A Markov-based Method for Military Analysis

A. H. Pincombe[a] and B. M. Pincombe[b]

Abstract

In the operations analysis of military engagements, an important task is the establishment of connections between low-level (detailed) variables and high-level outcomes, to estimate the values of various capabilities in defined scenarios. We define a family of Markov decision processes to provide bounds for these values, using standard military analysis methodology to simplify the problem and to reduce the number of Markov states that are required. The parameters that define the Markov transition matrix can be represented as analytical functions of lower-order variables, while the time dependence of the state probabilities can be obtained, from the Markov analysis, as a set of functions. Cases can be compared via costs, which represent losses and are thus indicative of the force size required for victory.

Introduction

The operations analysis of tactical military engagements is highly complex. Even if only equipment were considered, there would be a need to understand how each asset will perform against each opposition asset. But, overwhelmingly, tactical military engagements are about people, their plans, their decisions and their actions. The outcome of such an engagement will depend fundamentally on the tactics used by the people on each side.

For each capability there is likely to be a counter-capability that, more or less, negates it. In order to improve effectiveness, commanders use multiple capabilities in a coordinated fashion. This raises at least two questions. First, for a given scenario, with known opposition force concentrations and asset types, numbers and distribution, what capabilities will our forces need in order to attain their objectives? Second, what tactical coordinations are enabled by each combination of capability levels, and how do they influence the effectiveness of the armed force? Such an analysis, over all foreseeable scenarios, can be used for force planning, for equipment selection, or for the analysis of possible tactics. It involves the establishment of a connection between low-level factors and high-level results, such as the outcome of a battle.

Markov stochastic Lanchester models have been used, with considerable success [8,9] to estimate high-level results. Such models use the opposing numbers of a single resource, for example soldiers, to estimate the eventual outcome via processes such as attrition. But Lanchester models do not provide information on the relative contributions of various heterogeneous assets, or on the effectiveness of details of tactics. To analyse the contributions made by various factors to the achievement of goals, Doyle, Deckro, Jackson and Kloeber [5,6] used a top down approach based on extracting measurable sub-objectives from the fundamental objectives for an operation. The judgement of experienced military officers provided estimates of the level of achievement of each of the sub-objectives, and also provided the weights used to sum the sub-objectives. Despite a great deal of analysis, the situations being judged remain complex, and the validity of judgements is difficult to check. Simulation programs have been used to reduce the complexity of judgements, by using stochastic equations to represent the physical aspects of the conflict. Decisions can then be made by domain experts interacting with the program in a manner analogous to actual conflict, or they can be encapsulated in rule sets or intelligent agents. Outputs from the early programs typically were detailed logs of events and were particularly useful for the analysis of the detailed interactions of the two forces. But these were many-on-many models, with long run times and large numbers of variables that made sensitivity

[a] Land Operations Division, Defence Science and Technology Organisation, PO Box 1500, Edinburgh SA 5111, Australia.
[b] Intelligence, Surveillance and Reconnaissance Division, Defence Science and Technology Organisation, PO Box 1500, Edinburgh SA 5111, Australia.

information almost impossible to obtain [4]. Modern simulation programs tend to be used in conjunction with other mathematical tools. For example; simple interactions between many components can be used to produce emergent outcomes, as long as there are ways to recognise and measure these outcomes; and, if there is a way to reduce the number of parameter sets that must be considered, simulation can produce information on the probability density functions of output variables, giving information which is essential for the analysis of risk for any operation. An example of the highly effective use of a simulation model for the analysis of air operations is given in [4]. Problems experienced during missions were simulated using the EADSIM air defence simulation model, with analysts taking observations from the animated graphical display. They used five to ten runs for each scenario and then used spreadsheet models and databases to estimate the likely causes for the problems. Other types of mathematical models have also been used to analyse military engagements. Feigin, Pinkas and Shinar [7] used a Markov model to analyse few-on-few air engagements, but this model did not include any explicit representation of decisions, and generated large numbers of states for even small numbers of aircraft.

Military conflict evolves into a network of processes, many of which are coupled. Examples of processes include firefights, reinforcement, supply, maintenance and surveillance. While a study of the development of such networks is considered essential, analysis must begin at the lowest level of aggregation [1], to determine, for each scenario, those factors that are important and those that are unimportant. One of the factors involved in network formation is human decision making, but the probability that a particular decision will be made may be difficult to estimate. However, if the analyst selects a decision the consequences of that choice can be studied [1]. For complete information, all feasible combinations of choices must be studied, and this is possible only for low-level networks. In decision theory, similar decision situations have been modelled using Markov decision processes. These specifically provide for control actions, as well as having measures for costs and benefits.

In this work, we develop a Markov decision process model to determine the importance of the chosen factors involved in achieving victory in an isolated network of firefights. We also suggest ways in which the effect of interactions with other networks, for example, representing attempts to control asset replacement, might be modelled with decision processes. Because our interest is in the contributions of the various factors to the attainment of victory, we do not model situations to see which side would win, but instead we take an approach that is equivalent to the classical Lanchester approach of determining the force size that is required for victory, given some levels of the various factors. Like earlier studies, this work is based on military judgement. However, in this case, judgements are confined to the domain of expertise, such as the definition of scenarios, the selection of targets, and the provision of detailed guidelines on tactics.

We focus on limiting cases instead of considering myriad combinations of decisions and we place considerable emphasis on the issues that will be faced by an analyst in the application of Markov decision processes, including those involved in the definition of Markov states.

Model Development

Markov processes are comprised of a set of states $S = 1,2,\ldots,n$, and a transition probability matrix $\mathbf{P}=(p_{ji})$ governing the possible changes of state. If $x(t_k)$ is the state the process is in at time t_k, then

$$p_{ji} = Pr(x(t_{k+1})=i \mid x(t_k)=j).$$

In addition, a Markov decision process (MDP) has a set of control actions $U=\{u\}$ and some costs C, where the actions u change the values of the transition probabilities p_{ji}, and the costs are incurred in trying to change state. A vector of initial values for state probabilities completes the definition of a decision process. The transition probabilities take the form $p_{ji}(b_d,b_a;u,w(u))$, where b_d is a vector representing the capabilities and positions of the defending force and b_a a similar vector for the attacking force, u is the vector of control actions taken by the attacking force and $w(u)$ are the reactions of the defenders. In the scenarios considered here, the engagements are modelled as attacker-defender situations. The tactics of the aggressor become control actions and the defender's reactions serve to modify the effect of those control actions. Thus the defender is the system, and the states must represent the numbers and types of defence assets, and control is exercised through the actions of the attacking side. However, this is not as restrictive as it sounds. The main limitation is caused by time steps in the models being defined by the actions of the attacker, with every sortie creating a new time step. This is modified to some degree by the freedom both sides have to undertake any action, in between time steps, that will not have an effect until the next sortie. Thus we are not considering a situation where the defenders make a pre-emptive attack on the base of the attacking side. Such an attack would require a swapping of capabilities and, from a capability analysis viewpoint, the two situations can be considered separately. If the reactions of the defenders were less effective than they might have been, the attackers would receive a bonus. But this situation would just increase the difficulty of the capability assessment. In these models the capability assessment is based on the worst case for the attacking side, where the defenders always make decisions that are optimal for them.

To define states for the MDP we split the assets in the actual state into groups by function, such as surveillance and target allocation, or weapon control, or by the types of control action that can be taken against them. For surveillance and target allocation assets we need to know the different types of assets and the number of each type. These assets detect, identify and track attackers and pass their positions on to appropriate weapon controllers. Each type of surveillance asset has a limit to the number of attackers it can cope with at any time, as well as having some capacity to uncover deception. Weapon controllers also come in different types and the numbers of each type represents the state. These assets aim and control the weapons, with each asset being able to cope with a limited number of attackers at any time. Once this detailed state has been defined the critical vulnerabilities of the defence effort must be identified. For example, a subset of surveillance assets might be proposed as the critical vulnerabilities, because the destruction of these assets would cripple the responses of the defence system. The state of a Markov decision process would then be defined in terms of those surveillance assets, but this does not mean that the other assets are ignored. Each of the surveillance assets would manage a set of weapon controllers and, through them, a number of weapons. Surveillance assets thus have weapons capabilities, but these are not explicitly included in the state definition; as assets such as radars are destroyed, the associated weapon capability is removed or modified; any active surveillance assets that are not included in the state definition will have to be avoided in the model by choice of tactics. Also, the simplified states should have some ordering property, so that they can be expressed in some meaningful order. States can then be represented by the integers i, for $i=1,2,\ldots n,$ where n is the number of states. This ordering property might require the prioritisation of attacks against particular defence assets. The number of possibilities is enormous, even for a small number of assets, so we seek limiting cases. Even though it is possible to establish a prioritised list of targets, the actual order of destruction in a battle is likely to vary from this list, and the costs incurred will vary accordingly. From the viewpoint of capability planning it is the bounds on the costs that are important, and we use military expertise, informed by calculated probabilities of survival and success, to define two sets of states, a "best" case and a "worst" case. Each of the cases becomes a separate application. The aim in this preliminary analysis is to produce models with the smallest possible number of Markov states while retaining all of the capabilities of the defence. In these models, control actions are based on military doctrine and tactics. They will be selected by the analysts and will be completely defined, with, for example, the numbers and types of attackers; whether they attack as groups, or as individuals; how far they need to penetrate to successfully deliver their munitions; what actions are undertaken by each (such as jamming, launching self-directed missile, *etc.*) and with what timing; whether one defence asset or many assets are targeted in a single attack. For every action taken by the attacking force the defence will have a counter action, and whether or not they can take those counter actions will depend on the exact nature of the situation contrived by the combined actions of the attackers. Each of the options available to the defenders needs to be considered as a separate case.

The controller wants to move from the beginning state to state one, but when assets are destroyed the defenders might replace them, thus moving back to a higher state. Ancker [1] asks "*How does the timing* *of the reinforcements affect (results)*?" At one extreme, state assets that are destroyed during one time step are replaced before the next time step, and the Markov state never changes. Immediate but finite replacements can be modelled by defining extra states. Simple cases, such as a single replacement for each original defence asset, will have an obvious effect on the losses suffered by the attacking side. Cases where there are constant replacement probabilities for each asset will limit the possible level of success for the attacker and will prevent the attainment of some end conditions. Replacement of state assets is a cost multiplier, and the underlying problem is at least as significant as the attack problem. The two problems are complementary, but, on the question of the factors that are important, they are separable. The replacement process can be analysed to determine vulnerabilities and it can then be modelled, possibly using a Markov decision process, to uncover the levels of the various capabilities that will be needed in order to reduce or eliminate the replacement capacity, at an acceptable cost. The original attack problem can then be modelled without replacement, to determine the combinations of capabilities that can nullify, for an acceptable cost, the expected arrays of active defence assets. It is likely that the capabilities required in the two problems, attack and counter replacement, will be both qualitatively and quantitatively different. The illustrations used in the current work refer to the attack problem without replacement.

We use a fixed endpoint condition, with evaluation via costs, and, to ensure that the endpoint is reached, we allow the attack force to be as big as is needed. The attacking side will always reach its goal but the losses they suffer enable us to determine the potential contribution of each type of capability, and the effectiveness of each vector of control actions. This is equivalent to determining the force size that is required for victory.

The general form of the transition probability matrix **P** is triangular, and, for four possible states,

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ p_{21} & p_{22} & 0 & 0 \\ p_{31} & p_{32} & p_{33} & 0 \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix},$$

where p_{ji} is the probability of transition from state *j* to state *i*. However, we can also consider the case where the attacking force targets only one state asset at a time. For this case, the transition matrix will be of the bidiagonal form

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ p_{21} & p_{22} & 0 & 0 \\ 0 & p_{32} & p_{33} & 0 \\ 0 & 0 & p_{43} & p_{44} \end{pmatrix}.$$

These two forms allow the comparison of serial and parallel firefights [1], but it is the bidiagonal form that makes most sense in the context of limiting cases. Each of these cases has been defined on the basis of a particular order of attack on the critical assets of the defence. The bidiagonal form enforces this order of attack. In general we expect the "best" case to be close to optimal. In this work we do not consider cases where the attackers are many and are capable of a massed simultaneous attack, against all targets, which overwhelms the capabilities of the defence.

To illustrate the process of calculating probabilities, consider the bidiagonal case, where the transition is from state *j* to state *j-1*. This transition involves the destruction of a single defence asset, such as a particular weapon controller. For example, if the critical defence assets are two medium range, and one long range, weapon controllers; where the medium range weapons have non-overlapping territories; and where the long range controller is targeted first, let *1-p* be the probability of destruction of each medium range controller, once the long range controller is destroyed. Then the probability of destroying the long range controller will have a value lower than *1-p*, since the attackers must survive the projectiles from both long and medium range weapons. We can represent this probability as $1-\alpha$, and the transition matrix is of the form

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1-p & p & 0 & 0 \\ 0 & 1-p & p & 0 \\ 0 & 0 & 1-\alpha & \alpha \end{pmatrix},$$

which is totally defined by the two parameters α and *p*. Note that the medium range weapons do not interact in this scenario, and this should lead us to split the scenario into two Markov problems, one with three states and the other with two. This illustrates the emphasis on the lowest possible order of aggregation and also indicates that Markov processes might have a role in the analysis of higher levels of aggregation.

The probabilities of the system being in any particular state at time step k are represented by $\boldsymbol{s}_k = (s_{k1}, s_{k2},\ldots,s_{kn})$, where n is the number of states and s_{ki} is the probability of the system being in state i at time step k. The value of $\boldsymbol{s}_k$ can be calculated from the transition matrix $\mathbf{P}$ in the usual way $\boldsymbol{s}_k = \boldsymbol{s}_0\,\mathbf{P}^k$, where $\boldsymbol{s}_0=(0,0,\ldots,1)$ in these models.

There will be costs associated with any actions taken in an attempt to change the state. These costs are denoted by $C_l\,(i,u(i))$, where i is the state, $u(i)$ is the control vector and l represents the different types of attackers or other capabilities. When a particular set of tactics is being tested, there will be only a single control vector possible for each state, and we denote the costs as C_{li}. Generally the costs calculated in these models will be the expected values of the numbers of attack platforms destroyed, but in some cases the expenditure of munitions might also be accounted for. Thus we can define cost vectors $\boldsymbol{C}_l= (0, C_{l2},\ldots,C_{ln})$ and the expected value of cost at time step k is given by the dot product

$$E_{kl} = \boldsymbol{s}_k \bullet \boldsymbol{C}_l\ . \tag{1}$$

The elements s_{ki} of the state probability vector, at time step k, can be viewed as weights that are used to determine the expected values of costs for that time step. Estimates of total costs can be made either by summing each cost over the total number of time steps, or by summing the values of s_{ki}, over all the time steps, to form cumulative weights. If

$$W_{Ki} = \sum_{k=1}^{K} s_{ki}\ ,$$

then for small values of K, the weights will depend on attack priorities, but, for large K, they converge to some ultimate values W_i:

$$W_i = \lim_{K\to\infty} W_{Ki}.$$

Total costs can then be estimated by

$$T_l = \boldsymbol{W} \bullet \boldsymbol{C}_l\ ,$$

where $\boldsymbol{W} = (W_1, W_2,\ldots)$. It is also possible to define an end condition, such as $s_{k1} > 0.95$, and stop the model at the first time step for which the condition is true.

Each row of the transition matrix $\mathbf{P}$ represents an attempt to change from a particular state, and each element of the row will be a function of both the probability of survival for each type of attack platform and the probability that a platform of that type will destroy the targeted defence asset. The values of the elements determine the probabilities of being in each state at each time step, and ultimately they determine the number of time steps taken to reach the end condition. The expected value of state losses can be calculated in a manner analogous to Equation (1). Evaluation of losses on both sides might require the use of a scoring technique, if the heterogeneous assets are very different in costs and contributions. In each case, when the end condition is reached, a loss ratio (ratio of attack losses to defence losses) can be calculated. In parameter space, surfaces with constant loss ratios can be formed. This separates the parameter values into performance categories. All of the above can be done without reference to the values of lower-level factors.

Almost all of the more fundamental variables that might be considered by an analyst can be related explicitly, in a hierarchical fashion, to the survival probabilities and destructive capabilities for one or more types of attack platform. Much of this has already been done, and the equations below are derived from [2]. For example, the probability that a state asset survives an attack from a group of type i attack platforms is given by $p_{si,ass} = 1 - p_l\, p_{ki}$, where p_l is the probability that an attacker will reach weapon launch position, and p_{ki} is the kill probability for the number of weapons launched. The probability that the state asset will survive a coordinated attack from a number of different types of attackers is a little more complicated, but can still be written in an explicit form. Each type of attack can influence the probabilities of other coordinated attacks, so we need to calculate conditional survival probabilities $p_{si,ass,con}$ for the state asset relating to attacks from each type of attack platform. Then the probability of survival for the asset can be calculated as

$$p_{s,ass,con} = \prod_{i=1}^{q} p_{si,ass,con} \, ,$$

where q represents the number of different types of attack platform. The probability that a particular state asset will be destroyed during a particular time step is then given by

$$p_{k,ass} = 1 - p_{s,ass,con} .$$

The probability p_{ki} is calculated from information about more basic variables, such as the accuracy of placement or guidance of the weapon, the destructive capacity of the weapon, and the chance of the weapon reaching the target. Similar equations can be defined for the probability of destruction for each type of attack platform, based on variables such as the time that the attacker must spend in an engagement zone in order to launch its weapon, the use made of cover, the effect of manoeuvring, the effect of various types of jamming, the number of other threats the state assets have to deal with, and the priority given to each threat. For costs associated with each type of attacker, we need to calculate the probability of survival throughout the entire time step, not just to the point of weapon launch. Attackers entering an engagement zone might have had to transit an earlier engagement zone. If the state assets in the zones interact with each other (that is, if the zones overlap) all assets might need to be treated in a single model. A combined model might also be required if the credentials of defined attacker types are being tested, where the attacker type has capabilities that might be of benefit in each zone. Otherwise, the capability requirements can be assessed separately. The sensitivity of transition probabilities to changes in any of these variables is defined by such formulae, and we can thus explain all contributions of the independent variables to the cost per time step for each state, to the total costs, and to the loss ratios. The results might be used directly, or be used to define ranges of parameter values for simulation runs.

Discussion

It has been shown that Markov decision processes can provide a direct link between low-level variables and high-level outcomes for military engagements at the lowest level of aggregation. This link incorporates tactics and is expressed in terms of costs incurred in achieving goals. A hierarchy of functions can represent the underlying variables and at each level it is easy to obtain the sensitivity of variables to changes in the values of variables in the level below. In this way, the sensitivity of high-level costs to changes at any level can be obtained. Focusing on the upper and lower bounds for the costs associated with a scenario can produce an enormous reduction in the complexity of the modelling problem.

Future directions for research include the formal mathematical development of the process, including simplification of the equations for state probabilities, and the development of networking methods to deal with higher levels of aggregation. It might also be possible to link model outputs to the various measures of merit used in operations analysis. This could require the development of a methodology to estimate capability levels from the characteristics of given combinations of equipment, as well as a method to generalise the performance measures, possibly using cost effectiveness theory. Having a way to estimate capability levels for equipment would also allow these models to be used for equipment selection. The validity of particular models is an issue and it might be possible to develop test cases with known solutions to check the internal validity of models. At present the transition probabilities are calculated on a case-by-case basis. It appears likely that the appropriate structuring and definition of scenario and control action information could automate this. This would allow the models to be used to attack questions of optimisation. The application of the method to the analysis of specific scenarios will provide feedback for the development of methods as well as being the motivation for their development.

References

[1] C. J. Ancker (1995), “A proposed foundation for a theory of combat”, *Naval Research Logistics*, **42**, pp. 311-343.

[2] R. E. Ball (1985), *The fundamentals of aircraft combat survivability analysis and design*, American Institute of Aeronautics and Astronautics.

[3] F. T. Case, C. W. Hines, and S. N. Satchwell, (1995), “Analysis of air operations during Desert Shield and Desert Storm”, *Naval Research Logistics*, **42**, pp. 715-736.

[4] C. G. Cassandras, (1996), *Discrete event systems: modelling and performance analysis*, Irwin Publ.

[5] M. P. Doyle, R. F. Deckro, J. A. Jackson and J. M. Kloeber (1997), "A function approach to information operations moes: a preliminary study", Technical Report 97-04, Center for Modeling, Simulation and Analysis.

[6] M. P. Doyle, R. F. Deckro, J. A. Jackson and J. M. Kloeber (2000), "Measures of merit for offensive information operations courses of action", *Military Operations Research*, **5**, pp. 5-18.

[7] P. D. Feigin, O. Pinkas and J. Shinar (1984), "A simple Markov model for the analysis of multiple air combat", *Naval Research Logistics Quarterly,* **31,** pp. 413-429.

[8] M. Kress and I. Talmor (1999), "A new look at the 3:1 rule of combat through Markov stochastic Lanchester models", *Journal of the Operational Research Society*, **50,** pp. 733-744.

[9] M. P. Wiper, L. I. Pettit and K. D. S. Young (2000), "Bayesian inference for a Lanchester type combat model", *Naval Research Logistics*, **47,** pp. 541-55