n} s_n &= \beta_2 \\
 &\vdots \\
 &\vdots \\
 &\vdots \\
 x_m + \alpha_{m1} s_1 + \alpha_{m2} s_2 + \dots + \alpha_{mn} s_n &= \beta_m
 \end{aligned} \tag{2}$$

For each row in Table 1, one can write a corresponding Gomory constraint. Thus the number of Gomory constraints will be at most m . These constraints together with the LP output (2) can be expressed as:

$$\begin{aligned}
 &\text{Maximize } z = (\gamma - (\omega_1 s_1 + \omega_2 s_2 + \dots + \omega_n s_n)) \\
 &\text{such that} \\
 &x_1 + \alpha_{11} s_1 + \alpha_{12} s_2 + \dots + \alpha_{1n} s_n = \beta_1 \\
 &x_2 + \alpha_{21} s_1 + \alpha_{22} s_2 + \dots + \alpha_{2n} s_n = \beta_2 \\
 &\vdots \\
 &x_m + \alpha_{m1} s_1 + \alpha_{m2} s_2 + \dots + \alpha_{mn} s_n = \beta_m \\
 &f_{11} s_1 + f_{12} s_2 + \dots + f_{1j} s_j + \dots + f_{1n} s_n \geq f_1 \\
 &f_{21} s_1 + f_{22} s_2 + \dots + f_{2j} s_j + \dots + f_{2n} s_n \geq f_2 \\
 &\vdots \\
 &f_{m1} s_1 + f_{m2} s_2 + \dots + f_{mj} s_j + \dots + f_{mn} s_n \geq f_m
 \end{aligned} \tag{2a}$$

The above problem leads to an equivalent problem given in (3). Note that each Gomory constraint makes the given corresponding constraint redundant. This results in a model given by (3).

$$\begin{aligned}
 & \text{Minimize} && \omega_1 s_1 + \omega_2 s_2 + \dots + \omega_j s_j + \dots + \omega_n s_n \\
 & \text{Such that} && f_{11} s_1 + f_{12} s_2 + \dots + f_{1j} s_j + \dots + f_{1n} s_n \geq f_1 \\
 & && f_{21} s_1 + f_{22} s_2 + \dots + f_{2j} s_j + \dots + f_{2n} s_n \geq f_2 \\
 & && \cdot \\
 & && \cdot \\
 & && f_{m1} s_1 + f_{m2} s_2 + \dots + f_{mj} s_j + \dots + f_{mn} s_n \geq f_m \\
 & && s_j \geq 0 \text{ for } j=1,2,\dots,n.
 \end{aligned} \tag{3}$$

Note that if a slack variable is basic in Table 1, there is no need to develop a Gomory constraint with respect to that variable, as it reflects existence of a redundant constraint. Thus the number of constraints in (3) will be less than or equal to m . The f_{ij} and f_i are fractions such that:

$$0 \leq f_i \leq 1, \forall i, \quad 0 \leq f_{ij} \leq 1, \forall (i, j), i = 1, \dots, m; j = 1, \dots, n. \tag{4}$$

Each row in the problem (3) can be multiplied by a common denominator to obtain an alternative PIP with all variables restricted to integer values only and all coefficients are also integer constants. This is shown in (5).

$$\begin{aligned}
 & \text{Minimize} && \varpi_1 s_1 + \varpi_2 s_2 + \dots + \varpi_j s_j + \dots + \varpi_n s_n \\
 & \text{Such that} && F_{11} s_1 + F_{12} s_2 + \dots + F_{1j} s_j + \dots + F_{1n} s_n \geq F_1 \\
 & && F_{21} s_1 + F_{22} s_2 + \dots + F_{2j} s_j + \dots + F_{2n} s_n \geq F_2 \\
 & && \cdot \\
 & && \cdot \\
 & && \cdot \\
 & && F_{m1} s_1 + F_{m2} s_2 + \dots + F_{mj} s_j + \dots + F_{mn} s_n \geq F_m \\
 & && s_1, s_2, \dots, s_n \geq 0 \text{ and integers.}
 \end{aligned} \tag{5}$$

Note that ϖ_j, F_{ij} and F_i are discrete constants, hence it is also a PIP model. This model (5) is the required characteristic PIP model of the given PIP (1).

3. Relationship between the models (1) and (5)

The aim is to find suitable non-negative integer value of the non-basic variables, which currently in Table 1 are 0. The characteristic PIP model (5) finds these values. This characteristic model deals with variables that have relatively smaller integer values; hence it is likely to be less complex. In other words, the non-basic discrete variables are likely to have small ranges. The solution to the given PIP can be determined by substituting the solution from (5) in the relationships given by (2). If all basic variables satisfy integer requirements, one stops otherwise

one is required to find the next best solution and check integrality of that solution for the given PIP model from the relations (2).

3.2 Complexity reduction

Let the ranges of variable x_j be denoted by $range[x_j]$. The number of sub-problems needed to verify the optimal solution is proportional to $(\prod_{j=1}^n (range[x_j]))$. Thus when range of each variable is less, the number of sub-problems when dealing with the model (5) are likely to be less in number compared to the number of sub-problems created by (1). Hence the characteristic PIP is likely to be less complex.

4 The algorithm using the characteristic PIP

The steps taken to solve a given PIP by using its characteristic PIP can be stated as follows.

- Step 1:** Solve the relaxed PIP to obtain a continuous optimal solution.
- Step 2:** Generate a characteristic PIP model with discrete coefficients from the continuous optimal solution.
- Step 3:** Set $k=1$.
- Step 4:** Solve the characteristic PIP by the branch and bound method for the k th best solution.
- Step 5:** Test that solution for integral requirements by using the relations (2). If the solution satisfies integer requirement, go to Step 7; otherwise go to Step 6.
- Step 6:** Set $k = k+1$ and go to Step 4.
- Step 7:** Print the integer optimal solution to the given PIP.

5 Numerical example

$$\text{Maximize } Z = 6x_1 + 4x_2 + 11x_3 + 8x_4$$

$$\text{Such that } 3x_1 + 9x_2 + x_3 + 5x_4 \leq 80$$

$$11x_1 + 21x_2 + 19x_3 + 14x_4 \leq 200 \quad (6)$$

$$9x_1 - 8x_2 + 15x_3 + 12x_4 \leq 70$$

Where $x_1, x_2, x_3, x_4 \geq 0$ and discrete.

Using the automated branch and bound on Tora software [19, 20, 21], it generated **93** sub-problems to verify the optimal solution. The optimal integer solution is $Z = 82, x_1 = 0, x_2 = 4, x_3 = 6, x_4 = 0$

The continuous optimal solution of (6) is given in Table 2.

	x_1	x_2	x_3	x_3	s_1	s_2	s_3	<i>r.h.s.</i>
Z	$\frac{221}{467}$	0	0	$\frac{196}{467}$	0	$\frac{148}{467}$	$\frac{155}{467}$	$\frac{40450}{467}$
s_1	$\frac{1178}{467}$	0	0	$\frac{2133}{467}$	1	$-\frac{143}{467}$	$\frac{150}{467}$	$\frac{19260}{467}$
x_2	$-\frac{6}{467}$	0	0	$-\frac{18}{467}$	0	$\frac{15}{467}$	$-\frac{19}{467}$	$\frac{1670}{467}$
x_3	$\frac{277}{467}$	0	0	$\frac{364}{467}$	0	$\frac{8}{467}$	$\frac{21}{467}$	$\frac{3070}{467}$

Table 2: Continuous optimal solution of (6).

Here the slack variables, $s_1, s_2, s_3 \geq 0$ and restricted to integer values.

The Gomory constraints can be easily generated for each constraint, and an alternate PIP can be formulated as given in (7).

$$\begin{aligned} \text{Minimize } Z &= \frac{221}{467}x_1 + \frac{196}{467}x_4 + \frac{148}{467}s_2 + \frac{155}{467}s_3 \\ \text{Such that } \frac{244}{467}x_1 + \frac{265}{476}x_4 + \frac{324}{467}s_2 + \frac{150}{467}s_3 &\geq \frac{113}{467} \\ \frac{461}{467}x_1 + \frac{449}{476}x_4 + \frac{15}{467}s_2 + \frac{448}{467}s_3 &\geq \frac{269}{467} \\ \frac{277}{467}x_1 + \frac{364}{476}x_4 + \frac{8}{467}s_2 + \frac{21}{467}s_3 &\geq \frac{268}{467} \end{aligned} \quad (7)$$

Note from Table 2 that the variable s_1 is basic indicates that constraint number 1 is inactive. This constraint could have been dropped. However, if one multiplies through by a common denominator, one gets the characteristic PIP (8), given below.

$$\begin{aligned} \text{Minimize } z &= 221x_1 + 196x_4 + 148s_2 + 155s_3 \\ \text{Such that } 244x_1 + 265x_4 + 324s_2 + 150s_3 &\geq 113 \\ 461x_1 + 449x_4 + 15s_2 + 448s_3 &\geq 269 \\ 277x_1 + 364x_4 + 8s_2 + 21s_3 &\geq 268 \\ x_1, x_4, s_2, s_3 &\geq 0 \text{ and integer.} \end{aligned} \quad (8)$$

Solving (8) using the branch and bound on TORA, it took only 7 sub-problems and the best 4 solution are shown below:

Solutions	x_1	x_4	s_2	s_3
Best	0	1	0	0
Second	1	0	0	0
Third	0	0	0	13
Fourth	0	0	2	12 (Optimal integer solution)

These non-basic values when substituted in the relation equivalent to the relation (2) for the problem (7), an integer solution was obtained from the fourth best solution as given below. This was the same optimal solution that was obtained after solving 93 sub-problems when the given problem (6) was solved directly by the branch and bound method.

$$Z = 82, x_1 = 0, x_2 = 4, x_3 = 6, x_4 = 0.$$

Note the significant reduction was achieved as requirement of 93 sub-problems in the direct approach have been reduced to 7 sub-problems, when the corresponding characteristic PIP model was tackled using the same branch and bound approach.

Now consider the worst-case complexity of the models (6) and (8). This is given in Table 3.

Model	Range of variables	Worst-case complexity
(6)	The range of the four variables is given by 8, 10, 6 and 7, respectively.	3660, many of these integer points are in the infeasible region
(8)	The range of the four variables is given by 2, 2, 35 and 7, respectively.	980, once again many of these integer points are in the infeasible region.

Table 3: Complexity comparison.

6 Computational experiments

Consider the following two special examples before discussing the case of the randomly generated problems.

Problem 1: Taken from [22]

$$\text{Maximize } Z = 8x_1 + 5x_2$$

$$\text{Subject to } x_1 + x_2 \leq 6 \quad (9)$$

$$9x_1 + 5x_2 \leq 45$$

$$x_1, x_2 \geq 0 \text{ and are integers.}$$

This PIP created 7 sub-problems when solved by the branch and bound technique but through the characteristic model got the solution immediately.

Problem 2: Taken from [8]

$$\text{Maximize } Z = 3x_1 + 5x_2 + 7x_3$$

$$\text{Such that } 4x_1 + 9x_2 - 8x_3 \leq 81$$

$$5x_1 - 7x_2 + x_3 \leq 42 \quad (10)$$

$$-2x_1 + 9x_2 + 7x_3 \leq 10000$$

Where

$$x_1, x_2, x_3 \geq 0 \text{ and discrete.}$$

Using the automated branch and bound on Tora software [20], it generated 209 sub-problems, whereas on the new approach of using the characteristic PIP took only 63 sub-problems to reach the same optimal solution.

Motivated by these results, it was decided to experiment further on randomly generated problems. Computational results of the ordinary branch and bound versus the use of the characteristic PIP model are presented in Table 4. Ten PIP models were randomly generated for each class and TORA (automated branch and bound) was used to determine the number of sub-problems required to verify the optimal solution.

Randomly generated PIP models (No. of variables X No. of constraints)	Average % reduction in the No. of sub-problems when the characteristic was used.
10X10	51
20X20	48
30X30	42
40X40	56
50X50	54
60X80	56
70X70	64
80X80	52
90X90	46

Table 4: % reduction in the number of sub-problems when the characteristic approach was used.

From the above computational experience it may be reasonable to conclude that the effort required to find the optimal solution is consistently lower for the approach using the characteristic PIP model.

7. Computational experiments with some standard benchmarked PIP problems

More experiments were conducted with a few selected benchmarked problems that are available in the mixed integer-programming library (*MIPLIB*), see Bixby et al. (1998). These problems ranged from the moderately easy to the very difficult ones, making them good for testing the above algorithm discussed in this paper. The other reason for selecting *MIPLIB* was, that it provides information concerning the characteristics of the problems, see Table 5. However, most of these test problems are dealing with the binary variables, only a few are pure integer programs. A majority of these test problems are mixed integer programs. Since the proposed algorithm deals with the pure integer programs, consequently the models dealing with mixed integer variables could not be used for the current test purposes in this paper. It is because of this situation; the approach presented in this paper along with the other earlier approach developed by the authors [8] was used. The other approach in this case is a combination of the descending path and use of the characteristic *PIP* to get a tighter lower bound. The hybrid process was used on these test problems because the benchmarked problems contained both binary and pure integer variables. Even though the computations were limited to few benchmarked problems, the performance of the selected approaches on test models was pleasing. These are described in Tables 6.

Table 5: Specific characteristics of standard benchmark model

Problem	Optimal solution	Total number of variables	Number of constraints	Binary variables	Non-binary integer variables
enigma	0	100	22	100	0
gt2	21166	188	30	24	164
mod008	307	319	7	319	0
p0033	3089	33	17	5	28
p0201	7615	201	134	26	175
p0282	258411	282	242	197	85
stein27	18	27	119	27	0
stein45	30	45	332	45	0

- Bixby, R.E., Ceria, S., McZeal, C.M. and Savelsberg, M.W.P., (1998), An updated mixed integer library, *MIPLIP3.0*, *Optima* Vol. 58, pp12-15:
<http://www.ftp.caam.rice.edu/pub/people/bixby/miplib/miplip3>

For the branch and bound algorithm the software *lp_solve*, see Berkerlar and Dirks** (1996), was used to solve the benchmark problems before and after reducing the complexity using the ideas presented in this paper. The computational results are presented in Table 6.

Table 6: Branch and bound computations (without and with reduction in complexity)

Problem title	Solution effort as a function of the number of sub-problems to verify the optimal solution when the original PIP was used.	Solution effort in terms of the number of sub-problems when using the characteristic PIP (% Reduction)
enigma	9321	2335 (74.95)
gt2	Problem exploded to unmanageable levels	939761(Problem became solvable)
mod008	2848139	374668 (86.85)
p0033	7409	1481 (80.01)
p0201	10247	1707 (83.34)
p0282	Problem explode to unmanageable levels	1167457 (Problem became solvable)
stein27	12031	2406 (80.00)
stein45	235087	39181 (83.33)

From the above table, it is evident that reduction is significant when the characteristic PIP is used to solve the problem. .

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ARC Journal Ranking

ARC's first round draft journal rankings for operations research and related fields are provided below for ASOR Bulletin readers. The details rankings with subject areas can be found from http://www.arc.gov.au/era/submissions_ranking.htm

Journal Title	ISSN	Rank
Annals of Statistics	0090-5364	A*
Communications on Pure and Applied Mathematics	0010-3640	A*
Decision Support Systems	0167-9236	A*
Foundations of Computational Mathematics	1615-3375	A*
Fuzzy Sets and Systems	0165-0114	A*
Journal of Nonlinear Science	0938-8974	A*
Journal of Operations Management	0272-6963	A*
Journal of Risk and Uncertainty	0895-5646	A*
Journal of the Operational Research Society	0160-5682	A*
Management Science	0025-1909	A*
Mathematical Programming	0025-5610	A*
Mathematics of Computation	0025-5718	A*
Mathematics of Operations Research	0364-765X	A*
SIAM Journal on Applied Mathematics	0036-1399	A*
SIAM Journal on Numerical Analysis	0036-1429	A*
SIAM Journal on Optimization	1052-6234	A*
Transportation Research Part A-Policy and Practice	0965-8564	A*
Transportation Research Part B-Methodological	0191-2615	A*
Advances in Applied Mathematics	0196-8858	A
Advances in Applied Probability	0001-8678	A
Advances in Computational Mathematics	1019-7168	A
American Statistician	0003-1305	A
Annals of Mathematics and Artificial Intelligence	1012-2443	A
Annals of Operations Research	0254-5330	A
Applied Mathematical Modelling	0307-904X	A
Applied Mathematics and Computation	0096-3003	A
BIT Numerical Mathematics	0006-3835	A
Computers & Mathematics With Applications	0898-1221	A
Decision Sciences	0011-7315	A
Engineering Optimization	0305-215X	A
European Journal of Applied Mathematics	0956-7925	A
European Journal of Combinatorics	0195-6698	A
European Journal of Operational Research	0377-2217	A
IMA Journal of Applied Mathematics	0272-4960	A
IMA Journal of Numerical Analysis	0272-4979	A
International Journal of Forecasting	0169-2070	A
International Journal of Production Economics	0925-5273	A
Journal of Advanced Transportation	0197-6729	A
Journal of Artificial Intelligence Research	1076-9757	A
Journal of Computational and Applied Mathematics	0377-0427	A
Journal of Engineering Mathematics	0022-0833	A
Journal of Forecasting	0277-6693	A

Journal of Global Optimization	0925-5001	A
Journal of Heuristics	1381-1231	A
Journal of Scheduling	1094-6136	A
Linear Algebra and its Applications	0024-3795	A
Mathematics and Computers in Simulation	0378-4754	A
Nonlinearity	0951-7715	A
OMEGA - International Journal of Management Science	0305-0483	A
Operations Research	0030-364X	A
Operations Research Letters	0167-6377	A
Project Management Journal	8756-9728	A
Quarterly of Applied Mathematics	0033-569X	A
Queueing Systems	0257-0130	A
Siam Journal on Applied Dynamical Systems	1536-0040	A
SIAM Journal on Control and Optimization	0363-0129	A
SIAM Journal on Matrix Analysis and Applications	0895-4798	A
Stochastic Processes and their Applications	0304-4149	A
Structural and Multidisciplinary Optimization	1615-147X	A
Studies in Applied Mathematics	0022-2526	A
Supply Chain Management - An International Journal	1359-8546	A
Technological Forecasting and Social Change	0040-1625	A
Transportation	0049-4488	A
Transportation Research Part C-Emerging Technologies	0968-090X	A
Transportation Research Part D-Transport and Environment	1361-9209	A
Transportation Research Part E-Logistics and Transportation Review	1366-5545	A
Transportation Science	0041-1655	A
Advances in Nonlinear Variational Inequalities		B
Advances in Theoretical and Applied Mathematics		B
Annals of Combinatorics	0218-0006	B
ANZIAM Journal	1446-1811	B
Applied Mathematics Letters	0893-9659	B
Applied Numerical Mathematics	0168-9274	B
Applied Stochastic Models In Business And Industry	1524-1904	B
Asia-Pacific Journal of Operational Research	0217-5959	B
Canadian Journal of Administrative Sciences	0825-0383	B
Computational Management Science	1619-697X	B
Computational Optimization and Applications	0926-6003	B
Computers & Industrial Engineering	0360-8352	B
Computers & Operations Research	0305-0548	B
Discrete Mathematics and Theoretical Computer Science	1365-8050	B
Engineering Economics		B
Experimental Mathematics	1058-6458	B
Global Journal of Pure and Applied Mathematics		B
IEEE Transactions on Reliability	0018-9529	B
IEEE Transactions on Systems Man and Cybernetics Part C	1094-6977	B
Interface	0092-2102	B
International Journal of Game Theory	0020-7276	B
International Journal of Logistics	1367-5567	B
International Journal of Logistics Management	0957-4093	B

International Journal of Production Research	0020-7543	B
International Journal of Project Management	0263-7863	B
International Journal of Pure and Applied Mathematics		B
International Journal of Quality and Reliability Management	0263-671X	B
Journal of Business Logistics	0735-3766	B
Journal of Graph Theory	0364-9024	B
Journal of Information and Optimization Sciences	0252-2667	B
Journal of Multi-Criteria Decision Analysis	1057-9214	B
Journal of Nonlinear and Convex Analysis		B
Journal of Statistical Computation and Simulation	0094-9655	B
Journal of Supply Chain Management	1523-2409	B
Journal of Time Series Analysis	0143-9782	B
Managerial and Decision Economics	0143-6570	B
Mathematical and Computer Modelling	0895-7177	B
Naval Research Logistics	0894-069X	B
Networks	0028-3045	B
Numerical Functional Analysis and Optimization	0163-0563	B
Numerical Linear Algebra with Applications	1070-5325	B
Optimization	0233-1934	B
OR Spectrum	0171-6468	B
Production and Inventory Management Journal	0897-8336	B
Production and Operations Management	1059-1478	B
Production Planning & Control	0953-7287	B
Pure and Applied Mathematics Quarterly		B
Statistical Modelling	1471-082X	B
Stochastic Analysis and Applications	0736-2994	B
Stochastic Environmental Research and Risk Assessment	1436-3240	B
Stochastic Models	1532-6349	B
Stochastics and Dynamics	0219-4937	B
Transportation Planning and Technology	0308-1060	B
Transportation Research Record	0361-1981	B
ACM Transactions on Modeling and Computer Simulation	1049-3301	C
Advances in Fuzzy Mathematics		C
American Journal of Mathematical and Management Sciences	0196-6324	C
Applied Mathematical Finance	1350-486X	C
Applied Mathematics and Optimization	0095-4616	C
Applied Mathematics E-Notes		C
Applied Mathematics Research eXpress		C
Australasian Journal of Combinatorics		C
Australian and New Zealand Industrial and Applied Mathematics		C
Australian Journal of Mathematical Analysis and Applications		C
Australian Mathematical Society Journal		C
Austrian Journal of Statistics		C
Communications in Applied Mathematics and Computational Science		C
Communications in Nonlinear Science and Numerical Simulation	1007-5704	C
Communications in Statistics-Stochastic Models		C
Communications on Applied Nonlinear Analysis		C
Complex Systems		C

Computational and Applied Mathematics	0101-8205	C
Computational Methods in Applied Mathematics		C
Computers and Mathematics		C
European Journal of Industrial Engineering		C
Global Journal of Mathematical Sciences		C
Global Journal of Mathematics and Mathematical Sciences (GJMMS)		C
I A E N G International Journal of Applied Mathematics		C
IMA Journal of Management Mathematics		C
IMA Journal of Mathematics Applied in Business and Industry	0953-0061	C
Indian Journal of Pure & Applied Mathematics	0019-5588	C
Industrial Mathematics	0019-8528	C
INFOR	0315-5986	C
International Journal of Applied Mathematics		C
International Journal of Applied Mathematics & Statistics		C
International Journal of Applied Mathematics and Computer Sciences		C
International Journal of Applied Mathematics and Computing		C
International Journal of Applied Nonlinear Science		C
International Journal of Applied Quality Management	1742-2647	C
International Journal of Computer Mathematics	0020-7160	C
International Journal of Industrial and Systems Engineering		C
Int. J. of Industrial Engineering -Theory Applications and Practice	1072-4761	C
Int. Journal of Information and Operations Management Education		C
Int. Journal of Information Systems and Supply Chain Management		C
International Journal of Information Technology & Decision Making	0219-6220	C
International Journal of Logistics Economics and Globalisation		C
International Journal of Management and Decision Making	1462-4621	C
International Journal of Mathematical and Statistical Sciences		C
International Journal of Mathematics and Computer Science		C
International Journal of Mathematics and Mathematical Sciences	0161-1712	C
International Journal of Mathematics Sciences		C
International Journal of Numerical Analysis and Modeling		C
International Journal of Operations & Quantitative Management	1082-1910	C
International Journal of Services and Operations Management	1744-2370	C
International Journal of Systems Science	0020-7721	C
International Transactions in Operational Research	0969-6016	C
Japan Journal of Industrial and Applied Mathematics	0916-7005	C
Journal of Applied Mathematics	1110-757X	C
Journal of Applied Mathematics and Computing	1598-5865	C
Journal of Applied Mathematics and Decision Sciences	1173-9126	C
Journal of Applied Mathematics and Stochastic Analysis	1048-9533	C
Journal of Combinatorial Mathematics and Combinatorial Computing		C
Journal of Computational Mathematics	0254-9409	C
Journal of Computational Mathematics and Optimization		C
Journal of Decision Systems		C
Journal of Discrete Algorithms	1570-8667	C
Journal of Industrial and Management Optimization		C
Journal of Numerical Mathematics	1570-2820	C

Forthcoming Conferences

Smart Decision Making for Clean Skies (Modern Air Traffic Management and the Environment)

Date: 2-3rd of July, 2008; Organisers: UNSW@ADFA and Air Services Australia

More details and information on how to register can be found at

http://www.unsw.adfa.edu.au/dsarc/conferences/atm_conference.html

Operations Research in Australia: The Experts Speak

Date: 7-8 July, 2008; Organisers: UNSW@ADFA and DSTO

More details and information on how to register can be found at

http://www.unsw.adfa.edu.au/dsarc/conferences/or_conference.html

5th Int. Conference on Service Systems and Service Management (ICSSSM'08)

30 June - 2 July 2008, Melbourne

<http://www.infotech.monash.edu.au/about/news/conferences/icsssm08/>

IFORS2008: International Federation of Operational Research Societies Conference

13-18 July 2008, Sandown, Sandton, Zambia

<http://www.acitravel.co.za/event/index.php?eventID=3>

The 9th Int. Symposium on Generalized Convexity and Generalized Monotonicity

July 21-25, 2008, National Sun Yat-sen University, Kaohsiung, Taiwan

<http://www.math.nsysu.edu.tw/gcm9>

The 7th Int. Conference on the Practice and Theory of Automated Timetabling

19th - 22nd August 2008, Montreal, Canada

<http://www.asap.cs.nott.ac.uk/patat/patat-index.shtml>

15th International Symposium on Inventories

August 22-26, 2008 - Sofitel Budapest, Hungary

<http://www.diamond-congress.hu/isir2008>

The XIV Latin-Ibero American Congress on Operations Research (CLAIO 2008)

9-12 September 2008, Cartagena de Indias, Colombia

www.socio.org.co/CLAIO2008/index_eng.php

2008 IEEE International Conference on Systems, Man, and Cybernetics

October 12-15, 2008, Suntec Singapore

<http://www.smc2008.org/>

3rd Int. Conference on Bioinspired Optimization Methods and their Applications

(BIOMA2008), 13 - 14 October 2008, Ljubljana, Slovenia

<http://bioma.ijs.si/conference/2008>

9th Asia-Pacific Industrial Eng. and Management Systems (APIEMS) Conference

Bali, Indonesia, 3 - 5 December 2008

<http://www.apiems2008.org>

18th World IMACS Congress and International Congress on Modelling and Simulation

(MODSIM09) 13-17th July 2009, Cairns, Australia

<http://www.mssanz.org.au/modsim09/>

The 20th National Conference of the Australian Society for Operations Research 2009

28-30 September 2009, Gold Coast, Australia

<http://www.asor.org.au/page.php?page=13>

**The 20th National Conference
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BULLETIN

Editorial Policy

The ASOR Bulletin is published in March, June, September and December by the Australian Society of Operations Research Incorporated.

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Contributions and suggestions are welcomed, however it should be noted that technical articles should be brief and relate to specific applications. Detailed mathematical developments should be omitted from the main body of articles but can be included as an Appendix to the article. Both refereed and non-refereed papers are published. The refereed papers are *peer reviewed* by at least two independent experts in the field and published under the section 'Refereed Paper'.

Articles must contain an abstract of not more than 100 words. The author's correct title, name, position, department, and preferred address must be supplied. References should be specified and numbered in alphabetical order as illustrated in the following examples:

[1] Higgins, J.C. and Finn, R. Managerial Attitudes Towards Computer Models for Planning and Control. Long Range Planning, Vol. 4, pp 107-112. (Dec. 1976).

[2] Simon, H.A. The New Science of Management Decision. Rev. Ed. Prentice-Hall, N.J. (1977).

Contributions should be prepared in MSWord (doc or rtf file), suitable for IBM Compatible PC, and a soft copy should be submitted either as an email attachment or on a 3.5" diskette. The detailed instructions for preparing/formatting your manuscript can be found in the web:
<http://www.cs.adfa.edu.au/~ruhul/asor.html>

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