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Operations Research is the application of the methods of science to complex problems arising in the direction and management of large systems of men, machines, materials and money in industry, business government and defence. The distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as chance and risk, with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management determine its policy and actions scientifically.

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Guest Editorial

The 20th National Conference of Australian Society for Operations Research incorporating the 5th International Intelligent Logistics System Conference was held on the Gold Coast, Australia, in September 2009. It is our honour, on behalf of the Australian Society for Operations Research to present these special post-conference issues, which provide a unique opportunity to maintain currency with Operations Research issues in Australia and other parts of the world. An encouraging feature of the papers is the breadth they cover in both theory and application. These special issues contain a range of papers dealing with different areas relating to the theme of the conference “Making the Future better by Operations Research”. The majority of them deal with application and analysis. Some of the papers are theoretical and discuss the techniques required to analyse real life applications. As a result, the topics covered in these papers highlight the diversity of the applications of Operations Research techniques.

We have received 29 papers from more than fifty authors. Each paper was reviewed by two or more peers. Although the review process has not yet completed for all the papers, we are delighted to publish the first five papers in this issue. The contributions in these papers cover a range of topics such as OR application in manufacturing system, scheduling for sugarcane transportation, performance measurement, defence knowledge management and inventory modeling. Among these five papers, two were contributed from Australia, one was from Japan, one from Singapore and one from India.

The editors of the special issues wish to express their appreciation to all authors for the contribution of their latest findings to Operations Research. We would also like to thank the reviewers for the involvement of the reviewing process in ensuring the maintenance of the highest scientific standards for these special issues.

The reader is reminded that the contents prepared by the author were electronically reproduced for publication. Therefore, the views and opinions are those of the authors. Anyone with questions about a paper should contact the authors.

Guest Editors
Erhan Kozan and Andy Wong

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An Inverse Laplace Transform Solution Method of Continuous Markov Models Used in Manufacturing Systems

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Abstract
Continuous Markov models are often used in the performance evaluation of manufacturing systems. The parametric method has been successfully employed to solve continuous Markov models. However, this solution method is relatively difficult to implement when modelling manufacturing systems with quality failure or multiple failure modes. Therefore, as a general methodology, the inverse Laplace transform method is proposed for the solution of continuous Markov models of two-machine lines, which will form the basis of decomposing complex manufacturing systems. Compared with the parametric method, the solution procedure using the inverse Laplace transform approach is independent of the number of system states in the model. Therefore, models with additional states describing quality or multiple failure modes can be solved using this method.

Key Words: Stochastic systems, Markov models, Manufacturing systems, Performance evaluation, Solution methods, Continuous material flow

1. Introduction

Manufacturing systems are affected by stochastic events such as machine failures, starvation and blockage. These stochastic events make it difficult to predict the system performance (Li and Meerkov, 2009). Markov processes have been successfully used to model systems with such stochastic events. In this paper, we focus on the Markov models that have been developed for the analysis of manufacturing systems with continuous flow of material, e.g. chemical plants and oil refining systems (Tan, 2001). Due to the continuous nature of material flow, these models are often referred to, in the manufacturing system literature, as continuous Markov models or simply continuous models (Dallery and Gershwin, 1992). We use these two terms interchangeably in this paper. These continuous Markov models may also be referred to as fluid flow Markov models or fluid queue models in the literature (Gribaudo *et al*., 2008; Akar and Sohraby, 2004). In addition to representing system with continuous material flow, continuous Markov models have also been used to approximate high volume discrete part manufacturing systems, e.g. integrated circuit factories (Li *et al*., 2006; Altiok, 1997; Gershwin, 1994).

Continuous Markov models of manufacturing systems were first proposed for the simple two-machine lines (Gershwin, 1994). These models permit the estimation of the system performance measures, viz., production rate, WIP(Work-in-process), yield, etc., of the two machine lines. As building exact models for long line systems may be mathematically intractable, the performance of long lines is usually evaluated through approximation methods such as decomposition. In the decomposition approach, the continuous two-machine line models are used as fundamental building blocks for the analysis of long lines (Gershwin, 1994; Dallery *et al.*, 1989; Colledani and Tolio, 2009).

Other continuous Markov models have also been proposed to study additional characteristics of manufacturing systems such as quality control, multiple failure modes of machines, split/merge material flow, etc. The parametric method has been used in solving various continuous Markov models (Gershwin, 1994; Kim, 2006; Le Bihan and Dallery, 2000; Levantesi *et al.*, 2003). In this method, an exponential form with several parameters is assumed for the probability density function of the system states. Subsequently, several parametric equations may be obtained by applying the probability density function to the transition equations of continuous Markov models. For the simple model with machines having two states, it is not difficult to find the roots of these equations and then determine the probability density function.

However, the parametric method is rather difficult to extend to complex continuous models that incorporate integrated quality/quantity control, machines with multiple failure modes and other characteristics. In these models, additional states are required for the description of quality or multiple failure modes of machines (Kim, 2006). The number of parameters required for the probability density function in exponential form is then greatly increased, and results in higher order parametric equations with complex root regions. To solve these equations, the root regions have to be examined, and special root finding algorithms have to be developed. The root finding algorithm is the algorithm to find the roots of all possible regions and cases of the higher order parametric equations (Kim, 2006). Thus, much mathematical effort is involved in deriving the solution of complex continuous Markov models using the parametric method. The complexity of this method depends on the number of states in the model, and the solution procedure grows increasingly complex as the number of states increase. For other complex manufacturing systems of assembly/disassembly or multiple-part type lines, even more machine states are required for the model. Therefore, an alternative to the parametric solution method is required for continuous Markov models of these complex systems.

Any solution method of continuous Markov models requires solving state transition equations to obtain the probability density function. These transition equations consist of differential equations, which are also often encountered in the modelling of dynamical systems. In dynamical systems, the differential equations or state equations are solved using the inverse Laplace transform of the state transition matrix and the initial conditions of the system. The inverse Laplace transform method is an operational method that can be used advantageously for solving linear differential equations (Ogata, 2002). It replaces differential equations with relatively easily solvable algebraic equations (Dorf and Bishop, 2005). Using the inverse Laplace transform approach, the complexity of the solution procedure does not depend on the number of system states. In addition, solving the state equations does not require any root finding algorithm.

Based on the similarities of continuous Markov models and dynamical systems, the authors propose using the inverse Laplace transform approach as an alternate solution method. In the continuous Markov models of manufacturing systems, the transition equations consisting of differential equations are categorised into internal equations, and upper and lower boundary equations. If a continuous Markov model is treated as a dynamical system, it may be solved using the following simple procedure: (1) find the state matrix from the internal transition equations; (2) obtain the system states (the probability density function) using the inverse Laplace transform of the state transition matrix and the lower boundary equations (analogous to the initial conditions of a dynamical system); (3) use the upper boundary equations to calculate the unknowns in the transition equations. Thus, the solution procedure of continuous Markov models will not grow in complexity as the number of system states increase.

The major contribution of this paper is that a general methodology, i.e., the inverse Laplace transform approach is proposed for solving the continuous Markov models used in manufacturing systems. The solution procedure follows a fixed three-step procedure and is applicable to models with different machine states. It therefore facilitates the modeling of complex manufacturing systems involving integrated quality/quantity control, machines with multiple failure modes and other phenomena. In addition, numerical experiments have demonstrated that the proposed method reduces the computational time for solving these complex models. This may also lead to shorter computational time for the performance evaluation of long line systems through analytical approximation techniques, such as decomposition, where the performance of a large number of two-machine lines is evaluated iteratively.

2. State of art: continuous Markov models and solutions

Manufacturing systems are often modelled analytically with continuous material flow, continuous time Markov models. A continuous Markov model was proposed for an unreliable two-machine and one-buffer (or 2M1B) manufacturing system, as presented in Fig. 1, (Gershwin, 1994).

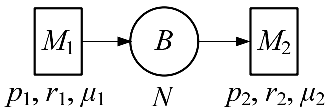

Fig. 1 A Two-Machine Line Manufacturing System

As shown in Fig.1, this system consists of two machines (M_1 and M_2) arranged serially and a buffer B separating the machines. The material is first processed in machine M_1 with a fixed processing rate μ_1. Subsequently, it enters the buffer and waits for the operation in M_2. The material leaves the system after being processed by M_2 with a fixed rate μ_2. The capacity of the buffer B, i.e., buffer size is N. When the amount of material in the buffer, i.e., buffer level x, reaches N, M_1 becomes blocked. Similarly, when the buffer level x is 0, M_2 is starved. In manufacturing systems, machines may also be subject to failures, e.g., tool breakdown, etc. Machines are thus modelled with two states, i.e., either up or down, as shown in Fig. 2 (Buzacott and Hanifin, 1978). When a machine is up and processing material, it may go to the down state with a failure rate p. When it is in the down state, it may be repaired and will transit back to the up state with a repair rate r.

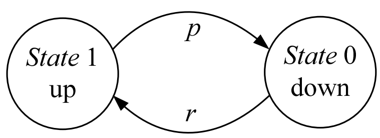

Fig. 2 An Unreliable Machine Model

This model captures the stochastic characteristics of manufacturing systems, viz., machine failures, blockage and starvation. It provides good results in performance evaluation of two-machine lines (Li *et al.*, 2006). It can be further extended to manufacturing systems of long lines through decomposition, where a long line is divided into a series of two-machine lines (Dallery *et al.*, 1989; Burman, 1995; Helber and Mehrtens, 2004; Li *et al.*, 2009).

Both continuous and discrete Markov models outperform simulation in the time it takes to calculate the performance measures, such as, production rate, WIP(work-in process), etc (Gershwin, 1994; Le Bihan and Dallery, 2000). This advantage is more pronounced in longer lines, and is useful for performance optimization (Nahas *et al.*, 2006). Compared with the flow of discrete parts in discrete Markov models, the flow of material is continuous in continuous Markov models (Altiok, 1997; Tan, 2001; Li and Meerkov 2009). This leads to a major difference between continuous and discrete Markov models of manufacturing systems. The difference is that the buffer level x is a real number in the range $[0, N]$ in continuous Markov models; while in discrete Markov models, the buffer level x is an integer number of $0,1,\ldots,N$ (Tolio *et al.*, 2002; Kim and Gershwin, 2005). The advantages of continuous Markov models over discrete Markov models were also analysed in the literature. One advantage is that continuous Markov models are applicable for the decomposition of both homogeneous and non-homogeneous lines (Dallery *et al.*, 1989, 1999). In addition, for the estimation of production rates of long lines, decomposition using continuous Markov models may be more accurate than using discrete Markov models (Le Bihan and Dallery, 2000; Levantesi *et al.*, 2003). Conversely, one possible disadvantage is that the mathematical effort required for solving the differential equations of continuous Markov models may be considerably more than solving the transition equations of discrete models (Kim, 2006).

Various continuous Markov models have also been developed to incorporate other phenomena of manufacturing systems. To evaluate the quality and quantity performance measures simultaneously, an integrated model with additional quality states is proposed for two-machine line manufacturing systems (Kim and Gershwin, 2005). Another two-machine model is proposed for machines with multiple failure modes, viz., machines may fail in different ways (Tolio *et al.*, 2002). Moreover, continuous Markov models are also used for systems with split/merge material flow (Tan, 2001; Helbe and Jusic, 2003).

The solution method for the continuous Markov model of a two-machine line manufacturing system (Fig. 1) (Gershwin, 1994), is described as the parametric method. As mentioned before, a machine has two states in this model. The system state is described as (x, α_1, α_2), where x represents the amount of material in the buffer B, α_1 and α_2 represent the states of machines M_1 and M_2 respectively. When $0 < x < N$, the system is in the internal states, and when $x = 0$ or N, the system is in the lower and upper boundary states respectively. According to this classification, the state transition equations are divided into internal, upper boundary and lower boundary equations.

The internal transition equations consist mainly of differential equations. We use the internal transition equation of state $(x, 0, 1)$ as an example to illustrate the derivation of these equations. In this model, the probability density function of state $(x, 0, 1)$ at time t is defined as $f(x, 0, 1, t)$. In a short time period δt, three states may transit to state $(x, 0, 1, t + \delta t)$:

- From state $(x + \mu_2 \delta t, 0, 1, t)$ with a probability $1 - (r_1 + p_2)\delta t$ (during the time period δt, M_1 is down and M_2 is up, thus the buffer level decreases from $x + \mu_2 \delta t$ to x).
- From state $(x + (\mu_2 - \mu_1)\delta t, 1, 1, t)$ with a probability $p_1 \delta t$ (during the time period δt, both M_1 and M_2 are up, thus the buffer level decreases from $x + (\mu_2 - \mu_1)\delta t$ to x).
- From state $(x, 0, 0, t)$ with a probability $r_2 \delta t$.

Thus, $f(x, 0, 1, t + \delta t)$ is calculated as:

$$f(x,0,1,t+\delta t)=\left[1-(r_1+p_2)\delta t\right]f(x+\mu_2\delta t,0,1,t)+p_1\delta t\, f(x+(\mu_2-\mu_1)\delta t,1,1,t)$$
$$+r_2\delta t\, f(x,0,0,t) \tag{1}$$

When $\delta t \to 0$, the above equation can be written in differential form (with t suppressed for simplicity) as:

$$\mu_2 \frac{\partial f}{\partial t}(x,0,1)=\mu_2 \frac{\partial f}{\partial x}(x,0,1)-(r_1+p_2)f(x,0,1)+p_1 f(x,1,1)+r_2 f(x,0,0) \tag{2}$$

In steady state, $\frac{\partial f}{\partial t}=0$, and hence,

$$\mu_2 \frac{df}{dx}(x,0,1)=(r_1+p_2)f(x,0,1)-p_1 f(x,1,1)-r_2 f(x,0,0) \tag{3}$$

Eqn (3) can be found in (Gershwin, 1994). The internal transition equations of the other states are also obtained similarly (Gershwin, 1994):

$$\begin{aligned}
\mu_1 \frac{df}{dx}(x,1,0) &= -(p_1+r_2)f(x,1,0)+p_2 f(x,1,1)+r_1 f(x,0,0)\\
(\mu_2-\mu_1)\frac{df}{dx}(x,1,1) &= -r_1 f(x,0,1)-r_2 f(x,1,0)+(p_1+p_2)f(x,1,1)\\
(r_1+r_2)f(x,0,0) &= p_2 f(x,0,1)+p_1 f(x,1,0)
\end{aligned} \tag{4}$$

Since the internal transition equations (3)-(4) consist mainly of first order linear differential equations, in the parametric method an exponential form solution may be assumed for the probability density function $f(x,\alpha_1,\alpha_2)$. For example, $f(x,\alpha_1,\alpha_2)$ for the case $\mu_1=\mu_2$ is described with an exponential form with four parameters (Gershwin, 1994):

$$f(x,\alpha_1,\alpha_2)=De^{\lambda x}Y_1^{\alpha_1}Y_2^{\alpha_2} \tag{5}$$

where D, λ, Y_1 and Y_2 are the parameters to be determined. For the case $\mu_1 \neq \mu_2$, even more parameters are required.

In the parametric method, the probability density function in Eqn (5) is then applied to the internal transition equations (3)-(4) of the model. Three second order equations are obtained with three unknowns:

$$\sum_{i=1}^{2}(p_i Y_i - r_i)=0 \tag{6}$$

$$-\mu_1\lambda=(p_1Y_1-r_1)\frac{1+Y_1}{Y_1} \tag{7}$$

$$-\mu_2\lambda=(p_2Y_2-r_2)\frac{1+Y_2}{Y_2} \tag{8}$$

Eqns (6)-(8) are the parametric equations. The root regions for the parametric equations are then examined. After solving these equations, the probability density function is applied to the normalization and boundary equations. All probability masses of boundary states and the probability density function are finally determined. The solutions of other continuous Markov models are developed based on the parametric method (Kim, 2006; Tolio *et al*., 2002; Le Bihan and Dallery, 2000).

The ease of use of the parametric method is rather limited only to the simple unreliable 2M1B model. For complex models involving phenomena such as, integrated quality/quantity control (Kim, 2006), machines with multiple failure modes (Tolio *et al*., 2002), machine repair rates with hyper-exponential distributions (Le Bihan and Dallery, 2000), the use of the parametric method is cumbersome. More machine states are used in the models to describe these complex phenomena. As a result, in the parametric method of solving these models, the number of parameters required for the probability density function is greatly increased. For example, in the integrated quality/quantity model, seven parameters are required for the probability density function (Kim, 2006), while the number of parameters required for the continuous Markov model with hyper-exponential machine repair rates is 15 (Le Bihan and Dallery, 2000). Even more parameters may be required for the models with multiple failure modes (Tolio *et al*., 2002). The use of the parametric method for these models may also lead to higher order parametric equations. For example, in the integrated quality/quantity model, nine parametric equations of higher orders are derived from the transition equations of the model (Kim, 2006). When solving this model using the parametric method, several additional steps are necessary to reduce the number of parametric equations to two. These two parametric equations are higher order polynomial equations. The root regions of these two equations are complex and an additional root finding algorithm has to be developed to determine the roots. Much mathematical effort is therefore involved in the solution of these complex models, and an alternative solution method is necessary to facilitate the modelling of complex manufacturing systems.

Solution methodologies have also been proposed for solving continuous Markov models used in telecommunication systems (Akar and Sohraby, 2004; Latouche and Takine, 2004; Serucola, 2001; Da Silva Soares and Latouche, 2009). A matrix exponential representation for the probability density function is usually obtained in these methodologies, rather than using an exponential form with a number of parameters in the parametric method. These methods are applicable to models with different numbers of states in telecommunication systems. However, these methods may not be used for models of manufacturing systems directly, since manufacturing systems may be subject to different types of machine failures which are not considered in the models for telecommunication systems.

In this paper, the authors propose the inverse Laplace transform method for a general methodology of continuous Markov models used in manufacturing systems. Inverse Laplace transforms are commonly used in the modelling of dynamical systems (Ogata, 2002; Dorf and Bishop, 2005). These systems are often described using differential equations, and are then re-expressed using state space models. The system states are then calculated using the inverse Laplace transform of the transition matrix and initial conditions of the system. No root finding algorithm is required to obtain the solution. Since continuous Markov models of manufacturing systems are also described by differential equations, the inverse Laplace transform method may be utilized for these models.

3. The Inverse Laplace transform solution method

The transition equations for continuous Markov models of manufacturing systems are also linear and first-order differential equations. Such differential equations are also used to describe linear dynamical systems. In linear systems, the differential equations are often written in terms of state equations, which are then solved using the inverse Laplace transform of the state transition matrix and the initial conditions of the system. Before attempting the inverse Laplace transform approach on continuous Markov models, we first illustrate this approach by solving the differential equations of a simple linear system. We will then extend this approach for the solution of continuous Markov models. The following notations will be used in the inverse

Laplace solution method for continuous Markov models:

M_1, M_2: Machine 1 and Machine 2 respectively.

α_1, α_2: The states of M_1 and M_2 respectively, where $\alpha_1, \alpha_2 \in \{0,1\}$ in the simple 2M1B model and $\alpha_1, \alpha_2 \in \{0,1,-1\}$ in the integrated quality/quantity model.

N: The buffer size, i.e., the maximum amount of material that can be held in the buffer.

x: The buffer level, i.e., the amount of material held in the buffer at a particular time t.

p_1, r_1, μ_1: The failure, repair, and processing rates of M_1 respectively.

p_2, r_2, μ_2: The failure, repair, and processing rates of M_2 respectively.

g_1, h_1: The quality failure and detection rates of M_1 in the integrated quality/quantity model, respectively.

g_2, h_2: The quality failure and detection rates of M_2 in the integrated quality/quantity model, respectively.

$f(x, \alpha_1, \alpha_2)$: The probability density function.

$P(0, \alpha_1, \alpha_2)$: The probability masses of the lower boundary states.

$P(N, \alpha_1, \alpha_2)$: The probability masses of the upper boundary states.

w_1, w_2: The number of states of M_1 and M_2 respectively, where $w_1 = 2$ and $w_2 = 2$ in the simple 2M1B model; $w_1 = 3$ and $w_2 = 3$ in the integrated quality/quantity model.

V: The total number of the two machine states (α_1, α_2), i.e., $V = w_1 w_2$.

$\phi_j(x)$: The simplified notation of the probability density function, $j = 1, 2, \ldots V$.

$\Psi_j(0), \Psi_j(N)$: The simplified notations of the probability masses of lower and upper boundary states $j = 1, 2, \ldots V$ respectively.

$\Phi_B(x)$: The vector of basic states.

v: The number of basic states.

3.1. Solution method of differential equations for linear dynamical systems

In this subsection, we present the fundamentals of the inverse Laplace transform approach. We shall first provide the definition of the Laplace transform and the inverse Laplace transform. We then describe the inverse Laplace transform method of solving a single differential equation and also a set of differential equations for linear systems. The equations (9)-(21) in this subsection can be found in (Dorf and Bishop, 2005; Ogata, 2002).

Suppose $x(t)$ is a function of time t such that $x(t) = 0$ for $t < 0$, the Laplace transform of $x(t)$ is given by

$$\mathcal{L}\left[x(t)\right] = X(s) = \int_0^\infty x(t) e^{-st} dt \tag{9}$$

where $\mathcal{L}$ is the operational symbol of the Laplace transform, s is a complex variable, and $X(s)$ is the Laplace transform of $x(t)$.

The inverse Laplace transform (with notation $\mathcal{L}^{-1}$) is the process of obtaining the time function $x(t)$ from the Laplace transform $X(s)$. Using Laplace transform, a differential equation may be replaced with an algebraic equation. Consider the differential equation,

$$\frac{dx(t)}{dt} - x(t) = 0 \tag{10}$$

where the initial condition is $x(0)$. Applying Laplace transform to the equation, we obtained the following equation:

$$sX(s) - x(0) - X(s) = 0 \tag{11}$$

This is a simple algebraic equation. Solving for $X(s)$, we have

$$X(s) = \frac{1}{s-1} x(0) \tag{12}$$

Using inverse Laplace transform, the solution is:

$$x(t) = \mathbf{L}^{-1}[X(s)] = e^t x(0) \tag{13}$$

Next, we present the inverse Laplace transform method for solving differential equations of a simple linear system. The system may be described using a set of first order differential equations of states as follows:

$$\begin{aligned} \dot{x}_1(t) &= a_{11}x_1(t) + a_{12}x_2(t) + \ldots + a_{1n}x_n(t) \\ &\vdots \\ \dot{x}_n(t) &= a_{n1}x_1(t) + a_{n2}x_2(t) + \ldots + a_{nn}x_n(t) \end{aligned} \tag{14}$$

where $x_i(t)$ is the i^{th} state of the system, and $\dot{x}_i(t) = \frac{dx_i(t)}{dt}$, $i = 1, 2, \ldots n$, and n is the number of system states. This set of differential equations may be written in matrix form as follows:

$$\begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} \tag{15}$$

A column vector $X(t)$ consisting of the states is referred to as the state vector and is written as:

$$X(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} \tag{16}$$

Then the system can be represented by the compact notation of the state differential equation as:

$$\dot{X}(t) = AX(t) \tag{17}$$

where A is the state matrix of $n \times n$ dimension.

For this system, if the initial condition is $X(t_0)$ at time $t = t_0$, then $X(t)$ may be calculated for all $t > t_0$ as:

$$X(t) = e^{A(t-t_0)} X(t_0) \tag{18}$$

where $e^{A(t-t_0)}$ is the state transition matrix.

When $t_0 = 0$, the Laplace transform of e^{At} is $(sI - A)^{-1}$, then,

$$e^{At} = \mathbf{L}^{-1}\left[(sI - A)^{-1}\right] \tag{19}$$

where $\mathbf{L}^{-1}\left[(sI - A)^{-1}\right]$ can be calculated as

$$\mathbf{L}^{-1}\left[(sI - A)^{-1}\right] = I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \ldots. \tag{20}$$

Thus, the solution for this system is found to be:

$$X(t) = e^{At}X(0) = \mathcal{L}^{-1}\left[(sI - A)^{-1}\right]X(0) \tag{21}$$

3.2. An inverse Laplace transform solution method for the 2M1B model

As mentioned in Section 2, in continuous Markov models differential equations may also be used to describe state transitions. In the 2M1B model, the transition equations for states $(x,0,1)$, $(x,1,0)$ and $(x,1,1)$ are differential equations, as indicated in Eqns (3) and (4). These three states are defined as basic states. For state $(x,0,0)$, $f(x,0,0)$ can be expressed by the probability density function of the basic states. Therefore, $(x,0,0)$ is defined as a non-basic state. The transition equations for the basic states are of typical differential equations as described in Section 3.1. The inverse Laplace transform method may be used to solve these equations.

Table 1 Notation Simplification for the Continuous 2M1B Markov Model

Notation	Simplified	Notation	Simplified	Notation	Simplified
$f(x,0,1)$	$\phi_1(x)$	$P(0,0,1)$	$\Psi_1(0)$	$P(N,0,1)$	$\Psi_1(N)$
$f(x,1,0)$	$\phi_2(x)$	$P(0,1,0)$	$\Psi_2(0)$	$P(N,1,0)$	$\Psi_2(N)$
$f(x,1,1)$	$\phi_3(x)$	$P(0,1,1)$	$\Psi_3(0)$	$P(N,1,1)$	$\Psi_3(N)$
$f(x,0,0)$	$\phi_4(x)$	$P(0,0,0)$	$\Psi_4(0)$	$P(N,0,0)$	$\Psi_4(N)$

We simplified the probability density function $f(x,\alpha_1,\alpha_2)$, the probability masses $P(0,\alpha_1,\alpha_2)$ (lower boundary) and $P(N,\alpha_1,\alpha_2)$ (upper boundary) with the notations in Table 1. Then the internal transition equations (3) and (4) are rewritten in matrix form as:

$$\begin{bmatrix} \dot{\phi}_1(x) \\ \dot{\phi}_2(x) \\ \dot{\phi}_3(x) \end{bmatrix} = \begin{bmatrix} \frac{r_1+p_2}{\mu_2} & 0 & \frac{-p_1}{\mu_2} \\ 0 & \frac{-(p_1+r_2)}{\mu_1} & \frac{p_2}{\mu_1} \\ \frac{-r_1}{\mu_2-\mu_1} & \frac{-r_2}{\mu_2-\mu_1} & \frac{p_1+p_2}{\mu_2-\mu_1} \end{bmatrix} \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \\ \phi_3(x) \end{bmatrix} + \begin{bmatrix} \frac{-r_2}{\mu_2} \\ \frac{r_1}{\mu_1} \\ 0 \end{bmatrix} \phi_4$$

$$\phi_4(x) = \begin{bmatrix} \frac{p_2}{r_1+r_2} & \frac{p_1}{r_1+r_2} & 0 \end{bmatrix} \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \\ \phi_3(x) \end{bmatrix} \tag{22}$$

If we define $A_1 = \begin{bmatrix} \frac{r_1+p_2}{\mu_2} & 0 & \frac{-p_1}{\mu_2} \\ 0 & \frac{-(p_1+r_2)}{\mu_1} & \frac{p_2}{\mu_1} \\ \frac{-r_1}{\mu_2-\mu_1} & \frac{-r_2}{\mu_2-\mu_1} & \frac{p_1+p_2}{\mu_2-\mu_1} \end{bmatrix}$, $A_2 = \begin{bmatrix} \frac{-r_2}{\mu_2} \\ \frac{r_1}{\mu_1} \\ 0 \end{bmatrix}$ and $A_3 = \begin{bmatrix} \frac{p_2}{r_1+r_2} & \frac{p_1}{r_1+r_2} & 0 \end{bmatrix}$, and define $\Phi_B(x) = \left[\phi_1(x)\ \phi_2(x)\ \phi_3(x)\right]^T$ as the state vector. Then Eqn (22) may be written as:

$$\dot{\Phi}_B(x) = A\Phi_B(x) \tag{23}$$

where the state matrix A is calculated as:

$$A = A_1 + A_2 A_3 \tag{24}$$

And

$$\phi_4(x) = A_3 \Phi_B(x) \tag{25}$$

Since the boundary transition equations may vary based on the processing rates of the machines, we will present the solution using the inverse Laplace transform method for three conditions:

1. $\mu_2 > \mu_1$
2. $\mu_2 < \mu_1$
3. $\mu_2 = \mu_1 = \mu$

Case 1*:* $\mu_2 > \mu_1$

The boundary transition equations are derived similarly with the internal transition equation as formulated in Eqn (3). These equations can be found in (Gershwin, 1994). For Case 1, the lower boundary equations ($x = 0$) are:

$$\begin{aligned} P(0,1,0) &= P(0,0,0) = 0 \\ \mu_2 f(0,0,1) &= r_1 P(0,0,1) - p_1 P(0,1,1) \\ \mu_1 f(0,1,0) &= p_2 \, \mu_1/\mu_2 \, P(0,1,1) \\ (\mu_2 - \mu_1) f(0,1,1) &= -r_1 P(0,0,1) + (p_1 + p_2 \, \mu_1/\mu_2) P(0,1,1) \end{aligned} \tag{26}$$

After simplifying with notations in Table 1, we obtain:

$$\begin{aligned} \Psi_2(0) &= \Psi_4(0) = 0 \\ \mu_2 \phi_1(0) &= r_1 \Psi_1(0) - p_1 \Psi_3(0) \\ \mu_1 \phi_2(0) &= p_2 \, \mu_1/\mu_2 \, \Psi_3(0) \\ (\mu_2 - \mu_1)\phi_3(0) &= -r_1 \Psi_1(0) + (p_1 + p_2 \, \mu_1/\mu_2) \Psi_3(0) \end{aligned} \tag{27}$$

Then, the lower boundary state vector $\Phi_B(0)$ can be found as:

$$\Phi_B(0) = \begin{bmatrix} \phi_1(0) \\ \phi_2(0) \\ \phi_3(0) \end{bmatrix} = \begin{bmatrix} \dfrac{r_1}{\mu_2} & \dfrac{-p_1}{\mu_2} \\ 0 & \dfrac{p_2}{\mu_1} \\ \dfrac{r_1}{\mu_2 - \mu_1} & \dfrac{p_1 \mu_2 + p_2 \mu_1}{(\mu_2 - \mu_1)\mu_2} \end{bmatrix} \begin{bmatrix} \Psi_1(0) \\ \Psi_3(0) \end{bmatrix} \tag{28}$$

Using the inverse Laplace transform as described in Eqn (21), $\Phi_B(x)$ can be calculated as:

$$\Phi_B(x) = \boldsymbol{L}_x^{-1}\left[(sI - A)^{-1}\right]\Phi_B(0) \tag{29}$$

Instead of a function of time t, $\Phi_B(x)$ is expressed by the inverse Laplace transform of a function in buffer level x. Similarly, the upper boundary state vector $\Phi_B(N)$ may be calculated as:

$$\Phi_B(N) = \mathbf{L}_{x=N}^{-1}\left[(sI-A)^{-1}\right]\Phi_B(0) \tag{30}$$

The upper boundary equations ($x = N$) of the 2M1B model are also found in (Gershwin, 1994):

$$\begin{aligned} P(N,0,0) = P(N,0,1) = P(N,1,1) = 0 \\ f(N,0,1) = 0 \\ \mu_1 f(N,1,0) = r_2 P(N,1,0) \\ (\mu_2 - \mu_1) f(N,1,1) = r_2 P(N,1,0) \end{aligned} \tag{31}$$

Using the notations in Table 1, we can simplify Eqn (31) as:

$$\begin{aligned} \Psi_1(N) = \Psi_3(N) = \Psi_4(N) = 0 \\ \phi_1(N) = 0 \\ \mu_1\phi_2(N) = r_2\Psi_2(N) \\ (\mu_2 - \mu_1)\phi_3(N) = r_2\Psi_2(N) \end{aligned} \tag{32}$$

The above equation can be rewritten using a matrix form in Eqn (33). Since the upper boundary states can be also calculated using Eqn (30), we have,

$$\Phi_B(N) = \begin{bmatrix} 0 \\ r_2/\mu_1 \\ r_2/(\mu_2-\mu_1) \end{bmatrix}\left[\Psi_2(N)\right] = \mathbf{L}_{x=N}^{-1}\left[(sI-A)^{-1}\right]\Phi_B(0) \tag{33}$$

The summation of the integral of the probability density function and all probability masses of boundary states is one. Thus, the normalization equation is,

$$\begin{aligned} 1 &= \int_0^N \left\{\left[\phi_1(x)+\phi_2(x)+\phi_3(x)\right]+\phi_4(x)\right\}dx + \sum_{i=0}^{4}\left[\Psi_i(0)+\Psi_i(N)\right] \\ &= \int_0^N \left\{\left[1 \quad 1 \quad 1\right]\Phi_B(x) + A_3\Phi_B(x)\right\}dx + \sum_{i=0}^{4}\left[\Psi_i(0)+\Psi_i(N)\right] \\ &= \int_0^N C\Phi_B(x)dx + \sum_{i=0}^{4}\left[\Psi_i(0)+\Psi_i(N)\right] \end{aligned} \tag{34}$$

where $C = \left[1 \quad 1 \quad 1\right] + A_3$ is the state summation matrix.

There are three unknowns namely, $\Psi_1(0), \Psi_3(0)$, and $\Psi_2(N)$ in the boundary equations of Eqns (28) and (33). In Eqn (33), three equations can be obtained. Using any two equations of Eqn (33) and the normalization equation (34), these three unknowns can be calculated, from which, the probability density function of all states can be calculated using Eqns (29) and (25).

When all probability masses and the probability density function are determined, all performance measures of interest can be calculated. In a production line, the performance measures of interest may be WIP (or average buffer level), production rates, etc. For example, the average buffer level is the amount of material in the buffer in the long run, and is calculated as:

$$\bar{x} = \int_0^N C\Phi_B(x)x dx + \sum_{i=0}^{4}\Psi_i(N)N \tag{35}$$

Case 2: $\mu_2 < \mu_1$

For Case 2, the solution follows the same procedure as with Case 1 previously.

Case 3: $\mu_2=\mu_1=\mu$

Since $\mu_2 = \mu_1 = \mu$, $\mu_2 - \mu_1$ is zero. The internal transition equations (3) and (4) become:

$$\begin{aligned} \dot{\phi}_1(x) &= \frac{r_1 + p_2}{\mu}\phi_1(x) + \begin{bmatrix} 0 & \frac{-p_1}{\mu} & \frac{-r_2}{\mu} \end{bmatrix} \begin{bmatrix} \phi_2(x) \\ \phi_3(x) \\ \phi_4(x) \end{bmatrix} \\ \begin{bmatrix} \phi_2(x) \\ \phi_3(x) \\ \phi_4(x) \end{bmatrix} &= \begin{bmatrix} 1 & \frac{r_1 + r_2}{p_1 + p_2} & \frac{p_1 + p_2}{r_1 + r_2} \end{bmatrix}^T \phi_1(x) \end{aligned} \tag{36}$$

States $\phi_2(x)$, $\phi_3(x)$ and $\phi_4(x)$ may be expressed in terms of state $\phi_1(x)$. We refer to state $\phi_1(x)$ as the basic state. States $\phi_2(x)$, $\phi_3(x)$ and $\phi_4(x)$ are referred to as the non-basic states. As described in Eqn (22), similarly, we may define:

$$A_1 = \frac{r_1 + p_2}{\mu}, A_2 = \begin{bmatrix} 0 & \frac{-p_1}{\mu} & \frac{-r_2}{\mu} \end{bmatrix}, A_3 = \begin{bmatrix} 1 & \frac{r_1 + r_2}{p_1 + p_2} & \frac{p_1 + p_2}{r_1 + r_2} \end{bmatrix}^T \tag{37}$$

The dimension of state matrix $A = A_1 + A_2 A_3$ is reduced to 1×1. The solution of the probability density function then follows the same procedure as with Case 1.

3.3. A general inverse Laplace transform solution method

In this subsection, we propose the use of the inverse Laplace transform as a general solution method for continuous two-machine line models. As described in Section 3.2, the solution procedure can be summarized into the following three steps:

1. Derive the state matrix from the internal transition equations.
2. Obtain the probability density function using the lower boundary equations.
3. Solve the upper boundary and normalization equations for the unknowns.

Let (x,α_1,α_2) denote the state of the two-machine line system. Suppose machine M_1 has w_1 states and machine M_2 has w_2 states, then the total number of the two machine state (α_1,α_2) is $V = w_1 w_2$. The probability density function $f(x,\alpha_1,\alpha_2)$, the boundary probability masses $P(0,\alpha_1,\alpha_2)$ and $P(N,\alpha_1,\alpha_2)$ are simplified as $\phi_j(x)$, $\Psi_j(0)$ and $\Psi_j(N)$, $j = 1,2,\ldots,V$.

Step 1. *Derive the state matrix from the internal transition equations*

The internal transition equations of the model may then be rewritten in the following form:

$$\begin{aligned} \begin{bmatrix} \dot{\phi}_1(x) \\ \vdots \\ \dot{\phi}_v(x) \end{bmatrix} &= \begin{bmatrix} a_{1,1} & \cdots & a_{1,v} \\ \vdots & \ddots & \vdots \\ a_{v,1} & \cdots & a_{v,v} \end{bmatrix} \begin{bmatrix} \phi_1(x) \\ \vdots \\ \phi_v(x) \end{bmatrix} + \begin{bmatrix} a_{1,v+1} & \cdots & a_{1,V} \\ \vdots & \ddots & \vdots \\ a_{v,v+1} & \cdots & a_{v,V} \end{bmatrix} \begin{bmatrix} \phi_{v+1}(x) \\ \vdots \\ \phi_V(x) \end{bmatrix} \\ \begin{bmatrix} \phi_{v+1}(x) \\ \vdots \\ \phi_V(x) \end{bmatrix} &= \begin{bmatrix} a_{v+1,1} & \cdots & a_{v+1,v} \\ \vdots & \ddots & \vdots \\ a_{V,1} & \cdots & a_{V,v} \end{bmatrix} \begin{bmatrix} \phi_1(x) \\ \vdots \\ \phi_v(x) \end{bmatrix} \end{aligned} \tag{38}$$

where $\dot{\phi}_i(x) = \dfrac{d\phi_i(x)}{dx}$, $i = 1, 2, \ldots, v$, these v states are the basic states. Since $\phi_{v+1}, \ldots, \phi_V(x)$ can be expressed in terms of the basic states, these $V - v$ states are referred as the non-basic states. We may also define the following matrices:

$$A_1 = \begin{bmatrix} a_{1,1} & \cdots & a_{1,v} \\ \vdots & \ddots & \vdots \\ a_{v,1} & \cdots & a_{v,v} \end{bmatrix}, A_2 = \begin{bmatrix} a_{1,v+1} & \cdots & a_{1,V} \\ \vdots & \ddots & \vdots \\ a_{v,v+1} & \cdots & a_{v,V} \end{bmatrix}, A_3 = \begin{bmatrix} a_{v+1,1} & \cdots & a_{v+1,v} \\ \vdots & \ddots & \vdots \\ a_{V,1} & \cdots & a_{V,v} \end{bmatrix} \tag{39}$$

We next choose the state vector $\Phi_B(x)$ to be composed of the basic states $[\phi_1(x) \ \ldots \ \phi_v(x)]^T$. The internal transition equations of the basic states can then be expressed in matrix form as:

$$\dot{\Phi}_B(x) = A\Phi_B(x) \tag{40}$$

where the state matrix $A = A_1 + A_2 A_3$

Step 2. *Obtain the probability density function using the lower boundary equations*

The lower boundary equations of a continuous Markov model are the transition equations for states when the buffer is empty ($x = 0$). Using these equations, the state vector $\Phi_B(0)$ consisting of $[\phi_1(0) \ \ldots \ \phi_v(0)]^T$ can be obtained in general matrix form as:

$$\Phi_B(0) = \begin{bmatrix} \phi_1(0) \\ \vdots \\ \phi_v(0) \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1V} \\ \vdots & \ddots & \vdots \\ \gamma_{v1} & \cdots & \gamma_{vV} \end{bmatrix} \begin{bmatrix} \Psi_1(0) \\ \vdots \\ \Psi_V(0) \end{bmatrix} \tag{41}$$

where $\Psi_1(0), \ldots, \Psi_V(0)$ are the lower boundary probability masses, and γ_{ij}, $i = 1, 2, \ldots v$, $j = 1, 2, \ldots V$ are determined by the lower boundary equations.

As before,

$$\Phi_B(x) = \boldsymbol{L}_x^{-1}\left[(sI - A)^{-1}\right]\Phi_B(0) \tag{42}$$

The upper boundary state vector $\Phi_B(N)$ can be calculated as:

$$\Phi_B(N) = \boldsymbol{L}_{x=N}^{-1}\left[(sI - A)^{-1}\right]\Phi_B(0) \tag{43}$$

Step 3. *Solve the upper boundary and normalization equations for the unknowns*

The upper boundary state vector $\Phi_B(N)$ can also be obtained from the upper boundary transition equations (when the buffer is full $x = N$) as:

$$\Phi_B(N) = \begin{bmatrix} \phi_1(N) \\ \vdots \\ \phi_v(N) \end{bmatrix} = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1V} \\ \vdots & \ddots & \vdots \\ \beta_{v1} & \cdots & \beta_{vV} \end{bmatrix} \begin{bmatrix} \Psi_1(N) \\ \vdots \\ \Psi_V(N) \end{bmatrix} \tag{44}$$

where $\Psi_1(N), \ldots, \Psi_V(N)$ are the upper boundary probability masses, and β_{ij}, $i = 1, 2, \ldots v$, $j = 1, 2, \ldots V$ are obtained from the upper boundary equations.

Using Eqns (43) and (44), we can obtain v equations for the basic states:

$$\boldsymbol{L}_{x=N}^{-1}\left[(sI-A)^{-1}\right]\Phi_B(0)=\begin{bmatrix}\beta_{11} & \cdots & \beta_{1V}\\ \vdots & \ddots & \vdots\\ \beta_{v1} & \cdots & \beta_{vV}\end{bmatrix}\begin{bmatrix}\Psi_1(N)\\ \vdots\\ \Psi_V(N)\end{bmatrix} \tag{45}$$

The normalization equation is then expressed as follows:

$$\begin{aligned}1&=\int_0^N\sum_{i=1}^{v}\phi_i(x)+\sum_{i=v+1}^{V}\phi_i(x)dx+\sum_{i=0}^{V}\left[\Psi_i(0)+\Psi_i(N)\right]\\ &=\int_0^N\left\{\begin{bmatrix}1 & \dots & 1\end{bmatrix}_{1\times v}\Phi_B(x)+\begin{bmatrix}1 & \dots & 1\end{bmatrix}_{1\times(V-v)}A_3\Phi_B(x)\right\}dx+\sum_{i=0}^{V}\left[\Psi_i(0)+\Psi_i(N)\right]\\ &=\int_0^N C\Phi_B(x)dx+\sum_{i=0}^{V}\left[\Psi_i(0)+\Psi_i(N)\right]\end{aligned} \tag{46}$$

where the state summation matrix $C=\begin{bmatrix}1 & \dots & 1\end{bmatrix}_{1\times v}+\begin{bmatrix}1 & \dots & 1\end{bmatrix}_{1\times(V-v)}A_3$.

Use any $v-1$ equations of Eqn (45) and the normalization equation (46) to calculate the unknowns in the boundary probability masses. Subsequently, the probability density function of all states can be calculated using Eqns (42) and (38). Finally, the performance measures, i.e., production rate, average buffer level, etc., are then calculated.

In this method, the formulae are further explained as follows:

- In Step 1, in Eqn (38), the basic states are identified, and the non-basic states are expressed in terms of the basic states. The state matrix is then obtained in Eqn (39).
- In Step 2, based on the lower boundary equation (41), the lower boundary probability density function is calculated. Then, the probability density function is determined in terms of the state matrix and the lower boundary probability density function, as described in Eqn (42).
- In Step 3, solve the upper boundary and normalization equations for the unknowns of the probability masses, as described in Eqns (45) and (46).

4. An application: solving the integrated quality/quantity model

The parametric method is difficult to extend to complex continuous Markov models involving phenomena such as integrated quality/quantity control, machines with multiple failure modes, etc. In these models, there are additional machine states. For example, in the integrated quality/quantity model, a machine is subject to both operational and quality failures, and an additional state is used to describe the quality status of the machine (Kim, 2006). The state transitions of this model are described in Fig. 3, and the machine states are defined as follows:

- State 1: The machine is operating and producing good parts.
- State -1: The machine is operating and producing bad quality parts, but the operator does not know this yet. This is the quality failure state.
- State 0: The machine is not operational.

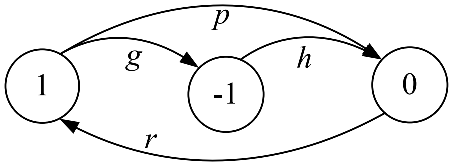

Fig. 3 A Three-State Machine Model

In Fig. 3, p and r are the operational failure and repair rates respectively, while g and h are the quality failure and detection rates respectively. It has also been assumed (Kim, 2006) that once the quality failure is detected the state of the machine transits to state 0.

This model is used to estimate performance measures such as total production rate, effective production rate, average buffer level, etc. It is also extended to the performance evaluation of long lines with quality failures through decomposition (Kim and Gershwin, 2008).

In pages 49-80 of (Kim, 2006), the parametric method is presented for solving this integrated quality/quantity model. However, the additional machine state for quality leads to nine higher order parametric equations. As a result, compared with the parametric method for the simple 2M1B model as discussed in Section 2, several extra solution procedures are required for solving this integrated quality/quantity model. Several steps are necessary to reduce the number of parametric equations to two. These two parametric equations are of fourth order. To find the roots of these two equations, the root regions have to be examined. There are four root regions, and each region may have 16 different cases depending on the machine parameters. For each case of one region, one specific root finding algorithm is developed. It is laborious to develop specific root finding algorithms for all roots of the parametric equations. Much mathematical effort is involved in obtaining the solution using the parametric method. Therefore, the parametric method is difficult to be generalized for solving these complex continuous Markov models.

On the other hand, the inverse Laplace transform approach (proposed in this paper) provides a much simpler procedure for solving these complex models. By following the three-step procedure as formulated in Section 3.3, we have solved several models found in the literature. We use the integrated quality/quantity model as an example to illustrate the application of this approach on these complex continuous Markov models.

4.1. Solving the Integrated Quality/Quantity Model

For brevity, we only discuss the case when $\mu_2 = \mu_1 = \mu$. The internal transition equations of the model (Kim, 2006) are:

$$
\begin{aligned}
\mu \frac{df}{dx}(x,0,1) &= (r_1 + p_2 + g_2) f(x,0,1) - r_2 f(x,0,0) - p_1 f(x,1,1) - h_1 f(x,-1,1) \\
\mu \frac{df}{dx}(x,0,-1) &= -g_2 f(x,0,1) + (r_1 + h_2) f(x,0,-1) - p_1 f(x,1,-1) - h_1 f(x,-1,-1) \\
\mu \frac{df}{dx}(x,1,0) &= -(p_1 + g_1 + r_2) f(x,1,0) + r_1 f(x,0,0) + p_2 f(x,1,1) + h_2 f(x,1,-1) \\
\mu \frac{df}{dx}(x,-1,0) &= g_1 f(x,1,0) - (r_2 + h_1) f(x,-1,0) + p_2 f(x,-1,1) + h_2 f(x,-1,-1) \\
(r_1 + r_2) f(x,0,0) &= p_2 f(x,0,1) + h_2 f(x,0,-1) + p_1 f(x,1,0) + h_1 f(x,-1,0) \qquad (47) \\
(p_1 + g_1 + p_2 + g_2) f(x,1,1) &= r_1 f(x,0,1) + r_2 f(x,1,0) \\
(p_1 + g_1 + h_2) f(x,1,-1) &= r_1 f(x,0,-1) + g_2 f(x,1,1) \\
(h_1 + p_2 + g_2) f(x,-1,1) &= r_2 f(x,-1,0) + g_1 f(x,1,1) \\
(h_1 + h_2) f(x,-1,-1) &= g_1 f(x,1,-1) + g_2 f(x,-1,1)
\end{aligned}
$$

Table 2 Notation Simplification for the Integrated Quality/Quantity Model

Notation	Simplified	Notation	Simplified	Notation	Simplified
$f(x,0,1)$	$\phi_1(x)$	$P(0,0,1)$	$\Psi_1(0)$	$P(N,0,1)$	$\Psi_1(N)$
$f(x,0,-1)$	$\phi_2(x)$	$P(0,0,-1)$	$\Psi_2(0)$	$P(N,0,-1)$	$\Psi_2(N)$
$f(x,1,0)$	$\phi_3(x)$	$P(0,1,0)$	$\Psi_3(0)$	$P(N,1,0)$	$\Psi_3(N)$
$f(x,-1,0)$	$\phi_4(x)$	$P(0,-1,0)$	$\Psi_4(0)$	$P(N,-1,0)$	$\Psi_4(N)$
$f(x,0,0)$	$\phi_5(x)$	$P(0,0,0)$	$\Psi_5(0)$	$P(N,0,0)$	$\Psi_5(N)$
$f(x,1,1)$	$\phi_6(x)$	$P(0,1,1)$	$\Psi_6(0)$	$P(N,1,1)$	$\Psi_6(N)$
$f(x,1,-1)$	$\phi_7(x)$	$P(0,1,-1)$	$\Psi_7(0)$	$P(N,1,-1)$	$\Psi_7(N)$
$f(x,-1,1)$	$\phi_8(x)$	$P(0,-1,1)$	$\Psi_8(0)$	$P(N,-1,1)$	$\Psi_8(N)$
$f(x,-1,-1)$	$\phi_9(x)$	$P(0,-1,-1)$	$\Psi_9(0)$	$P(N,-1,-1)$	$\Psi_9(N)$

The probability density function $f(x,\alpha_1,\alpha_2)$, and the probability masses $P(0,\alpha_1,\alpha_2)$ (lower boundary) and $P(N,\alpha_1,\alpha_2)$ (upper boundary), are simplified using the notations in Table 2.

Step 1. Derive the state matrix from the internal transition equations

The transition equations (47) can be rewritten in matrix form as:

$$\begin{bmatrix}\dot{\phi}_1(x)\\ \dot{\phi}_2(x)\\ \dot{\phi}_3(x)\\ \dot{\phi}_4(x)\end{bmatrix}=\overbrace{\begin{bmatrix}(r_1+p_2+g_2)/\mu & 0 & 0 & 0\\ -g_2/\mu & (r_1+h_2)/\mu & 0 & 0\\ 0 & 0 & -(p_1+g_1+r_2)/\mu & 0\\ 0 & 0 & g_1/\mu & -(r_2+h_1)/\mu\end{bmatrix}}^{A_1}\begin{bmatrix}\phi_1(x)\\ \phi_2(x)\\ \phi_3(x)\\ \phi_4(x)\end{bmatrix}$$

$$+\overbrace{\begin{bmatrix}-r_2/\mu & -p_1/\mu & 0 & -h_1/\mu & 0\\ 0 & 0 & -p_1/\mu & 0 & -h_1/\mu\\ r_1/\mu & p_2/\mu & h_2/\mu & 0 & 0\\ 0 & 0 & 0 & p_2/\mu & h_2/\mu\end{bmatrix}}^{A_2}\begin{bmatrix}\phi_5(x)\\ \phi_6(x)\\ \phi_7(x)\\ \phi_8(x)\\ \phi_9(x)\end{bmatrix} \quad (48)$$

$$\begin{bmatrix}\phi_5(x)\\ \phi_6(x)\\ \phi_7(x)\\ \phi_8(x)\\ \phi_9(x)\end{bmatrix}=\overbrace{\begin{bmatrix}A_3^1\\ A_3^2\\ A_3^3\\ A_3^4\\ A_3^5\end{bmatrix}}^{A_3}\begin{bmatrix}\phi_1(x)\\ \phi_2(x)\\ \phi_3(x)\\ \phi_4(x)\end{bmatrix}$$

$\phi_1(x),\ldots,\phi_4(x)$ in the LHS of Eqn (48) are the four basic states, and states $\phi_5(x),\ldots,\phi_9(x)$ are the non-basic states. As described in Section 3.3, we define three matrices, A_1, A_2 and A_3 as shown in Eqn (48). Matrix A_3 comprises of the submatrices $A_3^1, A_3^2, A_3^3, A_3^4$ and A_3^5, each of 1×4 dimension, which are defined as follows:

$$\begin{aligned}
A_3^1 &= \left[p_2/(r_1+r_2) \quad h_2/(r_1+r_2) \quad p_1/(r_1+r_2) \quad h_1/(r_1+r_2)\right]\\
A_3^2 &= \left[r_1/(p_1+g_1+p_2+g_2) \quad 0 \quad r_2/(p_1+g_1+p_2+g_2) \quad 0\right]\\
A_3^3 &= \left[0 \quad r_1/(p_1+g_1+h_2) \quad 0 \quad 0\right]+g_2/(p_1+g_1+h_2)\,A_3^2\\
A_3^4 &= \left[0 \quad 0 \quad 0 \quad r_2/(h_1+p_2+g_2)\right]+g_1/(h_1+p_2+g_2)\,A_3^2\\
A_3^5 &= g_1/(h_1+h_2)\,A_3^3+g_2/(h_1+h_2)\,A_3^4
\end{aligned} \quad (49)$$

In Eqn (49), the definitions of the submatrices A_3^3, A_3^4 and A_3^5 involve other submatrices. (e.g., A_3^3 involves A_3^2 in its definition). This is because the non-basic states ($\phi_7(x)$, $\phi_8(x)$, $\phi_9(x)$) have been expressed using the non-basic states ($\phi_6(x), \phi_7(x), \phi_8(x)$) in the transition equations (47).

The state vector is defined as $\Phi_B(x)=\left[\phi_1(x) \;\; \phi_2(x) \;\; \phi_3(x) \;\; \phi_4(x)\right]^T$. Then, we obtain:

$$\dot{\Phi}_B(x) = A\Phi_B(x) \quad (50)$$

where $A = A_1 + A_2A_3$.

Step 2. Obtain the probability density function using the lower boundary equations

The lower boundary transition equations of this model can be found in (Kim, 2006). After simplifying the equations using the notations in Table 2, we obtain the following:

$$
\begin{aligned}
\Psi_3(0) &= \Psi_4(0) = \Psi_5(0) = 0 \\
r_1\Psi_1(0) &= (p_1 + g_1 + p_2 + g_2)\Psi_6(0) \\
r_1\Psi_2(0) &= -g_2\Psi_6(0) + (p_1 + g_1 + h_2)\Psi_7(0) \\
(h_1 + p_2 + g_2)\Psi_8(0) &= g_2\Psi_6(0) \\
(h_1 + h_2)\Psi_9(0) &= g_1\Psi_7(0) + g_2\Psi_8(0) \\
\mu\phi_1(0) &= r_1\Psi_1(0) - p_1\Psi_6(0) - h_1\Psi_8(0) \\
\mu\phi_2(0) &= r_1\Psi_2(0) - p_1\Psi_7(0) - h_1\Psi_9(0) \\
\mu\phi_3(0) &= p_2\Psi_6(0) + h_2\Psi_7(0) \\
\mu\phi_4(0) &= p_2\Psi_8(0) + h_2\Psi_9(0)
\end{aligned}
\tag{51}
$$

By observing Eqn (51), we find that it is possible to express $\Psi_1(0)$, $\Psi_2(0)$, $\Psi_8(0)$ and $\Psi_9(0)$ in terms of $\Psi_6(0)$ and $\Psi_7(0)$. As before, the above boundary equations can be rewritten in matrix form as:

$$
\Phi_B(0) = \begin{bmatrix} \phi_1(0) \\ \phi_2(0) \\ \phi_3(0) \\ \phi_4(0) \end{bmatrix} = \begin{bmatrix} r_1/\mu & 0 & -p_1/\mu & 0 & -h_1/\mu & 0 \\ 0 & r_1/\mu & 0 & -p_1/\mu & 0 & -h_1/\mu \\ 0 & 0 & p_2/\mu & h_2/\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & p_2/\mu & h_2/\mu \end{bmatrix} \begin{bmatrix} \Psi_1(0) \\ \Psi_2(0) \\ \Psi_6(0) \\ \Psi_7(0) \\ \Psi_8(0) \\ \Psi_9(0) \end{bmatrix}
\tag{52}
$$

Then, $\Phi_B(x)$ can be calculated as:

$$
\Phi_B(x) = \boldsymbol{L}_x^{-1}\left[(sI - A)^{-1}\right]\Phi_B(0)
\tag{53}
$$

Step 3. Solve the upper boundary and normalization equations for the unknowns

We present the upper boundary transition equations of this model, which are provided in (Kim, 2006). Using the notations in Table 2 to simplify these equations, we can obtain the following:

$$
\begin{aligned}
\Psi_1(N) &= \Psi_2(N) = \Psi_5(N) = 0 \\
r_2\Psi_3(N) &= (p_1 + g_1 + p_2 + g_2)\Psi_6(N) \\
r_2\Psi_4(N) &= -g_1\Psi_6(N) + (h_1 + g_2 + p_2)\Psi_8(N) \\
(p_1 + g_1 + h_2)\Psi_7(N) &= g_2\Psi_6(N) \\
(h_1 + h_2)\Psi_9(N) &= g_1\Psi_7(N) + g_2\Psi_8(N) \\
\mu\phi_1(N) &= p_1\Psi_6(N) + h_1\Psi_8(N) \\
\mu\phi_2(N) &= p_1\Psi_7(N) + h_1\Psi_9(N) \\
\mu\phi_3(N) &= r_2\Psi_3(N) - p_2\Psi_6(N) - h_2\Psi_7(N) \\
\mu\phi_4(N) &= r_2\Psi_4(N) - p_2\Psi_8(N) - h_2\Psi_9(N)
\end{aligned}
\tag{54}
$$

Once again by observing Eqn (54), it is possible to rewrite $\Psi_3(N)$, $\Psi_4(N)$, $\Psi_7(N)$ and $\Psi_9(N)$ in terms of $\Psi_6(N)$ and $\Psi_8(N)$. As shown in Section 3.3, the upper boundary equations can also be written in matrix form as:

$$\Phi_B(N)=\begin{bmatrix}\phi_1(N)\\ \phi_2(N)\\ \phi_3(N)\\ \phi_4(N)\end{bmatrix}=\begin{bmatrix}0 & 0 & p_1/\mu & 0 & h_1/\mu & 0\\ 0 & 0 & 0 & p_1/\mu & 0 & h_1/\mu\\ r_2/\mu & 0 & -p_2/\mu & -h_2/\mu & 0 & 0\\ 0 & r_2/\mu & 0 & 0 & -p_2/\mu & -h_2/\mu\end{bmatrix}\begin{bmatrix}\Psi_3(N)\\ \Psi_4(N)\\ \Psi_6(N)\\ \Psi_7(N)\\ \Psi_8(N)\\ \Psi_9(N)\end{bmatrix} \tag{55}$$

$$=\boldsymbol{L}_{x=N}^{-1}\left[(sI-A)^{-1}\right]\Phi_B(0)$$

Using Eqn (46) in Section 3.3, the normalization equation is expressed as follows:

$$1=\int_0^N C\Phi_B(x)dx+\sum_{i=0}^{9}\left[\Psi_i(0)+\Psi_i(N)\right] \tag{56}$$

where the state summation matrix C is calculated as:

$$C=\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}+\begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix}A_3 \tag{57}$$

Using any three equations of Eqn (55) and the normalization equation (56), the unknowns ($\Psi_6(0)$, $\Psi_7(0)$, $\Psi_6(N)$ and $\Psi_8(N)$) can be calculated. Then, the probability density function of all states can be calculated using Eqns (53) and (48).

4.2. Performance evaluation

After determining the probability density function and all probability masses, the performance measures are calculated. The performance measures of interest for the integrated quality/quantity model include the average buffer level, total production rate and the effective production rate.

The average buffer level is calculated as:

$$\overline{x}=\int_0^N C\Phi_B(x)x dx+\sum_{i=0}^{9}\Psi_i(N)N \tag{58}$$

where the state summation matrix C is calculated using Eqn (57).

The total production rate is the amount of all parts(of both good and bad quality) processed per unit time in the long run. The total production rate of machine M_2 is calculated as:

$$P_{T2}=\mu_2\left\{\int_0^N C_{\alpha_2=1,-1}\Phi_B(x)dx+\sum_{i=6}^{9}\left[\Psi_i(0)+\Psi_i(N)\right]\right\} \tag{59}$$

where the state summation matrix $C_{\alpha_2=1,-1}=\begin{bmatrix}1 & 1 & 0 & 0\end{bmatrix}+\begin{bmatrix}0 & 1 & 1 & 1 & 1\end{bmatrix}A_3$.

The effective production rate is the production rate of good quality parts in the long run. The effective production rate of machine M_2 is calculated as:

$$P_{E2}=\mu_2\left\{\int_0^N C_{\alpha_2=1}\Phi_B(x)dx+\sum_{i=6,8}\left[\Psi_i(0)+\Psi_i(N)\right]\right\} \tag{60}$$

where the state summation matrix $C_{\alpha_2=1}=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}+\begin{bmatrix}0 & 1 & 0 & 1 & 0\end{bmatrix}A_3$.

To validate the inverse Laplace solution method, a large number of numerical experiments were performed. The solution programs of the two-machine line models analyzed in this paper were run on a Pentium(R) D computer (2.80GHz) with 2GB RAM. For the integrated quality/quantity models, we plot the performance measures (total production rate, effective production rate, and average buffer level, Eqns (58)-(60)) against buffer size, as shown in Fig. 4. The parameters for this experiment are shown in Table 3.

Table 3 Machine and Buffer Parameters

p_1	p_2	r_1	r_2	g_1	g_2	h_1	h_2	μ_1	μ_2	N
0.02	0.01	0.12	0.1	0.01	0.01	0.1	0.2	1.0	1.0	1~50

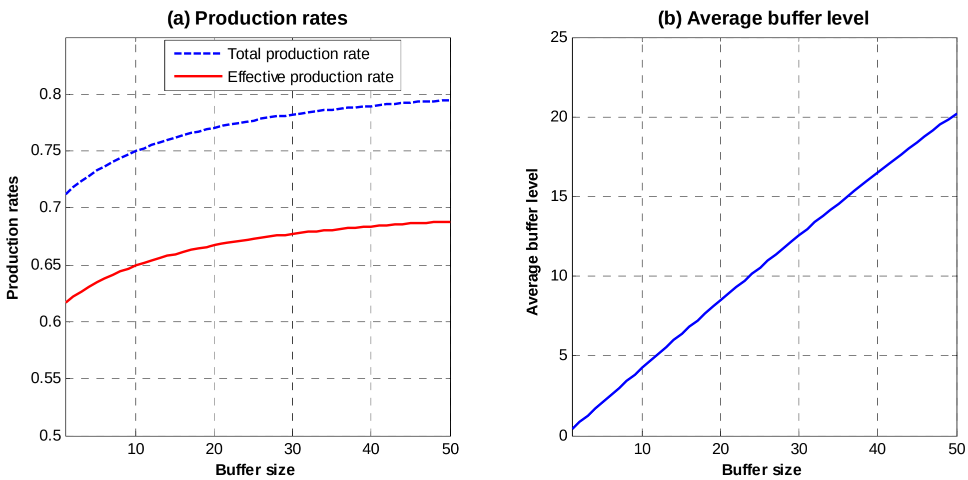

Fig. 4 Performance Measures of a 2M1B Integrated Quality/Quantity Model as Buffer Size is varied

We observe from Fig. 4 (b), that the average buffer level increases almost linearly as the buffer size increases. However, the total and effective production rates will reach saturated values (Fig. 4(a)), as the buffer size approaches infinity (the two saturated values are 0.81 and 0.70 respectively, and can be calculated using Eqns (59) and (60) by limiting the buffer size to infinity).

A large number of experiments with varied parameters of machines and the buffer were also performed. 40 cases of different combinations of these parameters are listed in Table 4. For all these cases, both the parametric method and the inverse Laplace solution method are used to calculate the performance measures, i.e., total production rate, effective production rate and average buffer level. The results of these performance measures are also shown in Table 4. All performance measures calculated using the inverse Laplace solution method are equal to those using the parametric method. This also indicates that the proposed method can be used to solve the continuous Markov models for manufacturing systems.

In addition, the total computational time for all 40 cases using the inverse Laplace solution method is about 0.035 seconds, while the total computational time using the parametric method is about 0.060 seconds. In the decomposition of long lines, the performance of a large number of two-machine lines has to be evaluated iteratively (Le behan and Dallery, 2000; Kim and Gershwin,. 2008; Colledani and Tolio, 2009). The inverse Laplace solution method may lead to shorter computational time of the long line decomposition, by reducing the computational time of solving two-machine line models.

5. Conclusions

This paper proposes the inverse Laplace transform approach as a general solution method for continuous Markov models of manufacturing systems. It is based on the similarities in the transition equations of continuous Markov models and the governing equations of dynamical systems. In these models, the transition equations include differential equations, which are categorised into internal, lower and upper boundary equations. Using the inverse Laplace transform approach, the solution can be obtained through a fairly simple and straightforward

Table 4 Parameters and Calculation Results of the Performance Measures

Case #	p_1	p_2	r_1	r_2	g_1	g_2	h_1	h_2	μ_1	μ_2	N	P_{T2}	P_{E2}	$\overline{x}$
1	0.01	0.01	0.1	0.1	0.01	0.01	0.1	0.1	1	1	1	0.739	0.611	0.500
2	0.01	0.01	0.1	0.1	0.01	0.01	0.1	0.1	1	1	5	0.758	0.626	2.500
3	0.01	0.01	0.1	0.1	0.01	0.01	0.1	0.1	1	1	25	0.799	0.660	12.500
4	0.01	0.01	0.1	0.1	0.01	0.01	0.1	0.1	1	1	50	0.816	0.675	25.000
5	0.01	0.01	0.2	0.1	0.005	0.005	0.1	0.1	1	1	1	0.829	0.752	0.541
6	0.01	0.01	0.2	0.1	0.005	0.005	0.1	0.1	1	1	5	0.843	0.765	2.893
7	0.01	0.01	0.2	0.1	0.005	0.005	0.1	0.1	1	1	25	0.868	0.787	17.384
8	0.01	0.01	0.2	0.1	0.005	0.005	0.1	0.1	1	1	50	0.873	0.792	39.187
9	0.005	0.02	0.1	0.2	0.01	0.01	0.1	0.1	1	1	1	0.792	0.655	0.621
10	0.005	0.02	0.1	0.2	0.01	0.01	0.1	0.1	1	1	5	0.811	0.670	2.976
11	0.005	0.02	0.1	0.2	0.01	0.01	0.1	0.1	1	1	25	0.847	0.700	13.650
12	0.005	0.02	0.1	0.2	0.01	0.01	0.1	0.1	1	1	50	0.860	0.711	26.397
13	0.02	0.01	0.2	0.1	0.02	0.01	0.2	0.1	1	1	1	0.741	0.612	0.384
14	0.02	0.01	0.2	0.1	0.02	0.01	0.2	0.1	1	1	5	0.763	0.631	2.046
15	0.02	0.01	0.2	0.1	0.02	0.01	0.2	0.1	1	1	25	0.806	0.666	11.407
16	0.02	0.01	0.2	0.1	0.02	0.01	0.2	0.1	1	1	50	0.822	0.679	23.674
17	0.02	0.02	0.2	0.12	0.012	0.015	0.14	0.2	1	1	1	0.714	0.612	0.571
18	0.02	0.02	0.2	0.12	0.012	0.015	0.14	0.2	1	1	5	0.738	0.633	3.010
19	0.02	0.02	0.2	0.12	0.012	0.015	0.14	0.2	1	1	25	0.776	0.665	17.723
20	0.02	0.02	0.2	0.12	0.012	0.015	0.14	0.2	1	1	50	0.785	0.672	39.749
21	0.005	0.01	0.1	0.1	0.01	0.005	0.1	0.2	1	2	1	0.832	0.738	0.061
22	0.005	0.01	0.1	0.1	0.01	0.005	0.1	0.2	1	2	5	0.848	0.752	0.295
23	0.005	0.01	0.1	0.1	0.01	0.005	0.1	0.2	1	2	25	0.875	0.776	1.014
24	0.005	0.01	0.1	0.1	0.01	0.005	0.1	0.2	1	2	50	0.880	0.780	1.230
25	0.022	0.03	0.1	0.1	0.015	0.01	0.15	0.18	2	2	1	1.174	1.011	0.527
26	0.022	0.03	0.1	0.1	0.015	0.01	0.15	0.18	2	2	5	1.204	1.037	2.639
27	0.022	0.03	0.1	0.1	0.015	0.01	0.15	0.18	2	2	25	1.293	1.113	13.363
28	0.022	0.03	0.1	0.1	0.015	0.01	0.15	0.18	2	2	50	1.345	1.158	27.160
29	0.03	0.01	0.19	0.1	0.03	0.02	0.1	0.15	2	2	1	1.336	0.907	0.417
30	0.03	0.01	0.19	0.1	0.03	0.02	0.1	0.15	2	2	5	1.369	0.929	2.155
31	0.03	0.01	0.19	0.1	0.03	0.02	0.1	0.15	2	2	25	1.457	0.989	11.798
32	0.03	0.01	0.19	0.1	0.03	0.02	0.1	0.15	2	2	50	1.503	1.020	24.892
33	0.02	0.015	0.12	0.2	0.02	0.01	0.25	0.15	2	1	1	0.798	0.693	0.878
34	0.02	0.015	0.12	0.2	0.02	0.01	0.25	0.15	2	1	5	0.833	0.723	4.395
35	0.02	0.015	0.12	0.2	0.02	0.01	0.25	0.15	2	1	25	0.887	0.770	22.920
36	0.02	0.015	0.12	0.2	0.02	0.01	0.25	0.15	2	1	50	0.894	0.776	47.491
37	0.01	0.02	0.15	0.2	0.01	0.02	0.15	0.2	3	2	1	1.589	1.354	0.932
38	0.01	0.02	0.15	0.2	0.01	0.02	0.15	0.2	3	2	5	1.616	1.378	4.631
39	0.01	0.02	0.15	0.2	0.01	0.02	0.15	0.2	3	2	25	1.675	1.427	23.368
40	0.01	0.02	0.15	0.2	0.01	0.02	0.15	0.2	3	2	50	1.689	1.440	47.731

procedure: (1) derive the state matrix from the internal transition equations; (2) obtain the probability density function from the lower boundary equations; (3) solve the upper boundary and normalization equations for the unknowns of the probability masses. Finally, all performance measures of interest can be calculated based on our proposed method.

The inverse Laplace transform method facilitates the solution of complex continuous Markov models of manufacturing systems that involve, for example, integrated quality/quantity control, machines with multiple failure modes, split/merge material flow, etc. Compared with the parametric method that may result in several extra solution procedures, this method follows the same simple three-step procedure in solving these complex models. In addition, this method may also lead to shorter computational time, compared with the parametric method. Experimental results demonstrate the applicability of our solution method and justify its use as a general analytical method of solving continuous Markov models for the performance evaluation of manufacturing systems.

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Scheduling Techniques to Optimise Sugarcane Rail Transport Systems

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Abstract

In Australia, railway systems play a vital role in transporting the sugarcane crop from farms to mills. In this paper, a novel job shop approach is proposed to create a more efficient sugarcane transport scheduling system to reduce the cost of sugarcane transport. There are several benefits that can be attained by treating the train scheduling problem as a job shop problem. Job shop is generic and suitable for train scheduling problems. The job shop technique prevents operating two trains on one track section at the same time because it considers that only one train (job in a conventional job shop) can operate on a track section (machine in a conventional job shop) at the same time. This technique is promising for finding better solutions in reasonable computation times than alternative methods.

Key Words: scheduling; rail transportation, sugarcane.

Introduction

Sugarcane transport is an important element in the raw sugar production system. Sugarcane is transported in specially designed wagons called cane bins. The efficiency of the transport system has a huge effect on the performance of the production system and on its overall costs. The total cost of the sugarcane transport operations is very high; over 35% of the total cost of sugarcane production in Australia is incurred in cane transport. Some of the potential effects of sugarcane transport on the production system are stopping the supply of cane to mills and causing interruptions to the raw sugar production process; delaying the arrival of sugarcane to the mill, allowing it to deteriorate and causing a reduction in sugar quality; delaying the arrival of empty cane bins to harvesters causing the harvesters to wait for empty bins and increasing their costs; and increasing costs through inefficiencies in the sugarcane transport system itself through the need for higher levels of labour and larger numbers of locomotives and cane bins.

In Australia, the railway system can generally operate for 24 hours a day while the harvesting period is generally limited to about 12 hours in the day. The cane railway system performs two main tasks: delivering empty bins from the mill to the harvesters at sidings; and collecting the full bins of cane from harvesters and transporting them to the mill. From the perspective of the transport system, the mill serves the function of converting full bins into empty bins while the harvesters convert empty bins into full bins.

The sugarcane transport system is very complex. It is managed through the use of a daily schedule, consisting of a set of locomotives runs, to satisfy the requirements of the mill and harvesters. Producing efficient schedules for sugarcane transport can reduce the cost and limit the negative effects that this system can have on the raw sugar production system.

Many researchers have developed optimization models to improve the efficiency of railway and sugarcane transport systems, although none have ever provided a workable daily schedule for a sugarcane rail system that can be directly used without considerable manual modification. There is still considerable work that can be done in this field to develop new techniques to achieve further efficiency improvements and get optimal solutions for the sugarcane transport problem.

An automatic cane railway scheduling system (ACRSS) was developed at James Cook University in the 1970s (Abel et al. 1981). The main aim of ACRSS was producing daily schedules for locomotives runs. This software solved the railway scheduling problem by dividing the problem into two parts: a routing problem that produced locomotives runs and a sequencing problem to determine the locomotive run start times that satisfy the harvester and mill requirements. Further refinement of the solution was then undertaken by adding locomotive runs to produce a daily schedule to satisfy the objective function. This function considered issues such as the number of cane bins, the cane age, a non-interrupted supply of full bins to the mill and the capacity of the sidings.

ACRSS has since been upgraded to be more efficient and easy to use (Everitt and Pinkney 1999) but it still has many limitations. This scheduler still depends on iterative techniques which produce feasible but not optimal solutions. ACRSS doesn't include all sugarcane transport system constraints, e.g. locomotive passing constraints, having two harvesters at one siding, and different speeds and load limits on different sections of track. Time-consuming, manual modifications are required to the produced schedules. Nonetheless, this model remains the only automatic cane railway scheduler in use in the Australian sugar industry.

Two subsequent and independent attempts to solve the sugarcane railway scheduling problem have been reported. Grimley and Horton (1997) used a mixed integer formulation and used AMPL modelling language and CPLEX solvers. The model did not prove practical for use by sugar mills and is not in use. Martin et al. (2002) proposed a new approach using Constraints Logic Programming (CLP) and solved the model using ECL^{i}PS^{e}, a Prolog language. This model was compared to ACRSS and could produce similar results to ACRSS but with longer solution times.

Higgins and Davies (2005) introduced a simulation model for the capacity of sugarcane rail transport systems in Australia. This capacity was determined by estimating some variables such as the number of locomotives used in the system and their shifts, number of bins, and the time spent waiting for the empty bins at farms. The fundamentals of stochastic simulation are used to develop this model to be flexible and easy to apply. Higgins et al. (2004) designed models for three regions in Australia to produce optimal harvest schedules of sugarcane. These models depended on the coordination between growers, harvesters, and mills. They developed software which was used to help the implementation process. These models achieved an increase in profits for all three regions. Higgins and Laredo (2006) designed a framework including mathematical models for harvesting and transport sectors. They used P-Median and spatial clustering formulations to build these models. In this research, the coordination and the integration between the parts and components of these models achieved good results to reduce the overall cost of the system. They applied this research on the north-east coast of Australia as a case study. As a result they could reduce the number of sidings and harvesters by increasing the period of harvesting to 24 hours per day.

Although there are peculiarities to the sugarcane rail transport system that prevent existing railway scheduling models from being used for this purpose, there are still aspects of railway scheduling models that can be applied to the sugarcane rail transport model.

Burdett and Kozan (2006) extended their previous work in railway capacity. In this study, they developed techniques to estimate the capacity for different railways systems to improve the network performance.

Ariano et al. (2007) developed a branch and bound method to solve a train scheduling problem as a blocking job shop problem. The branch and bound method improved the solutions to be near optimal and decreased the computation times. This research used a discrete event optimisation model to increase the efficiency of solutions. Zhou and Zhong (2007) proposed a new model for solving a single track train scheduling problem. They used a branch and bound method to reduce the total time of train trips. In this research, they solved the conflicting train problems through a single railway track.

Burdett and Kozan (2009) developed a new constructive technique to solve train scheduling problems. They used a job shop approach to solve this problem. This solution depended on a disjunctive graph model and constructive solutions methods. This technique achieved better results than meta-heuristics techniques for different criteria such as minimizing a makespan objective.

Liu and Kozan (2009) represented the train scheduling problem as a blocking parallel machine job shop scheduling problem. In this model, they solved the blocking problems in single and multiple railway tracks. They extended the classical disjunctive techniques and proposed a new heuristic method named the feasibility satisfaction procedure to solve this problem. The new technique treated many complex real situations in train crossing problems. A constraint programming approach was proposed for solving railway scheduling problems and obtaining solutions for many cases. (Rodriguez ,2007& Guel et al. 2009& Su Yun et al. 2002).

This paper reviews recent work in sugarcane transport and presents the starting point for a new model designed to produce efficient schedules by solving the cane transport scheduling problem. The new model includes the requirements of all major elements of the cane transport system to reduce the overall cost and optimise the performance of the system. Mathematical formulation proposed is solved by ILOG OPL software. Constraint programming (CP) search techniques such as depth first search and dichotomic search strategy are used inside OPL to solve the model.

The Model

In this model, a job shop scheduling approach is used. This scheduling approach is generic and suitable for railway operations scheduling problems. The mathematical model proposed is integrated with CP search techniques and a dichotomic search strategy.

The model defines the track sections as machines and the train activities as jobs. Each train activity (job) contains a number of operations defined by the start time and the processing time in a particular track section (machine). Some sections contain several parallel track sections, including sidings. The locomotive trip from the mill to the sidings and back to the mill is called a run. Each locomotive typically completes several runs per day. The total time for all operations during a run is equal to the run time. The total run time per day is the total completion time. At this stage of the research, the main objective of this model has been minimising the sum of total completion times for all locomotives.

The main model contains two main parts: the objective function and the constraints. The model was formulated as a scheduling model using binary integer programming. The main advantage of the scheduling model is reactiveness, where the harvester and mill requirements for cane bins are met throughout the modelling period. In addition, the accuracy of the results and locomotive passing are considered in this model.

Notation

The parameters are defined as following:

K	number of locomotives.
k, n	index of locomotives.
S	number of sections.
s	index of sections; s=1, 2, 3, ...,S.
R	maximum number of operations in each job.
r, a	index of operations in each job; r =1, 2, ...,R., a =1, 2, 3, ...,R.
O	total runs of each locomotive.
o, w	index of runs; o=1, 2, 3, ...,O. w=1, 2, 3,...,O.
t_{kors}	start time of locomotive k in run o on the section s during operation r.
f_{kors}	finish time of locomotive k on the section s during operation r in run o.
g_{ks}	processing time of locomotive k on the section s.
D	total work hours.
d	index of time; d=1,2,3,..., D.
α_{kosd}	total empty bins delivered to siding s by locomotive k during run o at time d.
β_{kosd}	total full bins collected from siding s by locomotive k during run o at time d.
A_s	total allotment of siding s per day.
C_{ke}	capacity of locomotive k of empty bins.
C_{kc}	capacity of loco k of full bins.
C_s	capacity of siding s.
h_s	harvesting rate at siding s *per hour*..
M	mill which works in the system.
h_M	crushing rate at mill M.
V	a big positive number.

The decisions variables are defined as following:

$$X_{kos} = \begin{cases} 1, & \text{if locomotive } k \text{ assigned to the section } s. \\ 0, & \text{otherwise.} \end{cases}$$

$$Z_{nks} = \begin{cases} 1, & n \text{ and } k \text{ are processed on the section } s \text{ and locomotive } k \text{ follows locomotive } n. \\ 0, & n \text{ and } k \text{ are processed on the section } s \text{ and locomotive } n \text{ follows locomotive } k. \end{cases}$$

q_{kors} = $\begin{cases} 1, \text{ if the operation } r \text{ of locomotive } k \text{ requires section } s \text{ during run } o. \\ 0, \text{ otherwise.} \end{cases}$

b_{knwas} = $\begin{cases} 1, \text{ if the locomotive } k \text{ requires the section } s\text{, but operation } a \text{ of locomotive } n \\ \quad \text{scheduled at the same section during run } w. \\ 0, \text{ otherwise.} \end{cases}$

$C_{\max}$ Makespan (the completion time of the last operation of the system for all runs).

The objective function

The main objective of this model is to minimise the completion time for all runs in one day. Each locomotive has runs which contain some activities or operations. Equation (1) gets the completion time of the last operation in the system (Makespan) for all runs.

$$\text{Minimise } C_{\max}$$

The model constraints

Rail operation constraints

Equations (2) to (13) explain the rail operation constraints responsible for passing all locomotives through the single track. These equations consider the delays during delivering and collecting the bins to and from sidings.

Equation (2) ensures the total duration of each operation is greater or equal to the processing time of that operation.

$$q_{kors}(f_{kors} - t_{kors}) \geq g_{ks} \qquad \forall\ k, o, r, s. \tag{2}$$

Equation (3) ensures operation $(r+1)$ of locomotive *k* cannot be processed before finishing operation r for locomotive *k* for inbound locomotives.

$$(t_{kors} + g_{ks})q_{kors} \leq q_{ko(r+1)s}t_{ko(r+1)s} - V(q_{kors} - 1) \quad \forall\ k \in K., o \in O., r = 1..., R-1., s \in S. \tag{3}$$

Equation (4) ensures operation r of locomotive *k* cannot be processed before finishing operation $(r+1)$ in locomotive *k* for outbound locomotives.

$$q_{ko(r+1)s}\ t_{ko(r+1)s} \leq (1 - q_{kors})(t_{kors} + g_{ks}) - Vq_{kors} \quad \forall\ k \in K., o \in O., r = 1..., R-1., s \in S. \tag{4}$$

Equation (5) ensures run $(o+1)$ of locomotive *k* cannot start before finishing run o of locomotive *k*. So, the last operation in run o has to finish before the first operation in run $(o+1)$ commences.

$$\sum_{s=1}^{S}(f_{koRs})q_{koR,s} \leq \sum_{s=1}^{S} t_{k(o+1),1,s}\ q_{k(o+1),1,s} \quad \forall\ k = 1,2,3,..., K., o = 1,2,3,..., O-1. \tag{5}$$

In equation (6), locomotives *n* and *k* respectively are processed on the section *s* and locomotive *k* is processed after locomotive *n*.

$$t_{kors} \geq t_{nwas} + g_{ns} Z_{nks} - V(1 - Z_{nks}) \quad \forall k,\ n \in K,\ k \neq n,\ s \in S,\ o, w \in O\ ,\ r, a \in R. \quad (6)$$

In equation (7), locomotives *n* and *k* respectively are processed on the section *s* and locomotive *n* after locomotive *k*.

$$t_{nwas} \geq t_{kors} + g_{ns}(1 - Z_{nks}) - VZ_{nks} \quad \forall k,\ n \in K,\ k \neq n,\ s \in S,\ o, w \in O\ ,\ r, a \in R. \quad (7)$$

Equation (8) ensures each section cannot process more than one locomotive at the same time.

$$\sum_{s=1}^{S} X_{kos} = 1 \quad \forall\ k \in K, \mathrm{o} \in \mathrm{O}., s\ \in\ S. \quad (8)$$

In equation (9), each operation of any locomotive requires only on*e* section *s* to be processed:

$$\sum_{s=1}^{S} q_{kors} = 1 \quad \forall\ k, o, r. \quad (9)$$

Equation (10) is a blocking condition. Operation *r* of locomotive *k* requires section *s* but the operation *a* of locomotive *n* is scheduled on the same section.

$$t_{kors} \geq b_{knwas} t_{nwas} \quad \forall k,\ n \in K,\ k \neq n,\ s \in S,\ o, w \in O\ ,\ r, a \in R. \quad (10)$$

Equation (11) ensures the non-negativity condition.

$$t_{kors} \geq 0 \quad , \ \mathrm{g}_{ks} \geq 0 \qquad \forall\ k, o, r, s. \quad (11)$$

Locomotive constraints

Equations (12) and (13) are locomotive capacity constraints.

In equation (12), the number of empty bins delivered for all sidings during run *o* at time d has to be less than or equal to the capacity of locomotive *k*.

$$\sum_{r=1}^{R} \sum_{s=1}^{S} q_{kors} \alpha_{kosd} \leq C_{ke} \ \forall k, o, d. \quad (12)$$

In equation (13), the number of full bins collected from all sidings during run *o* at time d has to be less than or equal to the capacity of locomotive *k*.

$$\sum_{r=1}^{R} \sum_{s=1}^{S} q_{kors} \beta_{kosd} \leq C_{kc} \ \forall k, o, d. \quad (13)$$

Bin constraints

Equations (14) and (15) show the relation between the total allotments for each siding and the number of empty and full bins delivered to or collected from each siding. These constraints will reduce the total number of bins used in the transport process.

In equation (14), the total number of empty bins delivered to siding s has to be equal to the allotment of this siding.

$$\sum_{k=1}^{K}\sum_{o=1}^{O}\sum_{r=1}^{R}\sum_{d=1}^{D} q_{kors}\alpha_{kosd} = A_s \ \forall\ s. \tag{14}$$

In equation (15), the total number of full bins collected from siding s has to be equal to the allotment of this siding.

$$\sum_{k=1}^{K}\sum_{o=1}^{O}\sum_{r=1}^{R}\sum_{d=1}^{D} q_{kors}\beta_{kosd} = A_s \ \forall\ s. \tag{15}$$

Equation (16) satisfies the harvesters' empty bin requirements without delay. Each harvester needs a specific number of empty bins to collect the cane, and this number has to be reached in a specific time. This equation considers the number of delivered bins and the harvester rate to prevent any interruption in the harvesting process. Also, it considers the siding capacity to prevent the overflow of empty bins.

$$h_s \le \sum_{k=1}^{K}\sum_{o=1}^{O}\alpha_{kosd} - d.h_s \le \mathrm{C}_s \qquad \forall\ s, d. \tag{16}$$

Equation (17) satisfies the mill's full bin requirements without delay. This equation considers the number of full bins delivered and the crushing rate to prevent any interruption in milling processes. Also, it considers the capacity of the mill to prevent any overflow of full bins maintain the quality of cane.

$$h_M \le \sum_{k=1}^{K}\sum_{o=1}^{O}\sum_{s=1}^{S}\beta_{kosd} \text{ - } d.h_M \le C_M \quad \forall\ d. \tag{17}$$

Computational Results

To demonstrate and validate the model in this research, an actual cane railway network will be used. A small section of the rail transport system at Kalamia mill, near Ayr south of Townsville, was used to in a case study to obtain the minimum completion time of all locomotive runs. Figure 1 shows the part of the rail network used, along with some information about the distances between sidings, siding capacity and the number of bins allocated to each siding (allotments. The case study involves 26 track sections, 15 sidings and 3 locomotives. OPL software was used to obtain the solution.

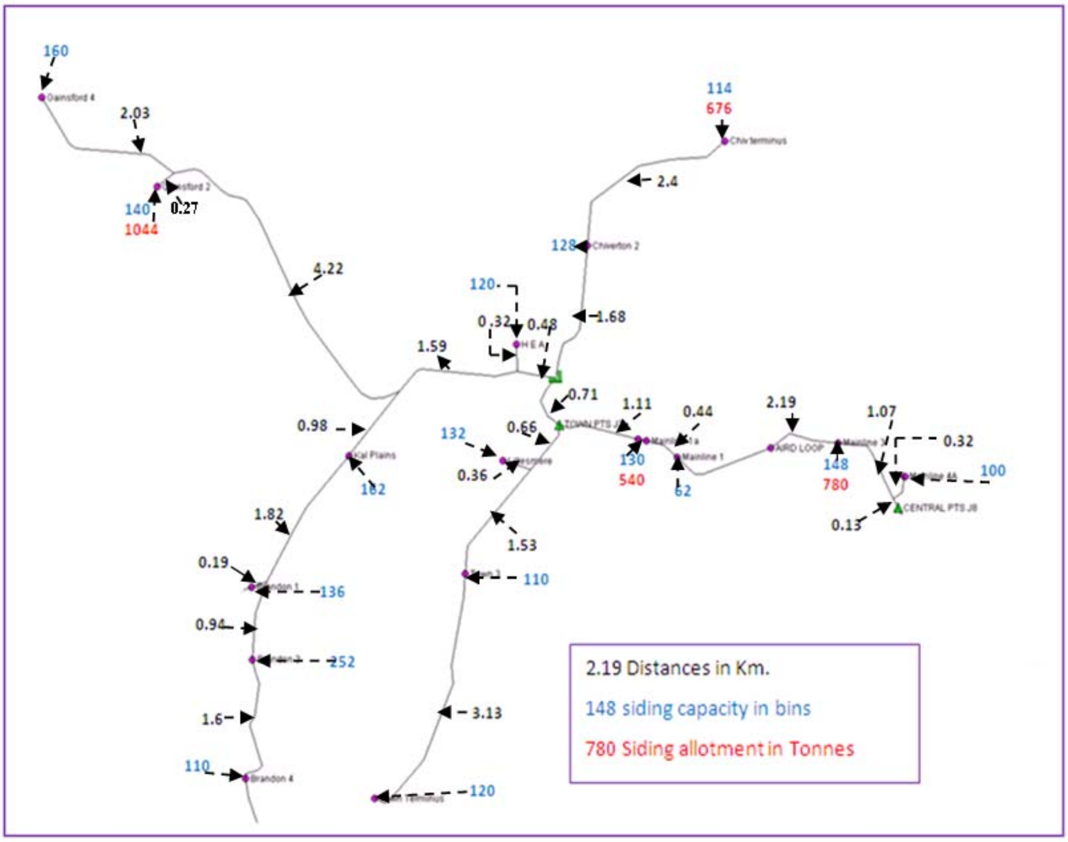

Figure 1 The case study: a small section of rail network at Kalamia mill

Table 1 shows the completion time of the last operation of the system (the Makespan) for one day and the CPU time. Also, as can be seen from this table, the number of variables and the constraints of the case study are very large, even though this case study only represents a small part of the Kalamia rail network. The full rail network is a relatively huge system. In addition, the changing dynamics of the system, such as the number of empty bins and the number of full bins at sidings, causes an increase in the number of mathematical constraints in the new model. All these constraints need a lot of memory to be executed on a PC as a software code (see Solver memory column in Table 1). The choice points column shows the explored points which can be branched and failure points which stop and can't be extended in the search tree. The optimal solution for this case under the Makespan objective was 3164 (s). This solution was obtained in CPU time 10.63(s), a reasonably short time.

Table 1: CPU time under the Makespan objective.

No. of variables	No. of constraints	No. of choice points	No. of failure points	Solution "Makespan" (s)	CPU Time (s)	Solver memory in bytes
5833	169288	2142	2121	3164	10.63	117917768

Table 2 shows the start and finish times of each locomotive run. From Table 2, runs 1 and 5 are assigned to locomotive 1 and start at time zero and 1582 respectively. The second and fourth runs are assigned to locomotive 2 and start at times 42 and 1116. The third run is assigned to locomotive 3 and starts at time 85.

Table 2: Start and finish times of locomotives' run for IP model.

Loco index	Run index	Run start time (s)	Run finish time (s)
1	1	0	1582
	5	1582	3164
2	2	42	1116
	4	1116	2190
3	3	85	1067

All locomotives have to deliver and collect the siding allotment for the day (deliver empty bins and collect full bins). Table 3 shows the actual visit times for all locomotives at each siding, the sidings visited, the number of empty bins delivered to and full bins collected from each siding and run time. Run time is the time between starting the run at the mill and returning to the mill after visiting the sidings.

Table 3: Collecting and delivering bins, and visit times

Index of Locomotive	Index of Run	Siding Number	Actual Visit time (s)	No. of empty bins delivered)	No. of full bins collected	Time (s)
1	1	6	785	120	90	1582
	5	6	2340	54	84	1582
2	2	20	128	90	34	1074
		22	315	10	90	
	4	22	1389	120	96	1074
3	3	14	287	112	112	982
Total	5			506	506	6294

Conclusion

The sugarcane railway operations are very complex and have a huge number of variables. The proposed scheduling model needs to be solved in a reasonable time, because of the dynamic nature of the system.

In the proposed model, the system constraints can be classified into two main categories. Firstly, there are constraints related to the rail operations system. These constraints are very important to ensure passing of locomotives without accidents or delays. Secondly, there are constraints related to sugarcane rail operations such as the locomotive, siding and mill capacity, harvesting rates, harvesting times, empty bins requirements, and full bins requirements. The main advantage of this model is reactiveness where the mill and harvester requirements for bins are achieved without interruption to their operations. The main outputs of the model are efficient schedules for sugarcane transport systems to optimise the performance of the system. Theses schedules considered locomotive passing constraints to satisfy safety conditions in the system. The accuracy of the solution is high compared with other models.

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Performance Measurement System for Value Improvement of Services

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Abstract

This paper provides a performance measurement system for value improvement of services that is regarded ill-defined problem with uncertainty. Building a methodology with systems approach involves the following processes: (a) selecting measures, and building a system recognition process for management problems, and (b) providing a performance measurement system for the value improvement of services based on the system recognition process. We call (a) and (b) PMS (Performance Measurement System) design process, also see it as a core decision-making process through the whole building process, because strategy and vision in the design process should be precisely interpreted, articulated together and then translated into a set of qualitative and/or quantitative measures under "means to purpose" relationship. In the proposed system, we apply fuzzy structural modelling to building up a structural model of PMS. We introduce Choquet integral to obtain the total value index integrating every value index with respect to functions, which composes the services by drawing an inference for individual links between the scores of PMS logically and analytically. In consequence, the system we propose provides decision makers with a mechanism to incorporate subjective understanding or insight into decision process, and also offers an adaptive support for changes with business environment or organizational structure. For showing how the system works and examining its effectiveness, a practical example is illustrated together with shifting in the study.

Key Words: System recognition process, Performance measurement system, Value improvement for Services

1.0 Introduction

In today's competitive business circumstances characterized by globalization, short product life cycles, open systems architecture and diversity of customers' preferences, a lot of managerial innovations such as just-in-time inventory management, total quality management, six sigma quality, customer-supplier partnership, business process reengineering, supply chain integration, etc. have been developed. The value improvement for services is also considered as a methodology of managerial innovation. It is indispensable for corporations to expedite the value improvement for services and provide fine products satisfying required function with reasonable cost.

This paper provides a performance measurement system for value improvement of services that is regarded ill-defined problem with uncertainty. To recognize some phenomenon as a problem and then solve it, it will be necessary to grasp the essence (real substance) of the problem. In particular, for value improvement problems discussing in this study, it can be defined as complicated ill-defined problem since uncertainty on views and experiences of decision makers, so-called fuzziness resides in.

Building the methodology involves the processes below: (a) selecting measures, and building a system recognition process for management problems, and (b) providing the performance measurement system for the value improvement of services based on the system recognition process. We call (a) and (b) PMS (Performance Measurement System) design process, also see it as a core decision-making process, because in the design process, strategy and vision are exactly interpreted, articulated with, and translated into a set of qualitative and/or quantitative measures under "means to purpose" relationship.

We propose, in this paper, a system recognition process which is based on system definition, system analysis and system synthesis to clarify the essence of ill-defined problem. Further, we propose and examine a performance measurement system based on system recognition process as a value improvement methodology for services, in which the system recognition process reflects the views of decision makers and enables to compute the value indexes for the resources. In the proposed system, we apply the fuzzy structural modelling to building up the structural model of PMS. We introduce fuzzy Choquet integral to obtain the total value index for services by drawing an inference for individual linkages between the scores of PMS logically and analytically. In consequence, the system we suggest provides decision makers with a mechanism to incorporate subjective understanding or insight about evaluation process, and also offers a flexible support for changes with business environment or organizational structure.

A practical example is illustrated to show how the system works and its effectiveness is examined.

2.0 System Recognition Process

Management systems are considered to include cover for large-scale complicated problems. However, for a decision maker, it is difficult to know where to start to solve ill-defined problems indwelling in uncertainty.

In general, the problem is classified broadly into two categories. One is a problem with preferable conditions and defined so-called well-defined problem (structured or programmable) which has an appropriate algorithm to solve the problem. The other one is a problem with non-preferable conditions and defined so-called ill-defined problem (unstructured or nonprogrammable) so that there may not exist any algorithm to solve the problem or there may exist partially even if it exists. Problems containing human decision making or large scale one with complicated nature are just applicable to that case. Therefore, uncertainties such as fuzziness (ambiguity in decision making) and randomness (uncertainty of probability event) reside in the ill-defined problem.

In this paper, the definition of management problems is extended to semi-structured and/or unstructured decision-making problems (Simon1977; Anthony1965; Gorry and Morton1971; Sprague and Carlson1982). It is extremely important and essential to consider the way to recognize the essence of an "object" when necessary to solve some problem in fields of social science, culture science and natural science etc.

This section will give a systems approach of the problem to find a way as a preliminary to propose the performance measurement system of value improvement for services. In this approach, the three steps taken in natural recognition pointed out by Taketani(1968) are generally applied to the process of recognition development. These steps which are phenomenal, substantial and essential regarding the system recognition are necessary processes to go through to recognize the object.

With the definitions and the concept of systems thinking, a conceptual diagram of system recognition can be described as in figure 1.

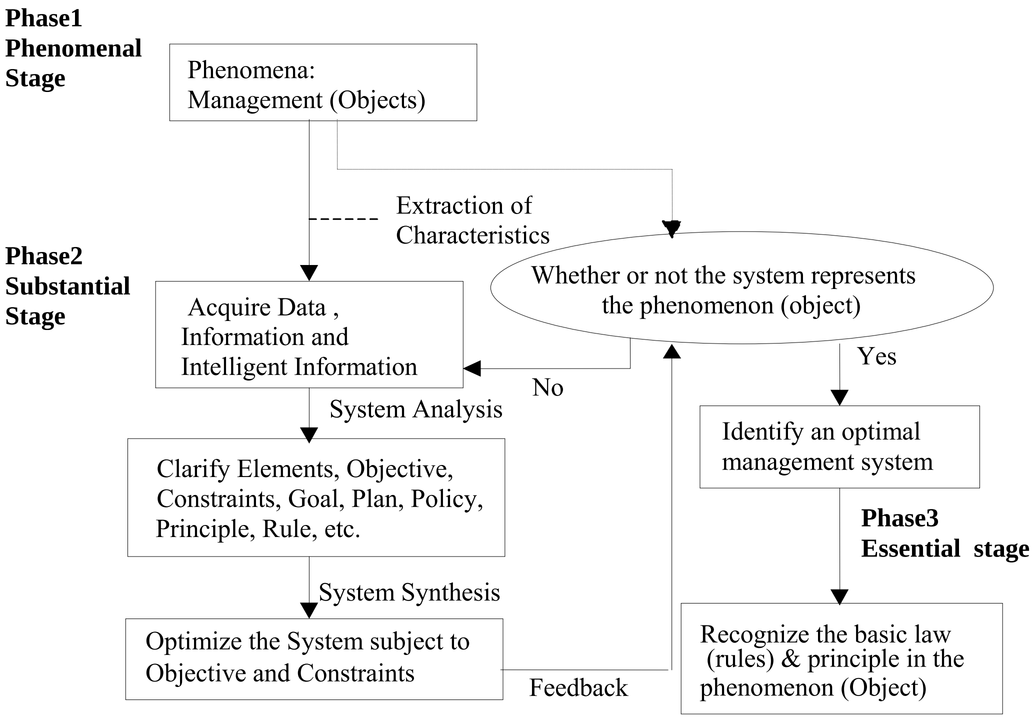

Figure 1. Conceptual Diagram of System Recognition Process

The conceptual diagram of system recognition will play an important role to practically design and develop the value improvement system for services. Phase 1, phase 2 and phase 3 in figure 1 correspond to the respective three steps of natural recognition described above. At the phenomenal stage of phase 1, we assume that there exists a management system as an object, for example, suppose a management problem concerning management strategy, human resource etc., and then extract characteristics of the problem. Then, in the substantial stage, we may recognize the characteristics of the problem as available information, which are extracted at the previous step, and we perform systems analysis to clarify the elements, objective, constraints, goal, plan, policy, and principle etc. concerning the problem. Next, the objective of the problem is optimized subject to constraints arising from the view point of systems synthesis so that the optimal management system can be obtained. The result of the optimization process, as feedback information, may be returned to the phase 1 if necessary, comparing with the phenomena at stage 1.

The decision maker examines whether or not the result will meet the management system he conceives in his mind (mental model). If the result meets the management system conceived in the phenomenal stage, it becomes the optimal management system and goes to the essential stage of phase 3. The essential stage is regarded a step to recognize the basic law (rules) and principle residing in the object. Otherwise, go back to the substantial stage, and the procedure is continued until the optimal system is obtained.

3.0 Performance Measurement System for Value Improvement of Services

A performance measurement system should act flexibly in compliance with changes in social and/or business environments. In this section, a performance measurement system for the value improvement of services is suggested as shown in figure 2 (2-1, 2-2):

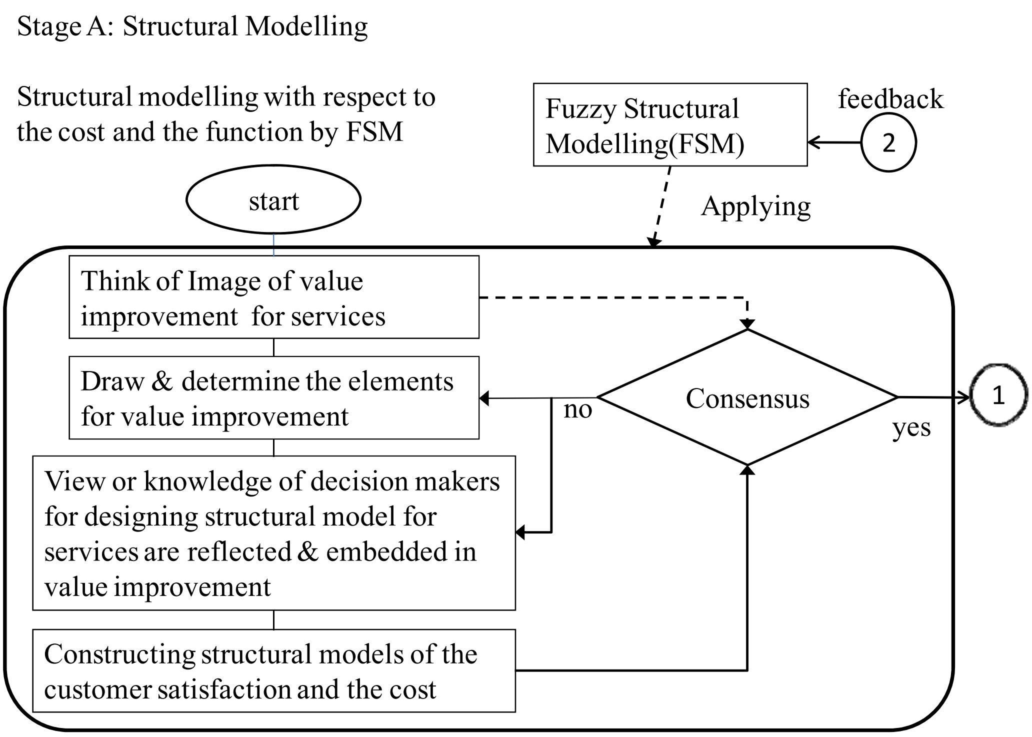

Figure 2.1 Performance Measurement System for Value Improvement of Services (Stage A)

At stage A, the algorithm starts at the initial stage termed structural modelling, in which each model of the function and the cost with respect to services is built up in its own way through the processes encircled with the dotted line in figure 2.1. For obtaining a concrete model for every individual case, we apply fuzzy structural modelling method (FSM) (Tazaki and Amagasa1979; Amagasa2004) to depicting an intuitively graphical hierarchy with well-preserved contextual relations among measured elements. For FSM binary fuzzy relation within the closed interval of [0, 1] is used to represent the subordination relations among the elements, and relaxes the transitivity constraint in contrast to ISM (Interpretive Structural Modelling) (Warfield1975) or DEMATEL (Decision Making Trial & Evaluation Laboratory) (Gabus and Fontela1975). The major advantage of those methodologies may be found in showing intuitive appeal of the graphical picture to decision makers.

Firstly, decision makers' mental model (imagination) on the given problem, which is the value improvement of services, is embedded in a subordination matrix and then reflected on a structural model. Here, the measured elements are identified by methods such as nominal group techniques (NGT) (Delbecq1975; Delbecq, Andrew and Gustafson1995), survey with questionnaire or interview depending on the operational conditions. Thus, we may apply NGT to extracting the measured elements composing the service value and regulating them, clarifying the measurement elements and the attributes. Then, the contextual relations among the elements are examined and represented on the assumption of “means to purpose”. And the hierarchy of the measurement system is constructed and regarded as an interpretative structural model. Furthermore, in order to compare the structural model with the mental model, a feedback for learning will be performed by group members (decision makers). If an agreement among the decision makers is obtained, then the process goes up to the next stage, and the result is considered to be the outcome of stage A. Otherwise, the modelling process restarts from the embedding process or from drawing out and representing the measurement elements process. Then, the process may continue to makes progress in the same way as illustrated in figure 2 until a structural model with some consent is obtained.

Thus, we obtain the models of the function and the cost for services as the outcome of stage A, which are useful for applying to the value improvement for services. Further, we extract and regulate the functions used to perform the value improvement of services by making of use NGT method described above.

(1) Structural model of functions composing of the customer satisfaction

We provides, as shown in Figure 3, an example of a structural model of function showing the relations between elements (functions) used to find the value of services, which is identified by making use of FSM. In this example, the customer satisfaction consists of a set of service functions such as “employee’s behaviour”, “management of a store”, “providing customers with information”, “response to customers”, “exchange of information” and “delivery service”. In addition, for each function “employee’s behaviour” is described as some functions such as “explainable ability for products”, “telephone’s manner” and “attitude to customers”. For “management of stores”, “sanitation control of stores”, “merchandise control” and “dealing with elderly and disabled persons” are enumerated. “Providing customers with information” includes “campaign information”, “information for new products” and “announcement in emergencies”. “Response to customers” consists of “cashier’s rapidity”, “use of credit cards ”, “discount for a point card system” and “settlement of complaints”. In “exchange of information”, “communication among staff members”, “contact with business acquaintances” and “Information exchange with customers” are included. Finally, “Delivery service” contains some functions of “set delivery charges”, ”delivery speed” , and “arrival conditions”.

(2) Structural model of resources composing of the Cost

Resources (sub-costs) composing the cost are also extracted and regulated with NGT method. An example is illustrated in Figure 4 to show the structural model with some resources (sub-costs) constituting the cost that is used to offer services in this paper. Resource (cost) consists of “human resources”, “material resources”, “financial resources” and “information resources”, each of which is also identified by using FSM in the same way as the customer satisfaction was identified. Further, a cost relevant to the human resources consists of “employee’s salaries”, “cost of study training for work” and “employment of new graduates / mid-carrier workers”. The “material resources” contain some sub-costs such as “Buying cost of products”, “rent and utilities”, “depreciation & amortization”. The “financial resources” consists of sub-costs that are “interest of payments”, “expenses incurred in raising funds”, and ”expenses incurred for a meeting of stock holders”. Sub-costs for “information resources” are “communication expenses”, “expenses for PR ” and “costs for installation of a system” .

With the structural models of the customer satisfaction and the resources (costs) mentioned above, we evaluate the value indexes of services.

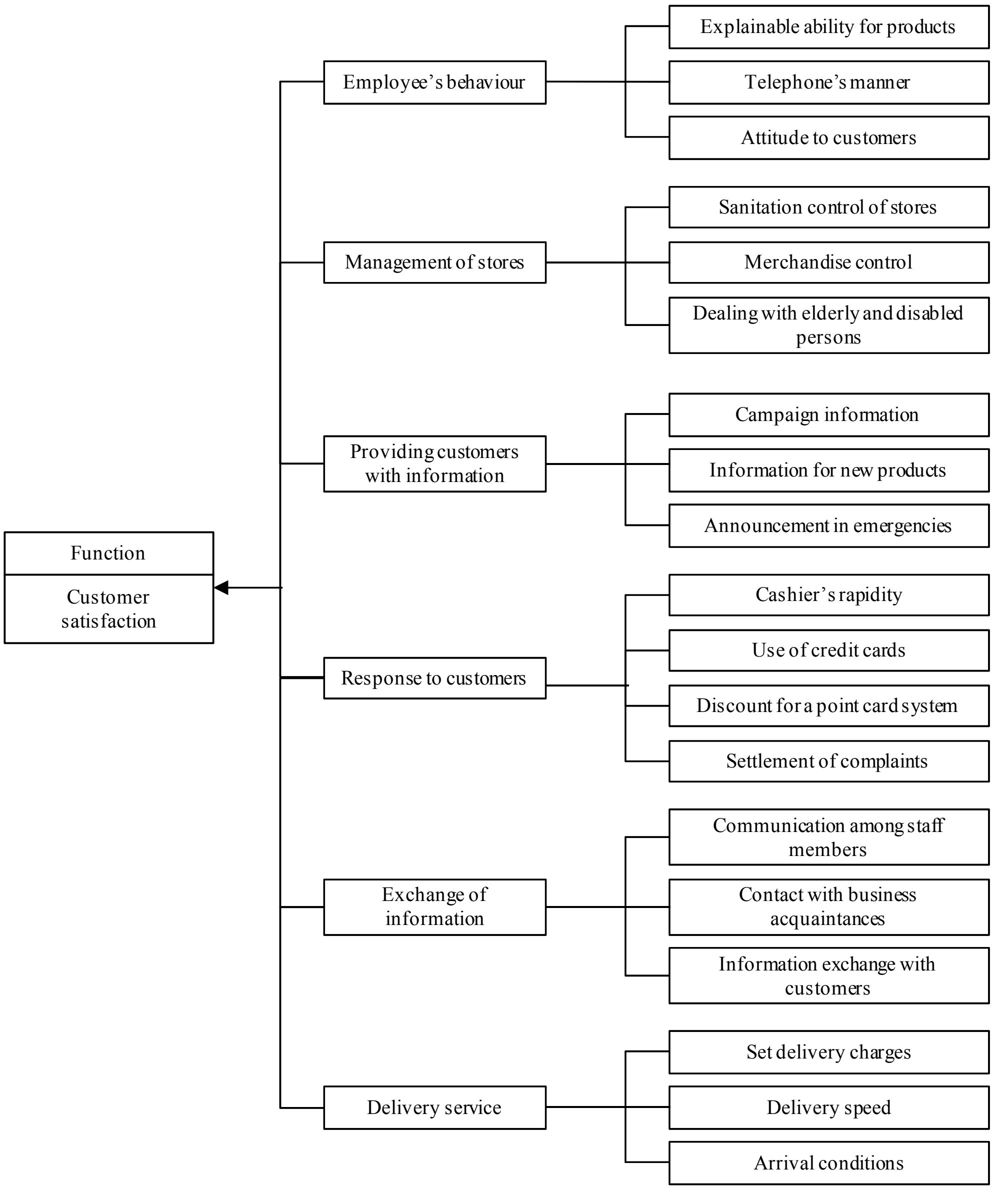

Figure 3. An Example of Structural Model of Customer Satisfaction

At stage B shown in Figure2.2, the value indexes for use of four resources which consist of the human resources(R_1), the material resources (R_2), the financial resources(R_3) and the information resources(R_4) are evaluated on the basis of the structural models identified at the stage A to perform the value improvement of services.

The weights can be computed by making use of Frobenius theorem or the ratio approach with transitive law (Furuya1957)(Amagasa and Cui2009). In this paper, we use the ratio approach to compute the weights of the function and the cost in the structural models shown in figures 3 and 4, and their weights are also used in Multi-Attribute Decision-Making.

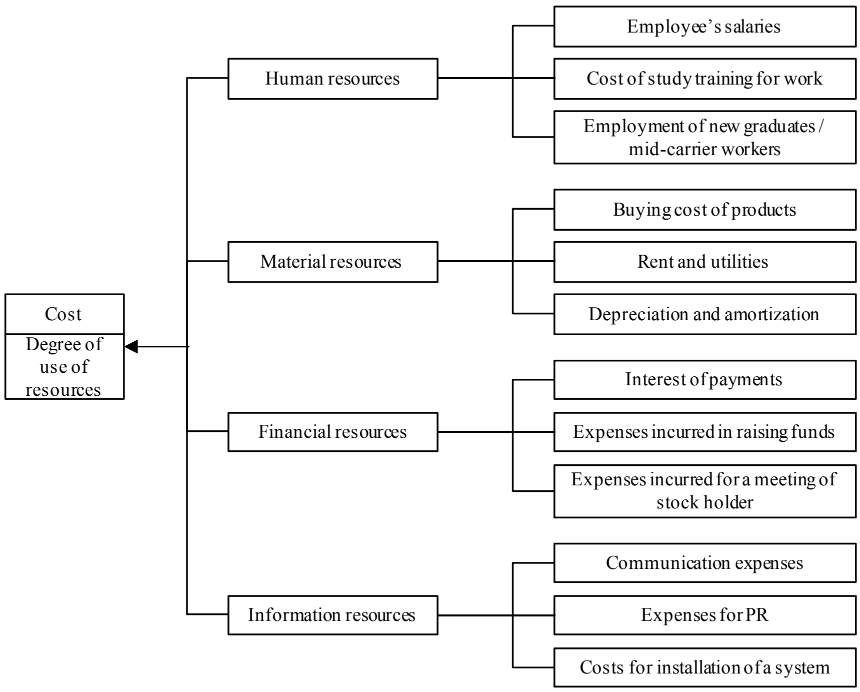

Figure 4. An Example of Structural Model of Cost

The ratio method:

The importance degrees of service functions are computed by using the ratio between the functions as follows:

Let F be a matrix determined by paired comparisons among the functions.
Assume that reflexive law is not satisfied in F, and only each element corresponding to $f_{i,i+1}$,$(i=1,2,\ldots n\text{-}1)$ of the matrix is given as an evaluation value,

	f_1	f_2	f_3	.	f_{n-1}	f_n
f_1	0	f_{12}	—	.	—	
f_2	f_{21}	0	f_{23}	.	—	—
f_3	0	f_{32}	0	.	—	—
.				.		
f_{n-1}	—	—	—	.	0	$f_{n-1,\,n}$
f_n	—	—	—	.	$f_{n,\,n-1}$	0

,where $0 \leqq f_{i,i+1} \leqq 1$ and $f_{i+1,i}$ satisfies the relation $f_{i+1,i} = 1 - f_{i,i+1}$ $(i=1,2,\ldots k,\ldots,n\text{-}1)$.

Then, the weight vector $E(=\{E_i,\ i=1,2,\ldots,n\})$ of functions $(F_i,\ i=1,2,\ldots,n)$ can be found below,

$$E_1 = \prod_{i=1}^{n-1} f_{i,i+1}$$

$$E_k = \prod_{i=1}^{k-1} (1 - f_{i,i+1}) \prod_{i=k}^{k-1} f_{i,i+1} \quad (1 < k < n\text{ , integer})$$

$$E_n = \prod_{i=1}^{n-1} (1 - f_{i,i+1}) \tag{1}$$

We apply formulas mentioned above to find the weights of functions used. Then, the matrix is constituted with paired comparisons by decision makers (specialists) who take part in the value improvement of services in the corporation.

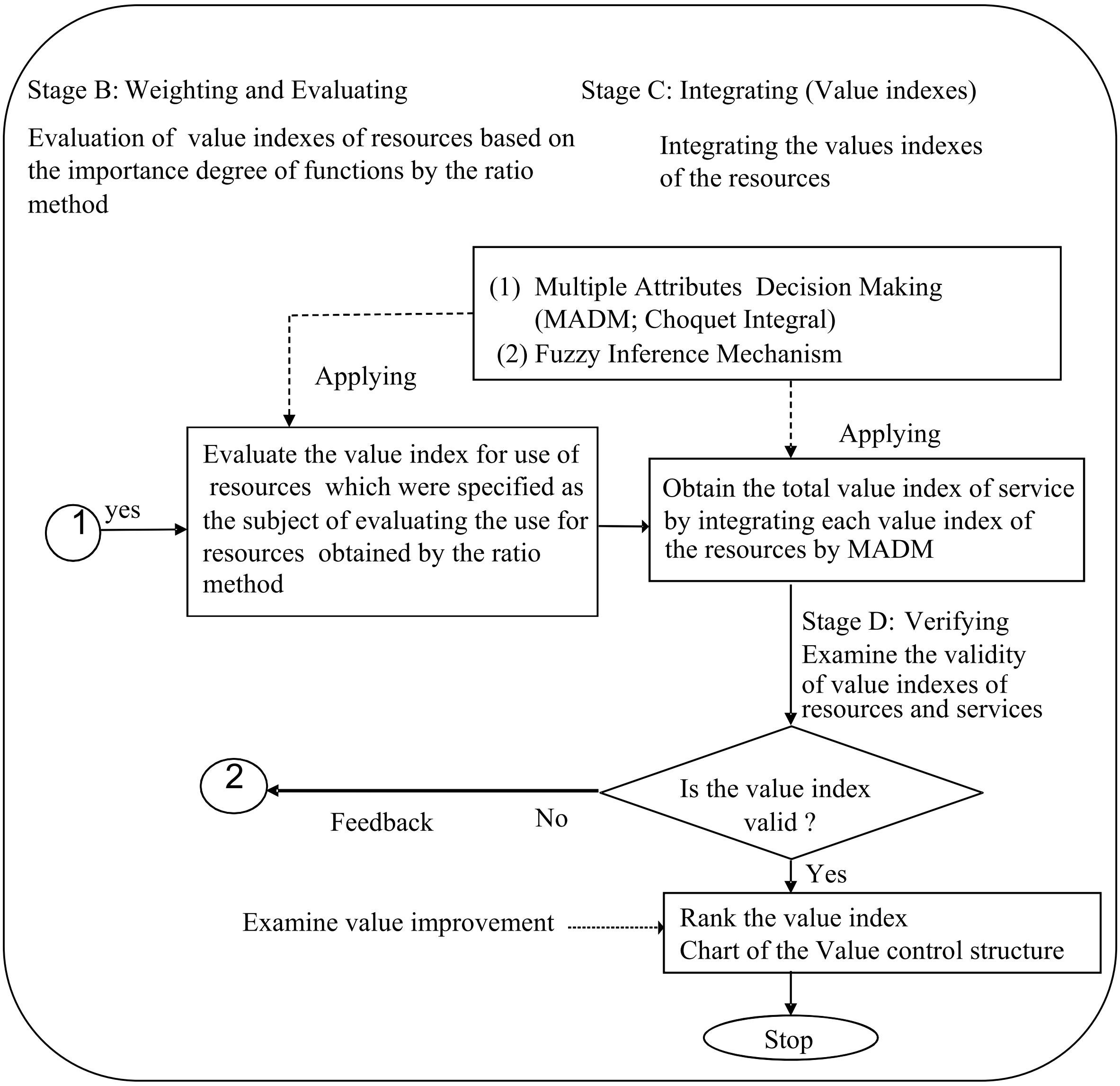

Figure 2.2 Performance Measurement System for Value Improvement of Services (Stages B and C)

(1) Importance degree of functions composing the customer satisfaction

Suppose, in this paper, that the functions composing the customer satisfaction are extracted and regulated as a set as follows;

$F = \{ F_i;\ i=1,2,\ldots,6 \}$
= {Employee's behaviour, Management of a store, Providing customers with information, Response to customers, Exchange of information, Delivery service}

Improvement of customer satisfaction becomes a main purpose of corporate management, and F_i, ($i=1,2,\ldots,6$) are respectively defined as the function to achieve customer satisfaction.

Then, for example, let each cell of the matrix be intuitively and empirically filled with paired comparison manner whose values are given by the ratio method by taking into consideration of knowledge and/or experiences of the decision makers (specialists);

	F_1	F_2	F_3	F_4	F_5	F_6
F_1	—	0.8			φ	
F_2		—	0.4			
F_3			—	0.3		
F_4				—	0.6	
F_5					—	0.8
F_6						—

Also, assume that as an evaluation standard to apply paired comparison, we specify five different degrees of grade based on the membership functions.

Not important: [0.0, 0.2)
Not so important: [0.2, 0.4)
Important: [0.4, 0.6)
Very important: [0.6, 0.8)
Critically important: [0.8, 1.0]

For instance, if F_i is believed to be critically important than F_j, the decision makers may make an entry of 0.9 in F_{ij}. Each value is empirically given by the decision-makers (or specialists) who have their experiences and knowledge with know-how for the value improvement. As the result of it, the values yielded by the ratio method are recognized as weights for the functions.

Thus, the weight vector E of functions (F_i, $i=1,2,\ldots,6$) is obtained as follows:

E = {0.046, 0.012, 0.017, 0.040, 0.027, 0.007}

Further, F can be standardized;

E = {0.31, 0.08, 0.11, 0.27, 0.18, 0.05}

***Importance degrees of constituent elements of " employee's behaviour** (F_1)"**:**

As it is clear from the structural model of the customer satisfaction shown in Figure 3, F_1 consists of all sub-functions F_{1i},($i=1,2,3$).

Here, we compute the importance degrees of {F_{1i},$i=1,2,3$} by the ratio method in the same way as F_1 was obtained.

Importance degrees for sub-functions of "employee's behaviour (F_1)*"*:

F_1= {F_{1i};$i=1,2,3$}

= {Explainable ability for products, Telephone's manner, Attitude to customers}

F_1	F_{11}	F_{12}	F_{13}
F_{11}	—	0.6	
F_{12}		—	0.3
F_{13}			—

Then the weight vector $E(=\{E_{1i};i=1,2,3\})$ for $\{F_{1i};i=1,2,3\}$ is found as follows:

$E=\{0.31, 0.21, 0.48\}$

From this, the importance degrees $\{E_{1i},i=1,2,3\}$ of sub-functions $\{F_{1i},i=1,2,3\}$ are also recomputed with weight of F_1 as follows;

E_{11}= *weight of* F_1 ×*weight of* $F_{11}=0.31\times0.31=0.096$
E_{12} = *weight of* F_1×*weight of* $F_{12}=0.31\times0.21=0.065$
E_{13}= *weight of* F_1×*weight of* $F_{13}=0.31\times0.48=0.149$

In a similar way, the weights of other functions F_i, $(i=2,3,\ldots,6)$ and the importance degrees for sub-functions of F_i, $(i=2,3,\ldots,6)$ are obtained by the ratio method. The computational results are summarized in table 1.

Table 1. Weights of Sub-Functions to Improve the Customer Satisfaction

	Sub-functions	Weights
F_1	F_{11} (Explainable ability for products)	0.096
	F_{12} (Telephone 's manner)	0.065
	F_{13} (Attitude to customers)	0.149
F_2	F_{21} (Sanitation control of stores)	0.026
	F_{22} (Merchandise control)	0.038
	F_{23} (Dealings with elderly & disabled persons)	0.016
F_3	F_{31} (Campaign information)	0.039
	F_{32} (Information for new products)	0.057
	F_{33} (Announcement in emergencies)	0.014
F_4	F_{41} (Cashier's rapidity)	0.059
	F_{42} (Use of credit cards)	0.024
	F_{43} (Discount for a point card system)	0.038
	F_{44} (Settlement of complaints)	0.149
F_5	F_{51} (Communication among staff members)	0.056
	F_{52} (Contact with business acquaintances)	0.038
	F_{53} (Information exchange with customers)	0.086
F_6	F_{61} (Set delivery charges)	0.031
	F_{62} (Delivery speed)	0.008
	F_{63} (Arrival conditions)	0.012

(2) Amount of the cost (resources) based on the structural model of cost

The cost is understood as the amount of resources utilized to provide the customers with the services. In order to calculate the cost for services, we prepare the questionnaire for the decision makers (specialists), that is, how much you utilized resources in every possible way to pursue/achieve the value of services.

Evaluation of cost (resources)

Let denote C by the amount utilized of four resources. These are expressed by C_i,($i=1,2,\ldots,4$) as below.

$C= \{C_i, i=1,2,\ldots,4\}$
={Human resources, Material resources, Financial resources, Information resources}

The degrees for use of resources is meant by the purpose of corporation for using resources effectively, and C_i,($i=1,2,\ldots,4$) are considered the costs to achieve the purpose.
The following matrix shows responses provided by the decision makers (specialists) answering to the questionnaire.

C	C_1	C_2	C_3	C_4
C_1	—	0.6		
C_2		—	0.7	
C_3			—	0.4
C_4				—

Applying ex.(1) to the matrix, we can obtain the sub-costs utilized to give services, that is,
$\{C_i, i=1,2,\ldots,4\}$ = {0.42, 0.28, 0.12, 0.18}.
For instance, "C_1= 0.42" shows the amount of human resources utilized to perform the services.

Evaluation of sub-cost composing the human resources:
$C_1= \{C_{1i}, i=1,2,3\}$
= {Employee' salaries, Cost of study training for work, Employment of new graduates/mid-carrier workers}

The following matrix is provided as similarly by the decision makers.
In analogous to the above, we can get the sub-costs utilized to give the services:

C_1	C_{11}	C_{12}	C_{13}
C_{11}	—	0.8	
C_{12}		—	0.4
C_{13}			—

$\{C_{1i}, i=1,2,3\}$ = {0.62, 0.16, 0.22}
Namely, the amount of cost, C_1 consists of those of sub-costs for human resources.
Then C_{11}= amount of $C_1 \times C_{11}$=0.42×0.62=0.26. "C_{11}=0.26" means the amount of sub-cost of human resources utilized to give the services.

Table 2. Weights of Sub-resources Composing Cost of Services

	Sub-resources composing cost	Sub-costs
C_1	C_{11} (Employee's salaries)	0.260
	C_{12} (Cost of study training for work)	0.067
	C_{13} (Employment of new graduates/mid-carrier workers)	0.092
C_2	C_{21} (Buying cost of products)	0.162
	C_{22} (Rent and utilities)	0.070
	C_{23} (Depreciation & amortization)	0.048
C_3	C_{31} (Interest of payments)	0.028
	C_{32} (Expenses incurred in raising funds)	0.065
	C_{33} (Expenses for meetings for stock holders)	0.028
C_4	C_{41} (Communication expenses)	0.027
	C_{42} (Expenses for PR)	0.108
	C_{43} (Costs for installation of a system)	0.045

Table 3. Importance Degrees of Resources from Functions of Customer Satisfaction

Function items / Resources	F_1	F_2	F_3	F_4	F_5	F_6	$\sum_{k=1}^{6} E_k = 100\%$
R_1	a_{11} RE_{11}	a_{12} RE_{12}	a_{13} RE_{13}	a_{14} RE_{14}	a_{15} RE_{15}	a_{16} RE_{16}	$\sum_{k=1}^{6} E_{1k}$
R_2	a_{21} RE_{21}	a_{22} RE_{22}	a_{23} RE_{23}	a_{24} RE_{24}	a_{25} RE_{25}	a_{26} RE_{26}	$\sum_{k=1}^{6} E_{2k}$
R_3	a_{31} RE_{31}	a_{32} RE_{32}	a_{33} RE_{33}	a_{34} RE_{34}	a_{35} RE_{35}	a_{36} RE_{36}	$\sum_{k=1}^{6} E_{3k}$
R_4	a_{41} RE_{41}	a_{42} RE_{42}	a_{43} RE_{43}	a_{44} RE_{44}	a_{45} RE_{45}	a_{46} RE_{46}	$\sum_{k=1}^{6} E_{4k}$

In a similar way, the amounts of other resources $\{C_i, i=2,3,4\}$ as well as sub-costs of $\{C_i, i=2,3,4\}$ are also computed by the ratio method. The computational result is found in table 2 which shows the sub-costs for the resources utilized to give the services.

In table 3, a_{ij} shows the degrees of resource R_i used to satisfy the function item $F_j, j=1,2,\ldots,m$. $RF_{ij} = E_i \times a_{ij} \times 10^{-2}, (j = 1,2,\ldots,6)$, $\sum_{j=1}^{6} a_{ij} = 100\ (\%), (i = 1,2,3,4)$. E_j, $(j=1,2,\ldots,6)$ shows the degree of importance of each function items. The costs of the resource $R_i, (i=1,2,3,4)$ will be computed and shown as $\sum_{k=1}^{6} RE_{ik}$ in table 3.

(3) Computing for the value indexes of four resources:

In general, the value index of object in value engineering is defined by the following formula.

Value index= Satisfaction for Necessity / Use of Resources (2)

The value index is interpreted to show the degree of satisfaction to fill necessity which is brought by the resources when resources are utilized. Based on this formula, in this study, we define the value of services composing four resources as below.

Value of services = Function of services / Cost of services (3)

Therefore, the value index, which is based on importance degree and cost concerning each resources used to give services, is obtained.

Value index of human resources = $\sum_{k=1}^{m} E_{1k}$ /cost of human resources

Value index of material resources = $\sum_{k=1}^{m} E_{2k}$ /cost of material resources

Value index of financial resources= $\sum_{k=1}^{m} E_{3k}$ /cost of financial resources

$$\text{Value index of information resources} = \sum_{k=1}^{m} E_{4k} \ \text{/cost of information resources} \qquad (4)$$

At stage C, the multi-attribute decision making method (MADM) based on Choquet integral (Grabish1995; Modave and Grabish1998) can be introduced and a total value index of services (service value) is found by integrating the value indexes of the human, the material, the financial and the information resources. Let $X_i, (i = 1,2)$ be fuzzy sets of universe of discourse *X*. Then the λ fuzzy measure *g* of the union of these fuzzy set, $X_1 \cup X_2$ can be defined as follows:

$$g(X_1 \cup X_2) = g(X_1) + g(X_2) + \lambda g(X_1) g(X_2)$$

,where λ is a parameter with values $-1<\lambda<\infty$, and note that $g(\cdot)$ becomes identical to probability measure when $\lambda=0$. Here, since it is assumed that when the assessment of corporation is considered the correlations between factors are usually independent, the fuzzy sets X_1 and X_2 are independent, that is, $\lambda=0$. Then, the total value index of services is expressed as in ex.(5).

Total value index of services
= g (value index of human resources, value index of material resources, value index of financial resources, value index of information resources) (5)
$= w_1 \times$ value index for human resources $+ w_2 \times$ value index for material resources
$+ w_3 \times$ value index for financial resources $+ w_4 \times$ value index for information resources, where w_i $(0 \le w_i \le 1; i = 1,2,3,4)$ are weights for respective resources.

At stage D, if the integrated evaluation value is examined and its validity is shown, the process goes to the final stage (stage E).

At stage E, we rank the integrated value index of services computed in the previous step by using fuzzy outranking method (Roy1991, Siskos and Oudiz1986) and draw the graphic structure of value control (Amagasa1986). Then the process terminates.
In this study, each of value indexes of services is represented in the graphic structure of value control depicted.

4.0 Simulation for Value Improvement System of Services

In this section, we carry out simulation of the procedure to perform the value improvement system of services and examine the effectiveness of the proposed value improvement system.

Here, as specific services trade, we take up a household appliance store DD Co., Ltd.
This store is said to be a representative example providing "a thing and services" to customers. The store sells "things" such as household electrical appliances which are essential to necessities of life and commercial items used in everyday life. In addition, it supplies customer services when customers purchase "thing" itself. DD Co., Ltd. was established in 1947 and the capital is 19,294million yen, a yearly turnover 275,9 00 million yen, total assets worth 144,795 million yen, the number of the stores 703 (the number of stores of franchise is 582, March 31, 2007) and the number of employees 3,401. The store is well known to the customers on the grounds that it would make a difference with other companies by which the management technique is designed for customer-oriented style, pursuing customer satisfaction. For example, salespersons have the sufficient knowledge about products they treat and give suitable advices and suggestions according to customers' requirements, which often happens at the sales floor. We conducted a

survey for DD Co., Ltd. Simulation is based on the result of questionary survey and performed by applying the performance measurement system for the value improvement of services shown in figure 2, which we proposed in Chapter 3.

In stage A: Structural modelling

Figures 3 and 4 illustrated in chapter 3 show the structural models with respect to the functions composing the customer satisfaction, and the cost showing the use of resources by making use of NGT and FSM.

In stage B: Weighting and Evaluating

Table2 shows the importance degrees of resources for functions of the customer satisfaction which is obtained by consensus among decision makers (specialists) with know-how deeply related to the value improvement of services.

By Table 4 it is understood that the distributed amount for four resources and the real ratios which are used to attain customer satisfaction related to six functions are provided with four resources.

From this, each of the value indexes with respect to the respective resources used to supply customer services, for which human resources, material resources, financial resources and information resources are considered, is obtained by using tables 1,2,3 and 4, and ex.(4).

(1) Value index of Human resources =45.64/42(=1.1)
(2) Value index of Material = 4.08/28(=0.14)
(3) Value index of Financial =13.19/12(=1.08)
(4) Value index of Information = 36.37/18(=2)

From the value indexes for the resources mentioned above, the chart of value control graphic structure is depicted as shown in figure5. Thus, it may be concluded by Figure 5 that the following results with respect to the value improvement for services from the view points of function and cost are ascertained.

(1) In this corporation, there is no need for doing the value improvement relating to each of human resources, the financial resources and the information resources because three of all four resources are below the curved line , implying the pretty good balance between the cost and the function of services

(2) For the material resources, it will be necessary to exert all possible efforts for the value improvement of the resource because the value index shows 0.04 which is much smaller than 1.00.

(3) On the whole, the total value index of services is counted 1. 23 as shown below, so that the value indexes for four resources are included within the optimal zone of the chart of value control graphic structure shown in figure5. Therefore, it seems to be concluded that the corporation may not have to improve the value of services in that organization.

Table 4. Importance Degrees of Resources for Functions of Customer Satisfaction Function (Customer satisfaction)

Use of resources \ Function (Customer satisfaction)		Employee's behaviour (0.31)			Management of stores (0.08)			Providing customers with information (0.11)			Response to customers (0.27)				Exchange of information (0.18)			Delivery service (0.05)			Integrated evaluation values
		Explainable ability for products	Telephone's manner	Attitude to customers	Sanitation control of stores	Merchandise control	Dealing with elderly and disabled persons	Campaign information	Information for new products	Announcement in emergencies	Cashier's rapidity	Use of credit cards	Discount for a point card system	Settlement of complaints	Communication among staff members	Contact with business acquaintances	Information exchange with customers	Set delivery charges	Delivery speed	Arrival conditions	
		0.096	0.065	0.149	0.026	0.038	0.016	0.039	0.057	0.014	0.059	0.024	0.038	0.015	0.056	0.038	0.086	0.031	0.008	0.012	
Human resources	Employee's salaries Cost of study training for work Employment of new graduates / mid-carrier workers	70	70	80	20	20	40			30	30	30	30	80	30	20	20		20	20	45.64 (%)
		6.72	4.55	11.92	0.52	0.76	0.64			0.42	1.77	0.72	1.14	11.92	1.68	0.76	1.72		0.16	0.24	
Material resources	Buying cost of products Rent and utilities Depreciation and amortization		10		10	30	20	20										30			4.08 (%)
			0.65		0.26	1.14	0.32	0.78										0.93			
Financial resources	Interest of payments Expenses incurred in raising funds Expenses incurred for a meeting of stock holder				20	20	20	20	20	20	30	30	30		30	30	20	20	30	30	13.19 (%)
					0.52	0.76	0.32	0.78	1.14	0.28	1.77	0.72	1.14		1.68	1.14	1.72	0.62	0.24	0.36	
Information resources	Communication expenses Expenses for PR Costs for installation of a system	30	20	20	50	30	40	60	60	50	40	40	40	20	40	50	60	50	50	50	36.37 (%)
		2.88	1.3	2.98	1.3	1.14	0.64	2.34	3.42	0.7	2.36	0.96	1.52	2.98	2.24	1.9	5.16	1.55	0.4	0.6	

In stage C: Integrating (Value indexes)

At integrating stage, multi-attribute decision making method (MADM) based on Choquet integral (Grabish1995; Modave and Grabish1998) can be introduced for the value improvement of services, and the total value index of services is obtained by integrating the value indexes of the four resources as follows:

$$\text{Total value index of services} = w_1 \times 1.1 + w_2 \times 0.14 + w_3 \times 1.08 + w_4 \times 2$$
$$= 0.46\times1.1+0.11\times0.14+0.17\times1.08+0.26\times2=1.23$$

As a result of simulation, the value of services of DD Co., Ltd. indicates considerably high level because the total value index becomes 1.23 (greater than 1.00) whose value belongs to the optimal region.

Nikkei Business announces the following comment about DD Co., Ltd. that it evaluates the store highly. The store advocates that customers “have trust and satisfaction with buying the products all the time". Also, the store supplies "goods having something attractive" at "a price of relief" as well as "superior services" as a household appliance retail trade based on this management philosophy. Furthermore, the store realizes a customer-oriented and community-oriented business and a supply of smooth services reflecting area features and scales advantages by controlling the total stock in the whole group. From this, it can be said that the validity propriety of proposed method was verified by the result of this simulation experiment, which corresponds to high evaluation for DD Co., Ltd. by Nikkei Business described above.

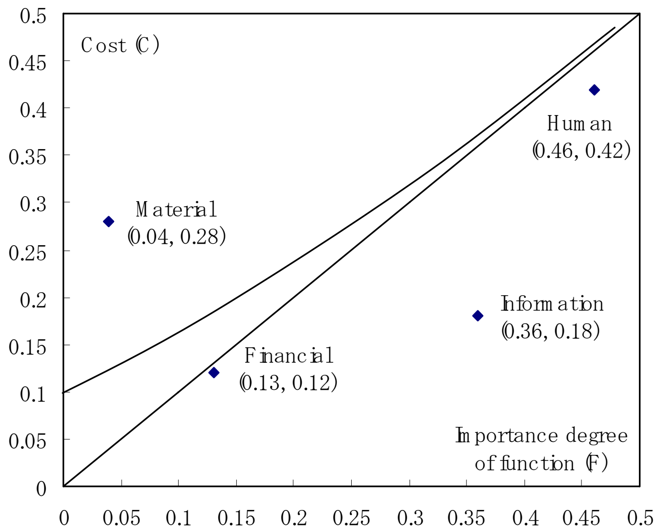

Figure5. Value Control Graphic Structure

5.0 Conclusion

It is very important for an administrative action to pursue profit of a corporation by making use of four resources effectively, which are capable persons, materials, capital and information. In addition, letting each employee attach great importance to services and then it is to be hoped that the employee would willingly improve services quality, and thus enhancing the degree of customer satisfaction is more important in services trade. These will be surely promised to bring about the profit improvement of the corporation.

We proposed, in this paper, a system recognition process which is based on system definition, system analysis and system synthesis, making clarify the essence of ill-defined problem. Further, we suggest the performance measurement system for the value improvement for services as a methodology and examined it, in which the system recognition process reflects the views of decision makers and enables to compute the effective service scores. As an illustrative example, we took up the evaluation problem of a household appliance store selected from the view point of service functions, and come up with a new value improvement methodology by which the value indexes of services are computed. In order to verify the effectiveness of the new methodology we suggested, we performed questionary survey about services functions for the household appliance store. As the results, it is observed that the proposed methodology is very significant for the value improvement of services in corporations.

Finally, it turns out that the soundness of this system was verified by the result of this simulation, which is consistent with the fact that Nikkei Business placed high value on DD Co., Ltd.
With this procedure it is possible to build performance measurement system for services, which is based on the realities. This part of study remains as a future subject.

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A Question of Time in Military Knowledge Management

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Abstract

For military operations, the importance of time in knowledge and information management has been largely overlooked. Time and knowledge have mostly been conceptualised in conventional archives as either 'timeless' histories of winning strategies and tactics, or time-stamped records of past events and states.

This paper seeks to address the problem of "time" with respect to information and decision-making by proposing that there often exists a moment of temporal convergence when shared human perceptions of events in time, and the time-depreciation (shelf-life) of knowledge in the face of opposition and uncertainty, may map onto shared intentions and a future goal-state.

A model of temporal convergence has been developed to help apply information and knowledge management theory to some of the more complex military processes addressed under Network-Centric Warfare, such as shared situational awareness, collaborative planning and cooperative action. This model may be useful in describing and reasoning about knowledge requirements and prerequisites for distributed decision-making through the sharing of situational knowledge and common intentions, with practical application to the planning and execution of operations. For the designers of knowledge systems seeking to address this space, it presents a framework that could generate novel temporal approaches to data and information management.

Introduction

For military operations, the importance of time in knowledge and information management has been largely overlooked. What was known at any particular point in time, by any particular individual or team who shared events in time and place, has mostly been captured in military archives as either 'timeless' histories of winning strategies or tactics, or time-stamped records of past events and data. But, on reflection, we sense that the timing of events and the unfolding experience are just as important in arriving at winning strategies. We sense that records are meaningless without an understanding of what it was like to someone there and then.

Although we intuitively know that time is important to understanding, it is difficult to clearly elucidate why, or how it is so. But we suspect that better understanding generally leads to better decision-making and, because understanding usually takes time, good decision-making also usually takes time (c.f. "rapid cognition" when there is insufficient time for reasoned analysis [1]). Consider the matter of time and change in law when a legal appeal is lodged against a past judgement made a decade or more earlier. In order to judge rightly in the appeal, the reviewing magistrate must reconstruct the then accepted body of law and social understanding, as it existed at the time of the original judgement, and assess whether or not someone else made a just decision there and then. Given the constant change in laws, precedents and social bias/interpretation, such Knowledge Management (KM) can be a very difficult task. A similar challenge confronts military Operations Research (OR) and KM practitioners as they seek to analyse and understand why certain conclusions and subsequent decisions were made by competent military commanders in past battles and campaigns; especially when such decisions may be judged to have been wrong by future generations holding quite different legal, ethical and social frameworks to those of the past.

Taking a step in the opposite direction leads us to the future. Is it possible to apply similar KM methodologies and tools to permit would-be decision-makers and associates to think, decide and act in an artificially constructed future scenario? Using present knowledge and projections of the unfolding “now”, is it possible to more reliably lock-onto and so achieve a desired future, despite concerted opposition? At the moment this process is called “Military Art”, but with the emergence of more and more capable Artificial Intelligence (AI) with Uncertainty (UAI) for KM systems, it may soon be possible to have computerised systems help Military Art along its way.

The conceptual model employed herein was developed to help articulate and unpack time and events for the operational-level staff officer, who must think deterministically for immediate control actions but also wilfully to affect and effect higher strategic intent; specifically, to both manage the unfolding moment-by-moment situation and to effect change in the military environment so as to shape desired future outcomes.

Although not specifically in the military context of Command decision-making, researchers such as Hutchins (1996) [2], have sought to study in detail temporally dispersed cognition and team consciousness (shared situational awareness). Steiger & Steiger (2008) [3] recently sought to address tacit-to-explicit knowledge externalisation and so describe instance-based cognitive mapping as a unique process involving multiple instances of decision-making. They used inductive AI learning algorithms to help model an individual’s mental state. Others researchers such as Fildes et al. (2008) [4], in their recent review of OR methods for practical commercial forecasting have indicated the need to develop models that better link forecasting methods to the organisational context. Andersen & Nielsen (2009) [5] explored concepts of commercial advantage from both emergent (stochastic) and intentional actions and report that the two phenomena, although rarely analysed together, are complementary processes.

So, although the topic of “Time in KM” is not widely addressed within the OR literature, it seems that much of the ground-work may well be in place to move forward in practically improving Information and Computer Technology (ICT) assisted decision-making. OR practitioners engaged within the military context and the domain of problem structuring and understanding for better decision-making are encouraged to reflect on the quintessential nature of time in the problem itself and to seek a robust way of talking about it.

In this paper we seek to explore the way ordinary people—those people served by information and KM systems—go about their decisions and business with regard to their perspectives of events in time. The authors trust that the adoption of common temporal descriptions and a simple cohesive lexicon will aid researchers in discussing time in military KM and so facilitate better Command decision-making systems.

Temporal Convergence

The Lexicon

Our general philosophical approach falls within the epistemological domain of "soft determinism" or "logical positivism", as characterised by the works of David Hume [6][7] who highlighted the complementary duality of causal-determinism (herein called Stochastic Future) and free-will (herein called Intentional Future), as in two sides of a coin. The Time-Value of Knowledge—the change in the functional value of knowledge with time and the delay between its acquisition and its use—was framed by Dalmaris, Hall & Philp (2006) in addressing the concept of Temporal Convergence [8]; that is, finding a cognitive implementation path between one's present state/condition in time and a desired state/condition at a future time. We developed this concept further by more closely examining the relevant literature [9] and using binary arguments [10] related to Temporal Convergence. These latter three references frame our concept and comparatively explore how it is derived from, yet is different to, other popular conventions. The terms Stochastic Future, Intentional Future, Calendar Time, Event-Relative Time, Moment of Temporal Convergence, Temporal Convergence and Temporal Divergence are defined in our philosophical paper [11], but by way of simple summary:

A *Stochastic Future* anticipates that after some interval of time from "now", the future state of the world resembles and is derived from the present and recent past. However, as the duration between anticipation and realisation increases, the realised future will increasingly diverge from that projected. Others [5] have described this in terms of "emergent actions".

An *Intentional Future* is based on a belief in a yet to be realised goal-state. It is assumed that one has the capability, means and opportunity to influence the unfolding world to achieve this preconceived or desired state.

Calendar Time is absolute (in terms of seconds, minutes and hours) and measures inexorable progress into the future as events unfold from the past to present, and as bounded by our mental ability (as in our event-horizon) to process and understand the microcosm of cause and effect.

Event-Relative Time addresses time as relative to a key future event in an envisaged future goal-state seen as a possible configuration of the future world. It is effectively a floating (but constrained) possible future environment. Time is mostly considered relative to the unfixed time of a key defined event (e.g. lift-off minus 8 hours).

Calendar Time and *Event-Relative Time* are not necessarily connected, but at some moment in the mind of an individual, these may converge – the *Moment of Temporal Convergence*.

After the *Moment of Temporal Convergence* it becomes possible to subsume *Event-Relative Time* by *Calendar-Time,* and the *Intentional* by the *Stochastic Future*. In this context we can start to gauge the probability of success or failure of our plans; that is, we have established a cognitive pattern tied to a criterion of success where reality gives substance to hope.

Temporal Convergence describes the cognitive state of a decision maker for whom an implementation path exists between his/her present state or condition in time and a desired state/condition at a future time. He/She is confident that they can "make it so".

Temporal Divergence describes the cognitive state of a decision maker for whom an implementation path cannot be found between his/her present state or condition in time and a desired state/condition at a future time. He/She is “at a loss”.

Most “achieved” successful outcomes in a complex environment are the manifestation of a chain of positive thoughts and human actions, by way of advantage or disadvantage, towards a desired goal and in the achievement of that goal. We hold the view that the average person naturally and reasonably operates within Temporal Convergence: the mix of both Stochastic and Intentional thinking in the daily management of their affairs and in the pursuit of their ambitions. The next question then becomes, “How?”

The Model

Consider the following model for the temporal aspects of operational tactical decision-making. We contrast two perspectives on the future that can be adopted for decision-making: a view of the future as stochastic, and a view of the future as intentional.

Stochastic Future View: A well-known model which helps to describe decision-making in a Stochastic Future is taken from USAF Col. John Boyd’s (1996) presentation titled “The Essence of Winning and Losing” [12]. Boyd’s Observe - Orient - Decide - Act (OODA) loop for military decision-making is upheld today by many practitioners [13], as depicted in Figure 1.

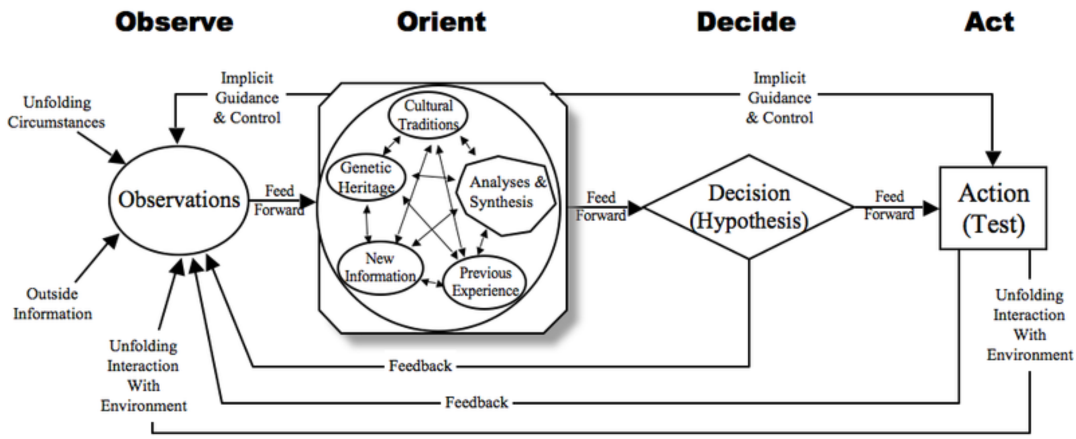

Figure 1. Boyd J. (1996)—OODA Loop.

In order to appreciate our present state “now” and to gauge what action we should take “next” (in *Calendar Time*), we may follow a series of deliberations in time. In a relentlessly unfolding world, four phases in time can be defined for a single OODA loop:

1. OBSERVE: the means by which one collects/registers information about the state of the external world. What is our current state with respect to the environment? What are the causes & effects that have brought us {where/when} to {here/now}? "t_1" is the end-time of a particular observation.

2. ORIENT: the internal processes by which observations are compared with prior knowledge and experience to update an understanding of the world. Given our state {here/now}, in what state are we likely to be {there/then}, all variables remaining the same in our present environment? "t_2" is the end-time of completed orientation and sense-making processes using observations at “t_1” to update one’s situational awareness

of the world.

3. DECIDE: the internal process by which various tentative solutions are assessed and one selected for action. Rationalise the projected change of state required, if any. "t_3" is the end-time of completed planning and decision-making processes to decide go/no go action.

4. ACT: the process by which the internally constructed solution is applied to the world. Initiate change action {here/now}, which by cause and effect is likely to bring about the required state else {where/when}. "t_4" is the end-time of effected action on the world.

5. Repeat. The then state of the world including results of action at "t_4" is observed in the next OODA loop at "t_1+i", where "i" is the overall duration or cycle time of the previous OODA loop process.

Boyd's loop is iterated in time to continuously update knowledge as depicted in Figure 2.

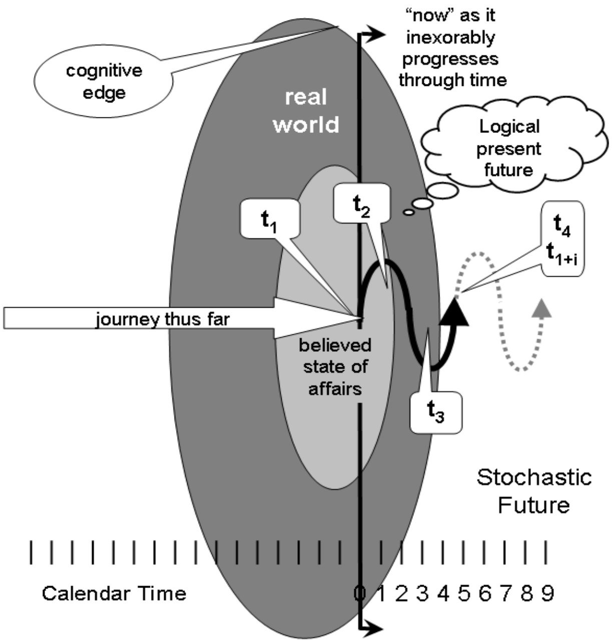

Figure 2. Projection of Boyd's OODA loop in Calendar Time within a Stochastic Future. (Heavy black wave represents the first OODA Loop and the grey wave the next reiteration.)

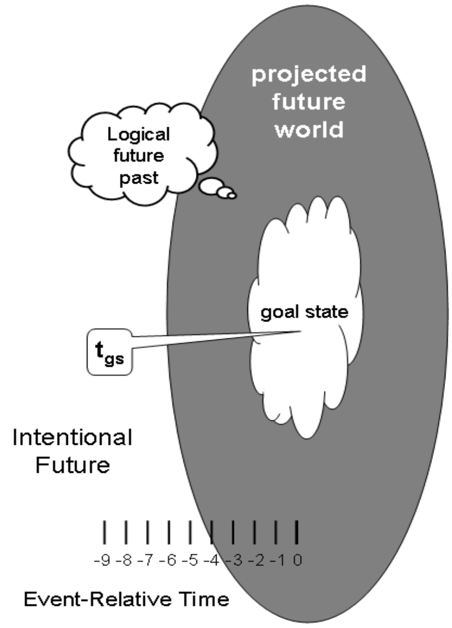

Figure 3. Projection of Event-Relative Time within an Intentional Future.

Intentional Future View: In contrast, an Intentional Future is anticipated by a belief in one's ability and opportunity to take action in shaping oneself and the environment to realise a goal-state. The time of achieving the goal-state, "t_{gs}", could be the next day, or the next decade, or unspecified; what matters is that at the present moment, the goal has yet to be realised, but "I intend to make it happen sometime." Within an Intentional Future it is possible to vividly imagine or plan exactly how you might expect events to unfold toward one's goal-state, as shown in Figure 3 above.

Once (and if) a pathway to actualisation can be envisaged and elucidated, a state of Temporal Convergence is achieved as illustrated in Figure 4.

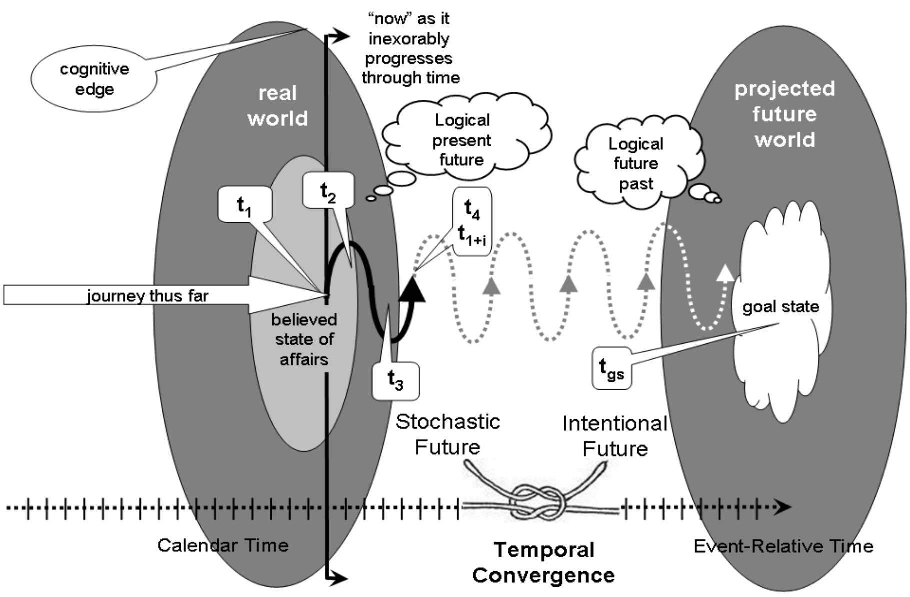

Figure 4. The fused-time of Stochastic and Intentional Futures in Temporal Convergence.

This Temporal Convergence model was developed to help articulate knowledge frameworks for military decision-making within the paradigm of Network-Centric Warfare (NCW). It seemed to the authors that two different knowledge-bases were needed to elucidate NCW distributed decision-making: (a) detailed, but short-lived, knowledge and change information about a particular tactical situation in a dynamically changing environment (Calendar Time); and (b) relevant (mostly enduring) knowledge and change updates about the cooperative and combined progress of different force-elements toward an operational-level military objective (Event-Relative Time) based on the Commander's end-game strategy in an Intentional Future. Both knowledge sets are spatially consistent but rarely the same, the temporal parameters and the scale/resolution of the two being quite different.

For example, consider a master chess-player versus a relative novice. A game would likely begin with a series of familiar moves based solely on the game-rules and the experience of the players. At some stage (sooner or later depending on the relative difference in expertise) the master chessplayer will recognise a familiar state and pattern of play which is known by experience (or other learning) to lead to victory. Once the current game-play can be projected in the mind of the master to a recognised winning end-game, a state of Temporal Convergence is in effect. The weaker opponent may have no such end-game in mind and so remains in Temporal Divergence. The point at which the master appreciates the pathway to the goal state—checkmate—is the Moment of Temporal Convergence or the "Tipping Point". The military term Culmination Point—the point in a battle/war after which an enemy is no longer able to mount or maintain a successful offensive—also captures this concept. Victory is assured, even though it may be a long time coming. Very good chess-players may sometimes concede the match long before a checkmate is declared simply because a known end-game is recognised by both as inevitable.

The chess-master may not be able to articulate how he/she recognises a pathway to victory. Rather, phrases like "I don't know exactly what it looks like, but I know it when I see it" may sometimes apply. As the chess-players are matched more closely in skill, the parameters for success may be interrelated in much more complex ways. The more complex the interrelation, the more difficult it becomes to predict the end-state of a sequence of parameter adjustments. So in mitigation to the polarity of success or failure there is also the softer position of "stance" or "poise", as being in a good (or better) position to achieve one's general goals; that is, Strategic Advantage. An expert chess player will often strive to obtain a situation of Strategic Advantage from which state a greater number and diversity of winning end-games may be possible.

Where the world includes entities in direct competition, as Boyd (1996) stresses many times, Strategic Advantage may accrue to an entity that can complete its OODA cycle and act to change the world (t_1–t_4) before competitors can likewise act to make their changes. Changing the world before others can complete their OODA processes enables faster decision-makers to impose Temporal Divergence on their slower adversaries. In any goal-orientated organisation, sustained Temporal Divergence leads to irrationality and loss of morale. Prepositioning (in the military sense of maintaining a Strategic Advantage) is understood as shaping the potential actualisation paths to achieve a (yet to be defined) goal-state.

Knowledge Management

Popular internet site <www.businessdictionary.com> defines Knowledge Management as: "Strategies and processes designed to identify, capture, structure, value, leverage, and share an organization's intellectual assets to enhance its performance and competitiveness. It is based on two critical activities: (1) capture and documentation of individual explicit and tacit knowledge, and (2) its dissemination within the organisation." Although there are many other more and less authoritative definitions for this new and emerging KM discipline, this one seems practical enough for our purposes. KM has been around as long as people have known anything (experience, skills and concepts) that they wish to share with others. But, what is new in KM is the potential to augment this basic human exchange with ICT in operational military doctrine and practice.

In military OR, KM is more closely related to Decision and Soft Systems Sciences than to records management; albeit the latter is an important part of the "Wicked Problem" [14]. In the case of a military headquarters (which often needs to enhance its performance and competitiveness in the face of strong opposition and deliberate frustration in achieving complex plans), KM is critical in sorting out "The Mess" [15]. At present KM is mostly a human function based on training, tools of trade (e.g. look-up-tables, spreadsheets and charts), military professional art and experience. In the main, the HQ Staff-system is tried, proven and works well under stress, given a competent Chief of Staff and Commanding Officer.

Supporting HQ Staff are divided into teams with distinct roles and responsibilities. The Commander directs effects and actions in the present to: (a) manage the unfolding moment-by-moment emergent situation (Stochastic Future); and (b) effect change in oneself and one's environment so as to shape future possibilities to converge on a goal-state (Intentional Future). Presently, it is through intensive dialogue that the Chief of Staff and Commander together seek to rationalise the Stochastic Future with the Intentional Future until a Moment of Temporal Convergence when the two futures intersect, or a Moment of Temporal Divergence when it becomes clear that the goal state cannot be realised in time. Before Temporal Convergence is achieved, the tasks of developing the Stochastic Future and the Intentional Future are considered separately. Using partitioned specialist processes for information filtered and derived from disparate sources, "what is" and "what is desired" are examined through the lens of the "Commander's Intent". Once Temporal Convergence is achieved, the Commander must fuse the "planned" world with the "real" world to thereafter modify, in Calendar Time, the trajectory of combined battlespace effects in the face of enemy opposition. The Operations-team has priority in bringing the plan to fruition. The Moment of Temporal Convergence, as Stochastic and Intentional Futures merge, is made evident by the switch in activity within the headquarters from deliberate planning to immediate planning.

The discipline of KM and the importance of time and epistemology in processing data, information and knowledge are yet to be rigorously addressed within the complexities of NCW, especially in the operational context. Since the key processes of decision-making are key considerations within OR, the temporal dimension of decision-making should rightly be well-considered by the OR community. We presently do not understand well, nor know how to manage, the Time-Value of Knowledge past, present or future. Yet intuitively we know that such is important in: realising the Commander's intent; sensing/directing courses of action; framing our situational awareness; and self-synchronising combatants during manoeuvres. Although we naturally seem to do it "OK", until we can articulate exactly what we are doing and understanding, it will be very hard for a machine to help us do these things better or faster. Getting ICT machines to help us is subliminally at the heart of modern KM.

Conclusion

In military Knowledge Management the matter of "time" is crucial because of the very tangible interface between present and future. In order to logically discuss this interface we have developed a lexicon of terms, beginning with Temporal Convergence, allowing the fusing of Calendar Time to Event-Relative Time when one's mind embraces "what is", "what is desired" and "a possible way to make it so". At some moment in time called the Moment of Temporal Convergence, or Culmination Point, a single new event or a cluster of interconnected events should enable an Operations-team and a Planning-team to fuse their Stochastic and Intentional Futures, as hope reveals its substance in reality. Thereafter one must wrestle with the probability of success or failure as intentions merge and buffer with experience of the unfolding present.

In this paper, we have presented a model which is useful in describing and reasoning about knowledge requirements and prerequisites for distributed decision-making through the sharing of situational knowledge and common intentions, with practical application to the planning and execution of operations. For the designers of knowledge systems seeking to address this space, it presents a real and practical challenge that could generate novel temporal approaches to data, information and KM.

Some practitioners of OR may consider their discipline to be largely centred on practice-based endeavour underpinned by scientific approaches. Usually, practice precedes theory. However when addressing the question of time, OR practitioners may discover that temporal awareness and projection is not just a matter of practical common-sense, but rather a fusion of current expectations and hypothetical cognitive futures. If OR is to help decision-makers make better decisions, it too should understand Temporal Convergence and learn to design and employ tools which can capture the Time-Value of Knowledge.

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An EOQ Model for Deteriorating Item with Demand Considering Shortage

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Abstracts

In this paper the Inventory models for deteriorating items are considered in which inventory is depleted not only by demand but also by decay; such as, direct spoilage as in fruits, vegetables and food products; physical depletion as in highly volatile liquids, drugs, pharmaceuticals, and chemicals etc., or deterioration as in electronic components. We have described different models in which depletion over time is the result of the combined effect of demand usage and decay. Economic order quantities under conditions of price-dependence of demand are considered for study.

In Model-1, the rate of deterioration is taken to be time-proportional and a power law form of the price-dependence of demand is considered. In Model-2, the rate of deterioration taken to be time-proportional and the time to deterioration is assumed to follow a two-parameter Weibull distribution. A power law form of the price-dependence of demand is considered. In Model-3, the inventory model has been formulated in stochastic environment. Here we have assumed that the time where the inventory depletes to zero (and subsequent starting of shortage) is a random point of time. In this situation the problem is no longer deterministic but stochastic in nature. If the probability distribution of the point is known, then we can calculate the expectation of the cost and minimizing the cost function we can find the optimal price. In all these models, average cost functions are derived and solved using a gradient based non-linear optimization technique (LINGO). These models are numerically illustrated and presented.

Keywords: EOQ, inventory analysis, perishable products

Introduction

Maintenance of inventories of perishable goods is a problem of major concern to a supply manager of a modern business organization. The effect of deterioration of physical goods cannot be disregarded in inventory systems because almost all the physical goods deteriorate over time. Deterioration is defined as decay, damage or spoilage. For some items such as steel, hardware, toys, glassware, etc. the process of deterioration is so slow that there is practically no need of considering deterioration in the determination of the economic lot size. On the other hand, food products, vegetables, fruits, photographic films, drugs, Pharmaceuticals**,** chemicals, electronic goods, radioactive substances and volatile liquids are examples of some items in which sufficient amount of deterioration takes place during the normal storage period and hence it should be taken into account in inventory modeling.

Researchers first started inventory modeling with a constant rate of deterioration. Aggarwal [1], Ghare *et al* [16], Whitin [29], etc. discussed inventory models with a constant demand, constant deterioration and instantaneous replenishment.

Dave *et al* [13] discussed an inventory model with time-proportional demand and instantaneous replenishment. Dave [14] extended the model of Dave *et al* [13] to include shortages in inventory. Bahari-Kashani [2] presented a heuristic model with time-proportional demand. These models also deal with a constant rate of deterioration.

Another class of models was developed taking the deterioration rate to he time-dependent, "Covert *et al* [3] worked with a two-parameter Weibull distribution deterioration. Philip [1974] extended this model to the case of a three parameter Weibull distribution. Deb *et al* [15] worked with a time-proportional deterioration and a finite rate of production. In all these models the demand rate was assumed to be uniform over time.

Various types of inventory models with a constant deterioration rate and a linearly time-varying demand rate were discussed by Chakrabarti *et al* [4],Chakrabarti[6], Chung *et al* [9], Giri *et al* [5], Goswami *et al* [20], Hariga [21], etc.

Jalan *et al* [24] extended the model of Covert *et al* [3] to include a linear time-dependent demand rate and shortages while Chakrabarti *et al*[5] made similar extensions to the model of Philip [1974].

Inventory models with a fixed deterioration rate and an exponentially time-varying demand were studied by Hariga *et al* [22], Wee[30] etc. Hariga [21] discussed some lot sizing policies by considering the general class of increasing and decreasing demand functions which vary in log-concave fashion with time and they assume the deterioration rate to be fixed.

Chakrabarti *et al* [7] discussed an order inventory model with variable rate of deteriorating and alternating replenishing rates considering shortage. Inventory models for deteriorating items with quadratic time varying demand and shortages in all cycles was studied by Chakrabarti *et al* [8].

Giri *et al* [17] developed an inventory model with a time-varying demand taking the deterioration rate, holding cost and ordering cost to be linearly time-dependent. In another paper, Giri *et al* [18] discussed two heuristic models in which the demand rate, deterioration rate, ordering cost, holding cost and shortage cost were all assumed to be linearly increasing functions of time. Giri, B.C.[19] developed a note on a lot sizing heuristic for deteriorating items with time-varying demands and shortages. Lin *et al* [27] developed an EOQ model with time-varying demand, partial back ordering and a linearly time-dependent deterioration rate.

While there is an abundance of inventory models with time-varying demand, models with price-dependent demand are surprisingly very few in spite of the fact that the selling price of an item can affect significantly the demand of an item. Kunreuther *et al* [25] discussed the joint pricing and ordering policy for non-seasonal products. Kunreuther *et al* [26] then extended the model to seasonal products. Cohen [10] formulated the joint pricing and ordering policy for an item deteriorating at a constant rate. Cheng [11] discussed a similar problem for a non-deteriorating item with limitations on storage space and inventory investment. Chen *et al* [12] revised the above model. All the above models assumed a linear form of the price-dependent demand rate. Hwang *et al* [23] considered the price-dependent demand rate to be non-linear and the deterioration rate to be constant. S.Mukhopadhyay *et al* [28] considered joint pricing and ordering policy for a deteriorating inventory.

In Model-1, we develop an inventory model taking a time-proportional deterioration rate and a power law form of the price-dependent demand rate. Here we have also considered shortage. It is seen that deterioration of a product increases with time. Hence we have taken the deterioration rate-to be time-proportional. The price-dependence of the demand function is taken to be nonlinear. The price-dependent demand is that when price of a commodity increases, demand decreases and when price of a commodity decreases, demand increases. The condition for the convexity of the cost function is established. A numerical example is discussed to illustrate the procedure of solving the model.

In Model-2, we develop an inventory model taking price-dependent demand pattern in a power law form and a two-parameter Weibull distribution to represent the deterioration. We have also considered shortage. The two parameter Weibull distribution is appropriate for an item with decreasing rate of deterioration only if the initial rate of deterioration is extremely high. Similarly, this distribution can also be used for an item with increasing rate of deterioration only if the initial rate is approximately zero.

In Model-3, we extend the inventory model of model-1 in stochastic environment. Here we have assumed that the point, where the inventory depletes to zero and shortage starts, is a random point. In this situation the problem is no longer deterministic but stochastic in nature. If the probability distribution of the point is known, then we can calculate the expectation of the cost and minimizing the cost function we can find the optimal price.

Model-I

Assumptions and Notations

We present here a continuous review, deterministic inventory model with time-proportional decay of the item in stock under the following assumptions and notations:

- ‘c’ is the constant purchase cost per unit item
- ‘K’ is the ordering cost per cycle
- ‘h’ stands for the inventory holding cost per unit per unit time
- ‘s’ is the shortage cost per unit per unit time
- ‘p’ is the selling price per unit item
- d(p) represents the price-dependent demand rate
- $\phi(t) = \theta t$, $0 < \theta < 1$, is the time-proportional decay rate of the stock. Since $\theta > 0$, ($d\phi(t)/dt$) $= \theta > 0$. Hence the decay-rate increases with time at a rate θ.
- T is the duration of each cycle
- Shortages are allowed

The Problem

The inventory system developed is depicted by the following figure-1.

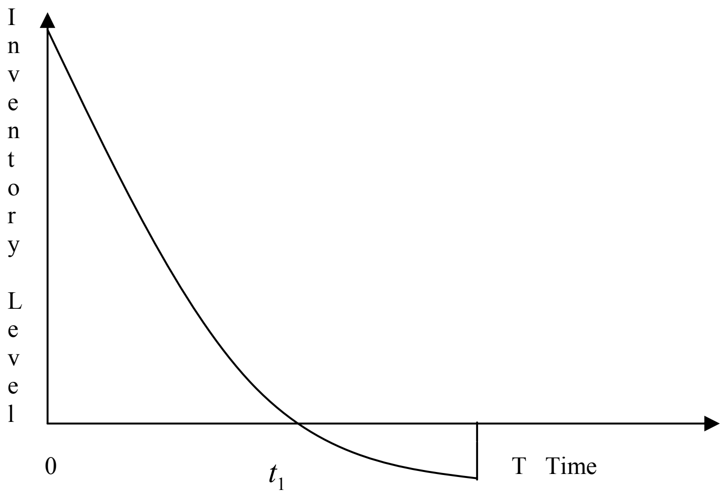

Figure-1: Inventory System

Let I(t) be the instantaneous inventory level at any time t($0\le t\le T$). Here the demand rate *d(p)* is assumed to be positive having a negative derivative in its entire domain. The inventory is depleting due to simultaneous occurrence of demand and deterioration. The inventory level drops to zero at t=t_1. Then shortage starts and accumulates up to T.

The differential equations for the instantaneous state of $I_1(t)$ over $(0,t_1)$ is given by

$$\frac{dI_1(t)}{dt}+\phi(t)I_1(t)=-d(p),\ 0\le t\le t_1$$

i.e. $\dfrac{dI_1(t)}{dt}+\theta tI_1(t)=-d(p),\ 0\le t\le t_1$ (1)

with the boundary condition $I_1(t)$=I(0) at t=0 (2)

and instantaneous state of shortage $I_2(t)$ over (t_1,T) is given by

$$\frac{d}{dt}I_2(t)=-d(p),\ t_1\le t\le T \quad (3)$$

with the boundary condition $I_2(t)$=0 at t=t_1 (4)

Let us suppose that the market price maintains a specific level, say p=p_1, during the time interval $[t_1,t_2]$.. Then there is a constant demand rate $d(p_1)$ $\forall t\in[t_1,t_2]$. If now the price changes to p=p_2 (say) during a subsequent time interval $[t_2,t_3]$ we have a constant demand rate $d(p_2)$ $\forall t\in[t_2,t_3]$. Thus the demand rate directly depends on p; rather than on *t.*

The solution of Eq. (1) is

$$I_1(t)=-d(p)(t+\frac{\theta t^3}{6})e^{\frac{-\theta t^2}{2}}+I(0)e^{\frac{-\theta t^2}{2}},0\le t\le t_1 \quad (5)$$

where I (0) is the initial stock and neglecting powers of 'θ' higher than the first as θ is small.

The solution of Eq. (2) is

$$I_2(t) = d(p)(t_1 - t), t_1 \le t \le T \tag{6}$$

Again at t=t_1, $I_1(t_1) = 0$. Therefore from (5),

$$I(0) = d(p)(t_1 + \frac{\theta t_1^3}{6})$$

Therefore

$$I_1(t) = d(p)\{(t_1 - t) + \frac{\theta}{6}(t_1^3 - t^3)\}e^{-\theta t^2/2}, 0 \le t \le t_1 \tag{7}$$

Let D(t) be the deterioration in the interval $[0, t_1]$ which is given as follows:

$$D(t_1) = d(p)\frac{\theta t_1^3}{6} \tag{8}$$

Since the only loss to the inventory system is due to either decay or demand, the quantity ordered in each cycle may be put in the following form

$$q = D(t_1) + \int_0^T d(p)dt$$

$$= d(p)[T + \frac{\theta t_1^3}{6}] \tag{9}$$

The total cost per cycle is as follows:

$$\widehat{C}(T, t_1, p) = K + c\, d(p)[T + \frac{\theta t_1^3}{6}] + hd(p)[\frac{t_1^2}{2} + \frac{\theta}{12}t_1^4] - sd(p)[t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}] \tag{10}$$

Hence the average system cost is

$$C(T, t_1, p) = \frac{\widehat{C}(T, t_1, p)}{T} = \frac{K}{T} + \frac{c}{T} d(p)[T + \frac{\theta t_1^3}{6}] + \frac{h}{T} d(p)[\frac{t_1^2}{2} + \frac{\theta}{12}t_1^4] - \frac{s}{T} d(p)[t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}] \tag{11}$$

Also the quantity ordered per unit time is (using (9) and $\phi = \theta$t)

$$\frac{q}{T} = d(p)[1 + \frac{\theta}{6}t_1^3] = d(p)[1 + \frac{\phi}{6}\frac{t_1^2}{T}]$$

The behavior of the quantity ordered per unit time with respect to changes in the deterioration rate is determined by $\frac{\partial}{\partial \phi}(\frac{q}{T}) = d(p)\frac{t_1^2}{6T} > 0$.

Also the change in the quantity ordered per unit time due to change in price is determined by

$$\frac{\partial}{\partial p}(\frac{q}{T}) = d'(p)[1 + \frac{\phi t_1^2}{6T}] \le 0, \text{ since } d'(p) \le 0 \tag{12}$$

We thus see that the optimal quantity ordered increases with increase in the deterioration rate ϕ and decreases with increase in selling price of the item.

Our problem is now to determine the values of T, t_1 and p which minimize C(T, t_1 ,p). For this purpose, the functional form of d(p) must be known explicitly. However, the functional form of d(p) must be prescribed to proceed further.

Solution of the problem

Taking d(p)=a p^{-b}, a>0,b>0 (13)

where a(>0) is a scale parameter and b(>0) is a shape parameter and $t_1 = \gamma T$ where 0< γ <1, we have

$$C(T,p)=\frac{K}{T}+\frac{c}{T}a\,p^{-b}\,[T+\frac{\theta\gamma^3T^3}{6}]+\frac{h}{T}a\,p^{-b}\,[\frac{\gamma^2T^2}{2}+\frac{\theta}{12}\gamma^4T^4]-\frac{s}{T}a\,p^{-b}\,[\gamma T^2-\frac{T^2}{2}-\frac{\gamma^2T^2}{2}]$$

$$=\frac{K}{T}+c\;a\,p^{-b}\;[1+\frac{\theta\gamma^3T^2}{6}]+h\;a\,p^{-b}\;[\frac{\gamma^2T}{2}+\frac{\theta}{12}\gamma^4T^3]-s\;a\,p^{-b}\;[\gamma T-\frac{T}{2}-\frac{\gamma^2T}{2}]$$

(14)

The necessary conditions for minimization of C(T,p) are

$$\frac{\partial C\,(T,p)}{\partial T}=0=\frac{\partial\,C(T,p)}{\partial p}$$

Now, $\frac{\partial C\,(T,p)}{\partial T}=0$ gives

$$-\frac{K}{T^2}+ca\,p^{-b}\,[\frac{\theta\gamma^3T}{3}]+ha\,p^{-b}\,[\frac{\gamma^2}{2}+\frac{\theta}{4}\gamma^4T^2]-s\,a\,p^{-b}\,[\gamma-\frac{1}{2}-\frac{\gamma^2}{2}]=0 \qquad (15)$$

and $\frac{\partial C\,(T,p)}{\partial p}=0$ gives

$$-c\,ab\,p^{-b-1}\,[1+\frac{\theta\gamma^3T^2}{6}]-h\,ab\,p^{-b-1}\,[\frac{\gamma^2T}{2}+\frac{\theta}{12}\gamma^4T^3]+s\,ab\,p^{-b-1}\,[\gamma T-\frac{T}{2}-\frac{\gamma^2T}{2}]=0 \qquad (16)$$

The sufficient condition for minimization of C(T,p) requires that it must be a convex function for T>0, p>0.

Now the function will C(T, p) be convex if

$$\begin{vmatrix}\frac{\partial^2C(T,p)}{\partial T^2} & \frac{\partial^2C(T,p)}{\partial T\partial p}\\ \frac{\partial^2C(T,p)}{\partial p\partial T} & \frac{\partial^2C(T,p)}{\partial p^2}\end{vmatrix}>0 \qquad (17)$$

Thus we get

$\{\frac{2K}{T^3}$ -ca p^{-b} $[\frac{\theta\gamma^3}{3}]$-ha p^{-b} $[\frac{\theta}{2}\gamma^4T]\}$ {-cab(-b-1) p^{-b-2} $[1+\frac{\theta\gamma^3T^2}{6}]$-hab(-b-

1) $p^{-b-2}[\frac{\gamma^2 T}{2}+\frac{\theta}{12}\gamma^4 T^3]$+s ab(-b-1) p^{-b-2} $[\gamma T-\frac{T}{2}-\frac{\gamma^2 T}{2}]\}$

$$>\{\text{-cab } p^{-b-1}[\frac{\theta\gamma^3 T}{3}]\text{-hab } p^{-b-1}[\frac{\gamma^2}{2}+\frac{\theta}{4}\gamma^4 T^2]\text{+sab } p^{-b-1}[\gamma-\frac{1}{2}-\frac{\gamma^2}{2}]\}^2 \quad (18)$$

Solving (15) and (16) simultaneously, therefore, implies minimizing cost. Eqs. (15) and (16) can be solved simultaneously by some computer oriented numerical technique to obtain optimal price p* and optimal cycle time T*. For this, we have to prescribe the values of the parameters a,b,h,K,θ,γ , c and s. As an illustration, we take up a numerical example. It may be noted from (16) that we must take b>1 to ensure real values of p and T. This implies that demand must be elastic to ensure minimization of cost.

Numerical Example

To illustrate the model the following example is considered.
Let $a=16\times 10^7$, $b=3.21$, $h=1.5$, $K=250$, $c=40, s=50, \gamma=o.6, \theta=0.3$ in appropriate units.

Equations (15) and (16) are now solved simultaneously for the above parameter values using a gradient based non-linear optimization technique (LINGO) and get the results shown in Table-1. It is verified that all the solutions in Table-1 for different values of θ, satisfy the convexity condition for C(T, p). The following points are noted from Table-1:

(i) The optimal price p* increases as the deterioration rate increases.
(ii) All of T*, q*, C* decreases with increase in the decay rate.
(iii) All of p*,T*,q* and C* being sensitive to changes in the value of the parameter θ, it must be estimated with sufficient care in the real market situations.

Table-1: Optimal solution for various values of 'θ'.

θ	Optimal price p*	Optimal cycle length T*	Optimal order rate q*	Optimal cost C*
0.1	69.28826	0.5348006	105.7405	8827.339
0.2	69.38081	0.5270395	103.8610	8799.639
0.3	69.46037	0.5197437	102.1406	8776.854
0.4	69.53044	0.5128813	100.5535	8757.598
0.5	69.59350	0.5064191	99.07973	8740.901
0.6	69.65114	0.5003223	97.70395	8726.147

Sensitivity Analysis

We now study sensitivity of the optimal solution to changes in the values of the different parameters associated with the model. The results are shown in Table-2. The following points are noted from Table-2:

i) p*, T*, C* and q* are very sensitive to changes in the value of the shape parameter b.
ii) T* is moderately sensitive to changes in the value of the parameter c while it has low sensitivity to changes in a, h and K.
iii) p* is insensitive to changers in h and K while it is sensitive to changes in c and a.

iv) C* is highly sensitive to changes in a and c while it is slightly sensitive to changes in h and K.
v) q* is slightly sensitive to changes in h and K while moderately sensitive to changes in a and c. It is noted that q* increases as a increases.

From the above analysis, it is clear that estimation of the parameters b and c should be done very carefully.

Table-2: Percentage changes in the optimal solutions corresponding to changes in the parameter values for the case θ = 0.3

Changing Parameter	Change (%)	Change in T*	Change in p*	Change in q*	Change in C*
a	50	0.4248009	69.12001	127.0908	13097.33
	25	0.465081	69.25893	115.251	10940.97
	-25	0.8449949	87.56174	59.50347	3364.97
	-50	1.608252	119.9441	28.04701	969.875
b	50	9.241268	69.46037	3.852261	53.11535
	25	47.12504	172.5523	3.591924	47.12504
	-25	0.1173189	80.29619	487.528	170435.6
	-50	0.0288865	124.1048	2015.062	2807572
h	50	0.5151356	69.6757	100.229	8708.477
	25	0.5173719	69.5645	101.1838	8743.899
	-25	0.5220941	69.34923	103.134	8812.652
	-50	0.534424	69.23115	104.1652	8851.354
K	50	0.6362307	69.90492	122.6759	8823.864
	25	0.5808731	69.68959	113.0348	8802.856
	-25	0.4504253	69.2103	89.48393	8744.238
	-50	0.3682001	68.92553	74.06947	8702.006
c	50	0.9435376	105.6423	48.58651	3560.296
	25	0.7257727	87.38809	68.44197	5350.339
	-25	0.3330115	51.80354	167.4962	16554.4
	-50	0.1751488	34.36899	328.5305	40344.21

Model-2

Assumptions and notation

Here all the assumptions and notations are same as in Model-1. The only change here is the rate of deterioration is taken to be two parameter Weibull distribution i.e., the instantaneous deterioration rate $\phi(t)$ at any time $t \geq 0$ follows a two-parameter Weibull distribution given by $\phi(t) = \alpha\beta t^{\beta-1}$, where α >0 is a scale parameter and $\beta > 0$ is a shape parameter. The implication of the Weibull rate (two parameter) is that the items in inventory start deteriorating at the instant when they are received into inventory. The rate of deterioration-time relationship is shown in figure-2.

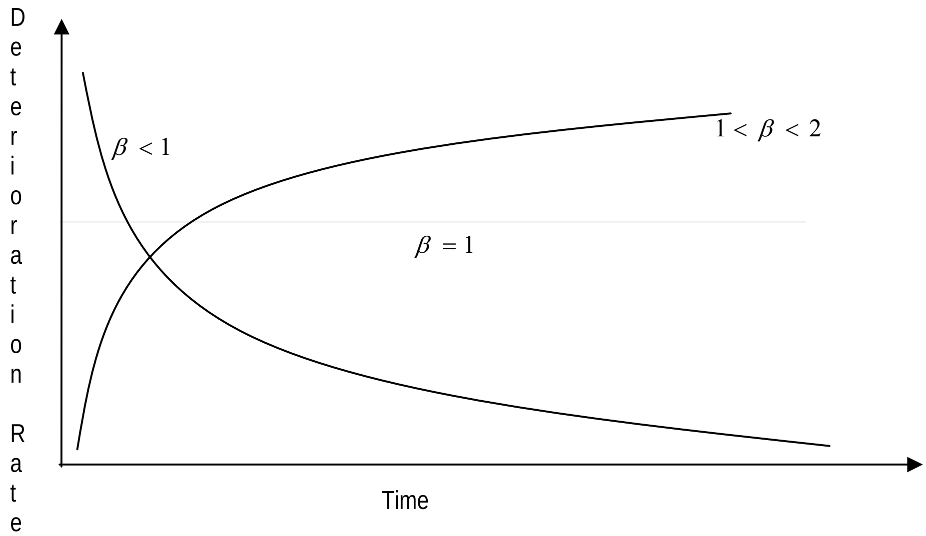

Figure-2: Rate of deterioration-time relationship

When $\beta=1$, ϕ (t) = α (a constant) which is the case of constant decay.
When $\beta<1$, rate of deterioration is decreasing with time.
When $\beta>1$, rate of deterioration is increasing with time.

In this model we have developed average cost function similar to model-1 and developed convexity condition of the cost function. A numerical example has been discussed to illustrate the solution procedure and the sensitivity of the solution is then carried out.

The Problem

The inventory system developed is depicted by figure-1 in model-1
Let I(t) be the instantaneous inventory level at any time t($0\le t\le T$). Here the demand rate *d(p)* is assumed to be positive having a negative derivative in its entire domain.

The inventory is depleting due to simultaneous occurrence of demand and deterioration. The inventory level drops to zero at t=t_1. Then shortage starts and accumulates up to T. The differential equations for the instantaneous state of $I_1(t)$ over $(0,t_1)$ is given by

$$\frac{dI_1(t)}{dt}+\phi(t)I_1(t)=-d(p),\ 0\le t\le t_1$$

$$\frac{dI_1(t)}{dt}+\alpha\beta t^{\beta+1}I_1(t)=-d(p),\quad 0\le t\le t_1 \tag{19}$$

with the boundary condition $I_1(t)$=I(0) at t=0 (20)

and instantaneous state of shortage $I_2(t)$ over (t_1,T) is given by

$$\frac{d}{dt}I_2(t)=-d(p),\ t_1\le t\le T \tag{21}$$

with the boundary condition $I_2(t)$=0 at t=t_1 (22)

The solution of Eq. (1) is

$$I_1(t) = d(p)\{(t_1 - t) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1})\}e^{-\alpha t^\beta}, 0 \le t \le t_1 \quad (23)$$

The solution of Eq. (2) is

$$I_2(t) = d(p)(t_1 - t), t_1 \le t \le T \quad (24)$$

Let D(t) be the deterioration in the interval $[0, t_1]$ which is given as follows

$$D(t_1) = d(p)\frac{\alpha t_1^{\beta+1}}{\beta+1} \quad (25)$$

Since the only loss to the inventory system is due to either decay or demand, the quantity ordered in each cycle may be put in the following form

$$q=d(p)[T+\frac{\alpha t_1^{\beta+1}}{\beta+1}] \quad (26)$$

The total cost per cycle is

$$\hat{C}(T, t_1, p)=K+cd(p)[T+\frac{\alpha t_1^{\beta+1}}{\beta+1}]+hd(p)[\frac{t_1^2}{2} + \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}]-sd(p)[t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}] \quad (27)$$

Hence the average system cost is

$$C(T, t_1, p)=\frac{K}{T}+\frac{c}{T}d(p)[T+\frac{\alpha t_1^{\beta+1}}{\beta+1}]+\frac{h}{T}d(p)[\frac{t_1^2}{2}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}]$$

$$-\frac{s}{T}d(p)[t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}] \quad (28)$$

Our problem is now to determine the values of T, t_1 and p which minimize C(T, t_1, p). However, the functional form of d(p) must be prescribed to proceed further.

<u>Solution of the Problem</u>

Taking d(p)=a p^{-b}, a>0,b>0 (29)

where a(>0) is a scale parameter and b(>0) is a shape parameter, and $t_1 = \gamma T$ where 0<γ<1, we have

$$C(T, p)= \frac{K}{T}+c\,a\,p^{-b}[1+\frac{\alpha\gamma^{\beta+1}T^\beta}{\beta+1}]+h\,a\,p^{-b}[\frac{\gamma^2 T}{2}+\frac{\alpha\beta\gamma^{\beta+2}T^{\beta+1}}{(\beta+1)(\beta+2)}]$$

$$-sa\,p^{-b}[\gamma T - \frac{T}{2} - \frac{\gamma^2 T}{2}] \quad (30)$$

Now $\frac{\partial C(T,p)}{\partial T} = 0$ gives

$$-\frac{K}{T^2}+cap^{-b}[\frac{\alpha\beta\gamma^{\beta+1}T^{\beta-1}}{\beta+1}]+hap^{-b}[\frac{\gamma^2}{2}+\frac{\alpha\beta\gamma^{\beta+2}T^\beta}{(\beta+2)}]-sap^{-b}[\gamma - \frac{1}{2} - \frac{\gamma^2}{2}]=0 \quad (31)$$

Also $\frac{\partial C(T,p)}{\partial p}=0$ gives

$$-\text{cab}\, p^{-b-1}[1+\frac{\alpha\gamma^{\beta+1}T^{\beta}}{\beta+1}]-\text{hab}\, p^{-b-1}\,[\frac{\gamma^2 T}{2}+\frac{\alpha\beta\gamma^{\beta+2}T^{\beta+1}}{(\beta+1)(\beta+2)}]+\text{sab}\, p^{-b-1}\,[\gamma T-\frac{T}{2}-\frac{\gamma^2 T}{2}]=0 \quad (32)$$

Equations (31) and (32) can now be solved simultaneously by using a gradient based non-linear optimization technique (LINGO),to find the optimal cycle length T* and the optimal selling price p* when the values of the parameters a, b, c, h, K, α,β,γ and s are prescribed. To illustrate the procedure, we take up the following numerical example. For convexity of C(T,p), we must have

$$\begin{vmatrix} \frac{\partial^2 C(T,p)}{\partial T^2} & \frac{\partial^2 C(T,p)}{\partial T\partial p} \\ \frac{\partial^2 C(T,p)}{\partial p\partial T} & \frac{\partial^2 C(T,p)}{\partial p^2} \end{vmatrix} > 0$$

implies

$$[\frac{2K}{T^3}+\text{cap}^{-b}\{\frac{\alpha\beta(\beta-1)\gamma^{\beta+1}T^{\beta-2}}{\beta+1}\}+\text{hap}^{-b}\{\frac{\alpha\beta^2\gamma^{\beta+2}T^{\beta-1}}{(\beta+2)}\}]*[\text{cab(b+1)}\, p^{-b-2}$$
$$\{1+\frac{\alpha\gamma^{\beta+1}T^{\beta}}{\beta+1}\}+\text{h ab(b+1)}\, p^{-b-2}\ \{\}$$
$$-\text{sab(b+1)}\, p^{-b-2}\{\gamma T-\frac{T}{2}-\frac{\gamma^2 T}{2}\}]>[-\text{cab}\{\frac{\alpha\beta\gamma^{\beta+1}T^{\beta-1}}{\beta+1}\ p^{-b-1}\}-\text{hab}\, p^{-b-1}$$
$$\{\frac{\gamma^2}{2}+\frac{\alpha\beta\gamma^{\beta+2}T^{\beta}}{(\beta+2)}\}+\text{sab}\, p^{-b-1}\ \{\gamma-\frac{1}{2}-\frac{\gamma^2}{2}\}]^2 \quad (33)$$

Numerical Example

To illustrate the model the following example is considered.
Let a=16×10^7, b=3.21, c=40, h=1.5, K=250, s=50, $\alpha=0.02, \beta=1.5, \gamma=0.6$ in appropriate units. Equations (31) and (32) are now solved simultaneously for the above parameter values using a gradient based non-linear optimization technique (LINGO), which yields the local optimal solution:

Optimal cycle time (T*) =0.5793512 unit, optimal price (p*) =72.56134 unit, optimal cost [C*(T, p)] =7672.253 unit, optimal quantity ordered (q*) =98.76997unit.

It is numerically verified that this solution satisfies the convexity condition for C (T, p).

Sensitivity Analysis

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by changes or errors in its input parameter values. In this model, we study the sensitivity of the optimal cycle length T*, price p*, cost C* with respect to the changes in the values of the parameters a, b, c, h, K, α,β . The results are shown in table-3. Careful study of table-3 reveals the following facts:

(i) The model is moderately sensitive to changes in parameters a and c.
(ii) The model is slightly sensitive to changes in the parameters K, α,β
(iii) The model is almost insensitive to changes in the parameter h.
(iv) The optimal outputs T*, C* and p* are highly sensitive to changes in the parameter b.

Table-3: Sensitivity analysis

Parameter	Change	T*	p*	C*
a	-50%	0.4684128	72.06377	11510.82
	-20%	0.5157896	72.32120	9572.668
	+20%	0.6733675	72.90068	5772.882
	+50%	0.8401869	73.82121	3815.452
b	-50%	426483.5	59511.04	0.000820682
	-20%	15.39282	163.1294	40.03532
	+20%	0.1123649	77.05114	187994.1
	+50%	0.0263893	110.82962	3364788.0
c	-50%	0.9210006	97.48950	4499.081
	-20%	0.7085670	82.48950	6344.033
	+20%	0.4621708	62.87859	9170.784
	+50%	0.3083254	48.75151	13829.06
h	-50%	0.5719094	72.66403	7652.671
	-20%	0.5762348	72.66403	7646.088
	+20%	0.5825542	72.45882	7698.473
	+50%	0.5852135	72.30740	7740.656
K	-50%	0.7189721	73.22282	7655.582
	-20%	0.6511887	72.8353	7686.468
	+20%	0.4989407	72.27031	7649.930
	+50%	0.4034117	71.78012	7671.196
α	-50%	0.5795055	72.83919	7587.301
	-20%	0.5791366	72.67753	7636.840
	+20%	0.5794923	72.43874	7709.967
	+50%	0.5787157	72.24394	7771.439
β	-50%	0.5729831	71.80545	7915.183
	-20%	0.5760224	72.15618	7800.709
	+20%	0.5859399	73.24236	7463.191
	+50%	0.5876647	73.51504	7388.859

Model-3

Assumptions and Notation

Here all the assumptions and notations are same as in Model-1. In this section the above inventory model in section-I has been formulated in stochastic environment. We now minimize the average cost per unit time C(T, t_1,p) under the situation

(i) t_1 is a random point of time.

The Problem and Solution

In model-1, the average cost function is

$$C(T,t_1,p) = \frac{K}{T}+\frac{c}{T} d(p)[T+\frac{\theta t_1^3}{6}] + \frac{h}{T} d(p)[\frac{t_1^2}{2}+\frac{\theta}{12}t_1^4] - \frac{s}{T} d(p)[t_1 T-\frac{T^2}{2}-\frac{t_1^2}{2}] \quad \text{\{from eqn.(11)\}} \quad (34)$$

Let us assume that T= γt_1 (γ>1). Therefore, the average cost function (AVC) becomes

$$AVC=C(t_1,p)=\frac{K}{\gamma t_1}+\frac{c}{\gamma t_1} d(p)[\gamma t_1+\frac{\theta t_1^3}{6}]+\frac{h}{\gamma t_1} d(p)[\frac{t_1^2}{2}+\frac{\theta t_1^4}{12}]$$
$$-\frac{s}{\gamma t_1} d(p)[\gamma t_1^2-\frac{\gamma^2 t_1^2}{2}-\frac{t_1^2}{2}] \quad (35)$$

We now take d(p)=ap^{-b}, a>0,b>0, where a(>0) is a scale parameter and b(>0) is a shape parameter. Therefore

$$AVC=\frac{K}{\gamma t_1}+\frac{c}{\gamma} a p^{-b} [\gamma+\frac{\theta t_1^2}{6}]+\frac{h}{\gamma} a p^{-b} [\frac{t_1}{2}+\frac{\theta t_1^3}{12}]-\frac{s}{\gamma} a p^{-b} [\gamma t_1-\frac{\gamma^2 t_1}{2}-\frac{t_1}{2}] \quad (36)$$

In this case, the average cost function C(t_1,p) is a random variable with respect to t_1.
So the expected average cost per unit time is

$$E(AVC)=\xi(p) = \frac{K}{\gamma}E(\frac{1}{t_1})+\frac{c}{\gamma} a p^{-b} [\gamma+ E(\frac{\theta t_1^2}{6})]+\frac{h}{\gamma} a p^{-b} [E(\frac{t_1}{2})+E(\frac{\theta t_1^3}{12})]$$
$$-\frac{s}{\gamma} a p^{-b} [(\gamma-\frac{\gamma^2}{2}-\frac{1}{2})E(t_1)] \quad (37)$$

Now, assume that the distribution function of t_1 to be Erlang distribution. Then its probability density function f(x) is given by

$$f(x)=\frac{\lambda^r x^{r-1}e^{-\lambda x}}{\Gamma(r)}, \qquad 0<x<\infty$$
$$=0, \qquad \text{otherwise.}$$

where λ and r are positive constants (given parameters) and $\Gamma(r)$ is a gamma function, defined by

$$\Gamma(r)=\int_0^{\infty} x^{r-1}e^{-x}dx$$

Since gamma function is tabulated, the value of $\Gamma(r)$ being a constant can be easily obtained for the given value of r. If r has an integer value, then $\Gamma(r)$=(r-1)! Therefore the expected average cost per unit time is as follows:

$$\text{E(AVC)}= \xi(p)=\frac{K}{\gamma}\frac{\lambda}{(r-1)}+\frac{c}{\gamma}\ a\, p^{-b}\ [\gamma+\ \frac{\theta}{6}\frac{r(r+1)}{\lambda^2})]+\frac{h}{\gamma}\ a\, p^{-b}\ [\frac{r}{2\lambda}+\frac{\theta}{2}\frac{r(r+1)(r+2)}{\lambda^3}]$$
$$-\frac{s}{\gamma}\ a\, p^{-b}\ [(\gamma-\frac{\gamma^2}{2}-\frac{1}{2})\frac{r}{\lambda}] \qquad (38)$$

The necessary condition for $\xi(p)$ to be minimum is that $\frac{\partial\xi(p)}{\partial p}=0$

This gives

$$\text{p}^{-b-1}[\frac{c}{\gamma}\{\gamma+\ \frac{\theta}{6}\frac{r(r+1)}{\lambda^2}\}+\frac{h}{\gamma}\{\frac{r}{2\lambda}+\frac{\theta}{2}\frac{r(r+1)(r+2)}{\lambda^3}\}-\frac{s}{\gamma}\{(\gamma-\frac{\gamma^2}{2}-\frac{1}{2})\frac{r}{\lambda}\}]=0 \qquad (39)$$

The sufficient condition for $\xi(p)$ to be minimum is that $\frac{\partial^2\xi(p)}{\partial p^2}>0$

<u>Numerical Example</u>

To illustrate the model the following example is considered.

Let $a=16\times10^7$, $b=3.21$, $h=1.5$, $K=250$, $c=40, s=50, \gamma=2, \theta=0.3, \lambda=10, r=5$ in appropriate units. An equation(39) is now solved for the above parameter values using a gradient based non-linear optimization technique (LINGO), which yields the Global optimal solution:

Optimal cost= $\xi(p)$ =6821.541, Optimal price= p*=43.348.

It is numerically verified that this solution satisfies the convexity condition for $\xi(p)$.

Conclusion

In the inventory literature, models with price-dependent dependent as well as deterioration are not very common. Many researchers discussed an inventory policy by taking a linear price-dependence demand and constant rate of deterioration. A linear price-dependence demand is rarely encountered in the real market. The constant rate of deterioration is also unrealistic. In model-1, we have taken the deterioration rate to be time-proportional and the price-dependence of demand to be non-linear. In model-2, the rate of deterioration is taken to be time- proportional and the time to deterioration is assumed to follow a two-parameter Weibull distribution and the price-dependence of demand to be non-linear. In model-3, the inventory model has been formulated in stochastic environment. In all these models shortages are also considered. All the models are solved using a gradient based non-linear optimization technique (LINGO), and are illustrated with numerical example.

The above models can be extended to Fuzzy environments. Fuzzy random variables (also referred to often as random fuzzy sets) were introduced as a valuable and well-formalized model to deal with probabilistic and statistical problems involving fuzzy data when this data are supplied by an existing fuzzy-valued quantification process. Formally, Fuzzy random variables have been presented as an extension of random sets (and, hence, of random variables and vectors), but in practice they correspond to an intermediate level of precision between either real or vectorial and set-valued random elements. They serve to handle several linguistic and imprecisely valued variables.

Here if deterioration is taken as imprecise in nature, then the above model becomes fuzzy in nature. In this situation by deriving the corresponding crisp model by de-fuzzyfication the corresponding fuzzy variable we can find the optimal order quantity, optimal price and optimal cost.

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