

## Editorial

In this issue, we have accepted two papers. The first paper is contributed by M.-T. Nguyen on *Strategic Planning Tool Suite: An Approach of Combining Scenario Analysis Approach* and the second paper is written by V. Sharma, K. Dahiya and V. Verma on *A Note on Two-Stage Interval Time Minimization Transportation Problem*. We are delighted to be publishing these papers here for Bulletin readers. We have provided a report for a special operations research conference held in Canberra in July 2008.

I am pleased to inform you that the electronic version of ASOR Bulletin is now available at the ASOR national web site: <http://www.asor.org.au/>. Currently, the electronic version is prepared only as one PDF. Your comments on the new electronic version is welcome.

ASOR Bulletin is only national publication of ASOR. I would like to request all ASOR members, ASOR Bulletin readers and OR organizations in the country to contribute to the ASOR Bulletin. The editorial policy is available either from the Bulletin web site or from the inside back cover of the Bulletin. The detailed instructions for preparing the manuscripts is available in the URL: <http://www.asor.org.au/> and <http://www.itee.adfa.edu.au/~ruhul/asor.html>

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# Strategic Planning Tool Suite: An Approach of Combining Scenario Analysis Methods

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## Abstract

Scenarios are an important tool in the strategic planning process, and are increasingly used in both Defence and the business world. This paper describes an approach of combining scenario analysis methods for systematically selecting and developing future scenarios. A strategic planning tool suite based on this approach is designed and implemented using public software to allow numerical calculations to be completely automated and to guide users through each step of the approach. A typical Defence strategic planning problem and numerical experiment are demonstrated with general guidelines to consider when using the tool.

## Introduction

Scenario analysis has emerged as a tool for strategic planning [7] when the future is perceived as surrounded by a high degree of uncertainty, complexity and paradigm shift. Scenario analysis techniques characteristically synthesise quantitative and qualitative information, constructing multiple scenarios or alternative portraits of the future.

Although scenarios are the important tool in the strategic planning process, and are increasingly used in both Defence [6, 10, 11, 21, 24] and the business world [12, 13], there is no single generally accepted 'best method' for constructing them. This paper describes a possible way for combining scenario analysis methods (*Non-Bayesian* method [2, 3, 18, 19, 28, 29] and *Bayesian* method [4, 22, 23]).

We demonstrate the approach of combining methods with a typical example and numerical experiment. Starting with relatively simple information from experts and problem-owners, the approach can help determine main scenarios, as well as a balanced mix of plausible futures. In

order to allow all numerical calculations to be completely automated and to guide users through each step of the approach, a strategic planning tool suite is also designed and implemented using public software [8, 15, 25].

After providing an overview of scenario analysis methodology, the paper presents a six-step approach of combining methods. The processes and mathematical formulation of the approach is then used to generate algorithms for developing the strategic planning tool suite. By introducing a typical Defence strategic planning problem for illustration, we then walk through the approach. The paper finally concludes by emphasising some general points to consider when using the strategic planning tool suite.

## Scenario Analysis Methodology: An Overview

Scenario analysis consists of the three basic stages:

1. *Problem analysis* to come up with an exact definition for the problem of the investigation,
2. *Subsystem analysis* to identify relevant external influences on the problem investigated and
3. *Synthesis process* to examine the existing interdependencies between the influencing factors and to establish alternative scenarios.

The problem analysis helps all experts and problem-owners gain a similar understanding of the problem at hand. Based on this consensus the problem can be further bounded and structured. The subsystem analysis expresses the problem as a system of inter-related dynamic components (subsystems), with the system itself linked to its external environment. From every subsystem, a number of representative influencing factors relevant to the

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problem is then identified. The synthesis process establishes a logical and systematic way for scanning the range of possible scenarios and for selecting main scenarios or a balanced mix of scenarios.

A variety of creative methods such as brainstorming, brainwriting, round table discussion, and the Delphi technique [14] can be employed in the first two analysis stages. There are two basic methodologies for implementing the second and especially the third stage of the scenario analysis:

- ▶ *Non-Bayesian* method (e.g. Morphological Analysis (MA) [29], Battelle approach [28], Field Anomaly Relaxation (FAR) [2, 3, 18–20]) and
- ▶ *Bayesian* method<sup>1</sup> (e.g. Cross-Impact Analysis using System of equations [22, 23], or Goal Programming (GP) [4]).

Some extensions based on both classes are also developed (Battelle approach with Cluster Analysis [1], GP with Integer Programming (IP) [9]).

The non-Bayesian method does not consider the probabilities of influencing factors on the problem investigated, therefore, the selected scenarios may have very small probabilities and could not practically be a basis of a meaningful planning effort. While the Bayesian method requires marginal and conditional probabilities for the pairs of factors as input. High demands are therefore placed on the expert's ability and willingness to make these estimates. Furthermore, the Bayesian method takes all scenarios into consideration. In consequence, the scenario probabilities are often

very small (see e.g. [17] for the review of the processes and mathematical formulation of each method, also the application issues of employing these methods).

The purpose of the strategic planning process is to reflect possible alternative developments which are constructed using quantitative data as well as the experience and intuition of experts and stakeholders. However, they are unlikely to be interested in the mathematical aspects of the scenario analysis. Hence the information required from them should be kept as simple as possible. We present next an approach which combines all the above methods in light of these requirements.

### An Approach of Combining Methods

We will use the structure of the non-Bayesian methods to break down the problem under examination, but adopt and use the FAR terminology throughout this section. Summary of the approach is given in Table 1.

#### Description of Future States

The first step in developing scenarios is to identify *sectors* (components or dimensions or environments) hypothesized to influence the future of the environmental subsystems investigated. The choice of sectors is critical and requires considerable thought which can be based on results of problem analysis (e.g. from brainstorming). They must also represent the whole system.

Table 1: Six-step Approach of Combining Methods

Step	Purpose	Method
1. Description of future states	Identify and select sectors and factors hypothesized to influence the future.	Brainstorm, MA, FAR, Battelle
2. Assessment of states' compatibilities	Evaluate compatibility/consistency values between pairwise factors.	Battelle
3. Determination of compatible scenarios	Define a criteria for plausible/compatible scenarios then enumerate all of them.	FAR, Battelle
4. Assessment of states' possibilities	Elicit marginal probabilities on the occurrence of factor.	Bayesian
5. Analysis of scenarios' possibilities	Obtain the likelihoods for the compatible scenarios and further prune scenarios due to their likelihood.	Modified GP
6. Determination of main scenarios	Group the selected scenarios into a few main ones or choose a balanced mix of plausible futures.	Cluster Analysis, IP, FAR

<sup>1</sup>The central idea of the Bayesian method is to elicit the likelihood distribution for future scenarios to be projected from the experts in the field.

Although the number of sectors should be kept to a minimum, the selected sectors need to be comprehensive enough to reflect all relevant concerns about the future and be thoroughly defined so that all experts understand relevant assumptions. Six to seven is usually recommended for the number of sectors [20].

Each sector can take on several *factors* (states or hypotheses). A given *scenario* is characterised by the choice of a specific factor for each of the sectors. There are as many possible scenarios as there are combinations of factors. Usually two to five possible future factors are designated for each sector by evaluating historical trends, current conditions, and expert opinion. These factor are mutually exclusive and technically exhaustive; in other words, other factors were thought to have a probability of occurrence so low as to justify their exclusion.

A symbolic name is also chosen in this step using particular letters from each of sector names and then uses these symbols to describe the scenarios.

#### Assessment of States' Compatibilities

The interdependencies between factors is considered in this step. According to the Battelle approach [1, 28], *compatibility ratings*,  $k_{ij}$ , are expressed on a scale from 1 to 5, by asking experts to answer the same question for each: 'Can we think of a scenario within which these two factors might coexist?'

A compatibility rating of 5 indicates two possible occurrences are very compatible, and a rating of 1 indicates they are not likely to occur together. Values of 2, 3, and 4 represent increasing compatibility.

#### Determination of Compatible Scenarios

The number of scenarios are exponentially growing with the number of factors. Some combinations of factors may not represent plausible scenarios. In order to decrease the complexity of computation and consider the real situations, the number of scenarios are selected by the following rules:

1. A compatibility rating existed between any two factors in a scenario must be different to 1 (not likely to occur together), **and**
2. The average of individual compatibilities between the factors in each scenario is greater than or equal to a lower limit  $L$ ,

**or** the number of compatibility ratings of 2 (low likelihood of occurring together) in a scenario is less than or equal to an upper limit  $U$ , where

- ▶  $L$  should be chosen to assure the remaining scenarios had an average scenario compatibility above 3 (in other words, above a neutral compatibility), and
- ▶  $U$  should be below half the number of the sectors in a scenario.

Under these two conditions, scenarios deemed to have a very low possibility of occurring are eliminated. In some cases, the participants have the option to further prune to a subset of these compatible scenarios or to also reintroduce any especially interesting scenarios which were excluded due to their incompatibility.

#### Assessment of States' Possibilities

This approach also requires marginal probabilities  $p(i)$  on the occurrence of Factors  $i$ . Because possible future states of each sector are considered to be exhaustive and mutually exclusive, the assigned marginal probabilities of each factors in each sector sum to 1. Also every sector usually only has 2 to 5 factors, these probabilities are quite easy to elicit.

The marginal probabilities and compatibility ratings obtained above are then used to estimate the joint probabilities between two factors and to serve as the basis to obtain cross-impact analysis and conduct the generation of scenarios.

#### Analysis of Scenarios' Possibilities

We now calculate the probabilities of the compatible scenario selected in the previous step using goal programming (GP) approach [4] with some modifications proposed in [1]. Let us denote

- ▶  $n$  the number of influencing factor
- ▶  $K$  the number of considered scenarios ( $K \ll 2^n$ )
- ▶  $a_i = (a_{is}), i = 1, \dots, n; s = 1, \dots, K$  the column vectors of 0's and 1's ( $a_{is} = 0$  if Factor  $i$  is not in Scenario  $s$  and  $a_{is} = 1$  if Factor  $i$  is in Scenario  $s$ )
- ▶  $y$  the column vector of the scenario probabilities  $y_s (s = 1, \dots, K)$
- ▶  $y^t$  the corresponding transposed vector of  $y$
- ▶ ' $\wedge$ ' operation indicates a component by component multiplication of two vectors.

The modified GP is of the form:

$$\text{minimise } \sum_{i,j} (\delta_{ij}^- + \delta_{ij}^+) + M\delta \quad (1a)$$

subject to:

$$y^t a_i \leq p(i), \quad (1b)$$

$$y^t (a_i \wedge a_j) \leq p^*(ij), \quad (1c)$$

$$\sum_{s=1}^K y_s \leq 1, \quad (1d)$$

$$p^*(ij) + \delta_{ij}^- - \delta_{ij}^+ = p(ij), \quad (1e)$$

$$p^*(ij) + p^*(i\bar{j}) = p(i), \quad (1f)$$

$$0 \leq \delta_{ij}^- \leq \delta; \quad 0 \leq \delta_{ij}^+ \leq \delta \quad (1g)$$

$$y_s \geq 0, \quad s = 1, \dots, K \quad (1h)$$

$$i = 1, \dots, n; \quad j > i \text{ and}$$

$M$  is a large value, say 10000,

where the joint probabilities  $p(ij)$  are defined by the transformation of the marginal probabilities  $p(i)$  and compatibility values  $k_{ij}$ , using the equations:

$$p(ij) \stackrel{\text{def}}{=} \begin{cases} p(i)p(j) - \frac{1}{2}(k_{ij} - 3) \times \\ [l_{ij} - p(i)p(j)], & k_{ij} \in [1, 3) \\ p(i)p(j) + \frac{1}{2}(k_{ij} - 3) \times \\ [u_{ij} - p(i)p(j)], & k_{ij} \in [3, 5] \end{cases}$$

$$i = 1, \dots, n; \quad j > i$$

$$l_{ij} \stackrel{\text{def}}{=} \max\{0, p(i) + p(j) - 1\},$$

$$u_{ij} \stackrel{\text{def}}{=} \min\{p(i), p(j)\}.$$

In equation (1e), the corrected (or final) joint probabilities  $p^*(ij)$  of the preliminary (or initial) joint probabilities  $p(ij)$  are adjusted by deviation variables  $\delta^-$  and  $\delta^+$ ;  $\delta$  is the maximum of all individual deviation variables; and  $p^*(i\bar{j})$  is the corrected joint probability that Factor  $i$  will occur and Factor  $j$  will not.

The modified GP model provides individual scenario probabilities, but because of the degenerate solution problem in linear programming, alternative probabilities exist. We should then solve the modified GP first to obtain the minimum possible deviation ( $m_{\text{dev}}$ ) and then to create a new objective function and one additional constraint for use in a post-optimality analysis. Using this suggestion, the new objective function is

$$\text{Min } y_s \text{ or Max } y_s, \quad (2)$$

and the additional constraint is

$$\sum_{i,j} (\delta_{ij}^- + \delta_{ij}^+) + M\delta = m_{\text{dev}}, \quad i = 1, \dots, n; \quad j > i. \quad (3)$$

This model is solved for each of the  $K$  scenarios to obtain their minimum and

maximum probability of scenario. The arithmetic mean of the upper and lower bound, after being adjusted by the summation of all scenarios so the probabilities summed to 1, defined the probability of each scenario.

### Determination of main scenarios

The objective of scenario analysis is to develop a manageable number of representative scenarios that can be used in strategic planning. The optimal number of scenario groupings is controlled by the ability of the end user (analysts, experts, stakeholders) to conceptualise the alternatives and use them in planning. The goal of finding a minimum number of scenarios is to support and limit the work of the scenario writer and reader.

Cluster analysis is used in the strategic planning context [1, 16, 26] to group together scenarios that are 'similar' while integer programming (IP) approaches [9] are developed to select a set of scenarios that includes all future states.

### Cluster Analysis - Representative Scenarios

The basis for clustering is similarity defined by a distance between pairs of scenarios and the method of grouping scenarios. Here we use the user-defined inter-scenario compatibility distance and the standard complete linkage method [26].

► *Inter-scenario compatibility* distance is determined by comparing the compatibility ratings between the factors in one scenario with each factor in another scenario, summing all of these compatibility levels, and dividing by the number of factors levels compared.

► *Complete Linkage* method (based on the maximum distance between scenarios, one from each cluster) finds similar clusters as all scenarios in a cluster are within some maximum distance of each other.

However, alternative clustering distance and method are possible (e.g. squared Euclidean distance with Ward's minimum variance method [16]).

### Integer Linear Programming - Balanced Mix of Scenarios

Selecting a minimum number of plausible alternate scenario, to be expanded into scenario descriptions, can be formulated in such a way that each state (factor) of each environment (sector) will be represented at

least once (or twice, or three times; chosen by the user).

Denote by  $S_i$  the set of all scenarios in which Factor  $i$  occurs. Using the decision variable  $z_k$ , taking binary value 0 or 1 according to whether Scenario  $k$  (among  $q$  accepted scenarios,  $q \leq K$ ) is selected for scenario development, the IP can be written as:

$$\begin{aligned} & \text{Minimise} && \sum_{k=1}^q z_k \\ & \text{subject to} && (4) \\ & && \sum_{k \in S_i} z_k \geq N_i, \quad \forall i = 1, \dots, n \end{aligned}$$

where  $N_i$  is an integer denoting the minimum number of times Factor  $i$  should be included in a scenario definition.

The formulation has the attraction that it can be modified and extended easily by adding a variety of constraints to the formulation. For example, the requirement to select:

- ▶ a particular scenario can be represented simply by setting  $z_k = 1$  for that scenario
- ▶ a particular combination of Factor  $i_1$  and Factor  $i_2$  to be at least  $R$  times could be formulated by denoting the set of scenarios that contain the combination  $S_{i_1 i_2}$  and adding the constraint  $\sum_{k \in S_{i_1 i_2}} z_k \geq R$ .

A similar formulation results if, rather than requiring a factor to be represented at least  $N_i$  times, the aim is that the total probability of scenarios in which Factor  $i$  occurs is set to be  $P_i$ . This obviously requires a probability estimate  $Y_k$  for scenario  $k$  as input data. We can use the arithmetic mean of the upper ( $\max y_k$ ) and lower ( $\min y_k$ ) values probability estimates in the previous step for the probability  $Y_k$ . This formulation can be written as:

$$\begin{aligned} & \text{Minimise} && \sum_{k=1}^q z_k \\ & \text{subject to} && (5) \\ & && \sum_{k \in S_i} Y_k z_k \geq P_i, \quad \forall i = 1, \dots, n, \end{aligned}$$

where  $Y_k = \frac{1}{2}(\max y_k + \min y_k)$ .

Note that the IP frequently has multiple optima. Alternative solutions should be found and presented to the end user. This can significantly increase the flexibility for making a decision. For finding an alternative solution of an IP problem (see e.g.

[27]) involving only binary variable ( $z_k \in \{0, 1\}$  for all  $k$ ), we just add the following constraint to exclude an existing solution:

$$\sum_{k \in B} z_k - \sum_{k \in N} z_k \leq |B| - 1, \quad (6)$$

where  $B = \{k | z_k = 1\}$ ,  $N = \{k | z_k = 0\}$  and  $|B|$  is the cardinality of set  $B$ .

## Illustrative Example & Strategic Planning Tool Suite

### Typical Example

As an example to be used for illustrating the combining methods, we consider the following strategic question in Defence planning:

Australia's Joint Operations for the 21st century states regional factors (such as state fragility, poor governance and economic underdevelopment) may affect Australia's security interests, both directly and indirectly. As a result, a key task for Australia's Defence Force is to contribute to a stable regional environment.

Contributing to a stable regional environment includes being able to defend Australian territory against credible threats without relying on the combat forces of other countries, providing joint forces to contribute to, or lead, coalition operations in Australia's neighbourhood as well as contributing to crisis response as part of a coalition effort in humanitarian assistance and disaster relief.

This leads to the question, what will Australia's regional environment look like in 2030 and what types of operations will Australia be required to respond to in this timeframe in our region?

### Numerical Experiment & Tool Suite

The free, open-source Integrated Development Environment (IDE) NetBeans [25] was used in the creation of the tool suite [5]. For the illustrative example, Australia's Regional Environment in 2030, the description of future states (Step 1) is recorded using Morphology Analysis (MA) tool, and captured in the form of Table 2. The symbolic name is chosen as *PESTHAC* from the 7 sectors (listed in the far left boxes). Each sector has 3 factors except the Sector A (Type of Operation required by ADF) which has 4.

Table 2: Australia's Regional Environment in 2030: A Morphological Analysis

Political Governance	P1: Political stability in most regions	P2: Unstable political environment	P3: Collapse or change in major players
Economic Growth	E1: Developing	E2: Declining	E3: Collapse
Social Cohesion	S1: Tolerance between groups	S2: Factionalisation between groups	S3: Conflict and uprising between group
Implications of S&T	T1: Overwhelming rate of change or development of technology	T2: Continuing (comparable) advancement of technology	T3: Lagging advancement of technology.
Health and Habitat	H1: Improving/Sustainable	H2: Degradation	H3: Collapse, meltdown
Type of Operation required by ADF	A1: Peacekeeping/Peace enforcement ADF role	A2: Counter Insurgency/Counter Terrorism	A3: Conventional warfare A4: Humanitarian assistance
ADF Concurrent Obligations	C1: Minor commitment to regional Operations	C2: Major commitment to regional Operations	C3: Commitment to Operations further afield

Table 3: Sample Data

Compatibility & Probability data	P			E			S			T			H			A				C			
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	4	1	2	3	
<b>P</b>	1	0.3																					
	2	1	0.2																				
	3	1	1	0.5																			
<b>E</b>	1	5	1	3	0.5																		
	2	1	5	3	1	0.4																	
	3	1	3	2	1	1	0.1																
<b>S</b>	1	2	3	4	3	3	2	0.5															
	2	4	3	2	3	3	2	1	0.4														
	3	2	1	1	1	1	1	1	1	0.1													
<b>T</b>	1	5	3	2	3	3	1	1	4	1	0.6												
	2	1	3	5	3	3	1	4	2	1	1	0.3											
	3	1	2	2	2	2	1	1	1	1	1	1	0.1										
<b>H</b>	1	2	5	3	3	3	2	5	2	2	4	3	2	0.8									
	2	4	2	3	3	3	2	2	5	2	2	3	2	1	0.1								
	3	1	1	2	2	2	1	1	1	1	1	2	1	1	1	0.1							
<b>A</b>	1	2	4	3	2	4	1	5	1	1	2	4	1	5	1	1	0.1						
	2	4	2	3	3	3	1	2	4	1	3	3	1	3	3	1	1	0.7					
	3	2	2	3	1	3	1	2	1	1	2	3	1	3	1	1	1	1	0.1				
	4	1	1	2	1	2	1	1	1	1	1	2	1	2	1	1	1	1	1	0.1			
<b>C</b>	1	2	3	2	2	4	2	2	4	2	2	3	2	2	4	1	1	5	1	1	0.3		
	2	3	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	1	0.5	
	3	4	3	3	4	2	2	4	2	2	3	3	1	4	2	1	5	1	1	1	1	1	0.2

All the necessary data<sup>2</sup> to run through the approach (Step 2 and Step 4) is presented in Table 3 with:

- ▶ a list of all factors  $i$  ( $i = 1, \dots, 22$ ) corresponding to  $P_1, P_2, \dots, C_2$  and  $C_3$  respectively,
- ▶ the compatibility ratings  $k_{ij}$  for every two factors  $i$  and  $j$ , where  $k_{ij} \in \{1, 2, 3, 4, 5\}$  which is represented as a triangular matrix ( $k_{ij} = k_{ji}$ ), and

▶ the estimated probabilities  $p(i)$  for the individual factor  $i$  (values on the diagonal, e.g.  $p(E_1) = 0.5$ ).

In Step 3, all compatible scenarios (i.e. those without a value of 1) are selected using the following value of  $L$  and  $U$ :

- ▶ Minimum average compatibility value,  $L = 3.285$ ,
- ▶ Maximum number of "2" ratings,  $U = 3$ .

<sup>2</sup>Note that all data presented here is fictitious and used for illustrative purposes only.

Table 4: Selected Compatible Scenarios

Scenario	Factor					Number of Rating 2	Average Compatibility Value	Scenario	Factor					Number of Rating 2	Average Compatibility Value			
	P	E	S	T	H				A	C	P	E	S			T	H	A
1	1	1	2	1	1	2	1	6	3.286	18	3	1	1	2	1	1	3	3.714
2	1	1	2	1	1	2	2	2	3.381	19	3	1	1	2	1	2	2	3.286
3	1	1	2	1	2	2	1	4	3.524	20	3	1	1	2	2	2	2	3.143
4	1	1	2	1	2	2	2	1	3.524	21	3	1	2	1	1	2	2	3.095
5	2	2	1	2	1	1	2	0	3.667	22	3	1	2	1	2	2	2	3.143
6	2	2	1	2	1	1	3	1	3.810	23	3	1	2	2	1	2	2	3.048
7	2	2	1	2	1	2	1	4	3.286	24	3	1	2	2	2	2	2	3.190
8	2	2	1	2	1	2	2	2	3.238	25	3	2	1	2	1	1	2	3.619
9	2	2	1	2	1	3	2	2	3.238	26	3	2	1	2	1	1	3	3.714
10	2	2	2	1	1	2	1	4	3.333	27	3	2	1	2	1	2	2	3.286
11	2	2	2	1	1	2	2	2	3.286	28	3	2	1	2	1	3	2	3.286
12	2	2	2	1	2	2	1	4	3.333	29	3	2	1	2	2	2	2	3.143
13	2	2	2	1	2	2	2	3	3.190	30	3	2	2	1	1	2	2	3.095
14	2	2	2	2	1	2	2	3	3.095	31	3	2	2	1	2	2	2	3.143
15	2	2	2	2	2	2	1	3	3.333	32	3	2	2	2	1	2	2	3.048
16	2	2	2	2	2	2	2	3	3.095	33	3	2	2	2	2	1	3	3.333
17	3	1	1	2	1	1	2	1	3.524	34	3	2	2	2	2	2	2	3.190

Table 5: Scenarios Probabilities

Scenario	Probability	Scenario	Probability	Scenario	Probability	Scenario	Probability
1	5.90%	10	5.31%	19	8.85%	28	1.77%
2	8.55%	11	5.31%	20	0.00%	29	0.00%
3	3.24%	12	0.00%	21	6.49%	30	6.49%
4	2.95%	13	0.00%	22	2.95%	31	2.65%
5	2.95%	14	1.18%	23	1.18%	32	1.18%
6	2.36%	15	0.00%	24	1.18%	33	1.18%
7	3.54%	16	0.00%	25	2.95%	34	1.18%
8	3.54%	17	2.36%	26	2.36%		
9	1.77%	18	2.36%	27	8.26%		

Using 'Battelle' tool, the result of this selection process is shown in Table 4 where the 'Factor' column lists all accepted scenarios (e.g. Scenario 6:  $P_2E_2S_1T_2H_1A_1C_3$ ).

We now calculate the probabilities of the scenarios (Step 5) selected in the previous step. Here, 'Bayesian' tool will call the external mathematical programming solver GLPK [15], to find a solution for the modified GP (1a)–(1h).

Based on the solution of this modified GP, the upper and lower bounds for all selected scenario probabilities are then obtained by re-solving the modified GP with new objective functions (2) with one extra constraint (3). The arithmetic mean of these probabilities is calculated and shown in Table 5.

Note that Scenario 12, 13, 15, 16, 20 and 29 were computed to have probability 0 throughout the parametric analysis, this is strong indication of these scenarios being implausible. So, subject to expert commen-

tary, these scenarios could be omitted from further consideration.

In the final step, 'Clustering' tool for choosing representative scenarios or 'IP' tool for searching a balanced mix of plausible scenarios can be used.

### Cluster Analysis

The 'Clustering' tool will call the statistical package R [8] to run several sets of trials (e.g. with 3, 4 and 5 clusters). Table 6 shows the results of an analysis with three clusters. Table 7 displays the various statistical indicators for each cluster and also proposes a representative scenario.

To determine which set of clusters are optimal, an average compatibility rating for all scenarios within each cluster is calculated, and subsequently compared to determine which number obtains a maximum average compatibility rating.



Table 6: Sample of Cluster Analysis

Cluster	Scenario	Average Compability Value	Probability
1	1, 4, 5, 7, 8, 9, 10, 14, 17, 18, 19, 21, 22, 23, 24, 25, 27, 28, 30, 31, 32, 34	3.280	77.0%
2	2, 26, 33	3.480	12.1%
3	3, 6, 11	3.440	10.9%

Table 7: Cluster Statistics and Representative Scenarios

	Factor	Mean	Mode	Median	Maximum	Minimum	Representative Scenario
Cluster 1	P	2.43	3	3	3	1	3
	E	1.57	2	2	2	1	2
	S	1.57	2	2	2	1	2
	T	1.64	2	2	2	1	2
	H	1.29	1	1	2	1	1
	A	1.79	2	2	3	1	2
	C	1.86	2	2	3	1	2
Cluster 2	P	2.38	2	2	3	1	2
	E	1.77	2	2	2	1	2
	S	1.54	2	2	2	1	2
	T	1.69	2	2	2	1	2
	H	1.46	1	1	2	1	1
	A	2.00	2	2	3	1	2
	C	2.08	2	2	3	2	2
Cluster 3	P	2.75	3	3	3	2	3
	E	1.25	1	1	2	1	1
	S	1.75	2	2	2	1	2
	T	1.75	2	2	2	1	2
	H	1.50	2	1.5	2	1	2
	A	1.75	2	2	2	1	2
	C	2.00	2	2	3	1	2

The representative scenarios (Table 7) may not correspond entirely to possible real scenarios. We may use them as end-state scenarios and others in their cluster as transition scenarios while the clusters might represent different branches on a scenario tree (e.g. [20]).

be insignificant in the futures (no scenario has these factors). In IP model (4),  $N_i$  thus takes the value  $\{2, 2, 0, 2, 2, 0, 2, 2, 0, 2, 2, 0, 2, 2, 0, 2, 2, 0, 2, 2, 0\}$  for each factor respectively. The 'IP' tool will call GLPK solver [15] to find a solution to this model. Scenario 3, 4, 6, 14, 18 and 33 are listed for this smallest set of scenarios.

### Integer Programming's

To illustrate another possibility for selecting a minimum number of plausible alternate futures, to be expanded into scenario theme descriptions, we use the selected scenarios in Table 4, with the omission of implausible Scenario 12, 13, 15, 16, 20 and 29.

If we want to find a smallest number of scenarios that cover each factor twice except that the factors  $P_3, E_3, S_3, T_3, H_3, A_3, A_4,$  and  $C_3$  (see Table 2 for the description of the factors) which are believed to

If the total probability of the futures in which the factor occurs is set by the user, then IP model (5) must be used. For example, we set  $P_i$  (where  $\sum_{i=1}^{22} P_i \leq 1$ ) to the value  $\{\frac{1}{14}, \frac{1}{14}, 0, \frac{1}{14}, \frac{1}{14}, 0, \frac{1}{14}, \frac{1}{14}, 0, \frac{1}{14}, \frac{1}{14}, 0, \frac{1}{14}, \frac{1}{14}, 0, \frac{1}{14}, 0, 0, \frac{1}{14}, \frac{1}{14}, 0\}$ , respectively, for each total probability of factor and use the corresponding scenario probabilities listed in Table 5 as the values for  $Y_k$ . The IP tool calls GLPK solver which outputs Scenario 1, 3, 5, 6, 24 and 32 as a solution.

## Conclusion

The approach of combining scenario analysis methods offers an attraction that starting with relatively simple information from experts and problem-owners, main scenarios, as well as a balanced mix of these plausible futures for scenario development can be determined. Using systematic approach, strategic planning can rationalise the often ad hoc process of selecting futures for scenario development.

A computer decision support tool, similar to the strategic planning tool suite presented in this paper, needs to be used to automate all numerical calculations in each step of the combining methods. Although, analysing and interpreting data and results must be cautiously scrutinised by experts.

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## References

- [1] J. Brauers and M. Weber. A New Method of Scenario Analysis for Strategic Planning. *Journal of Forecasting*, **7**:31–47, 1988.
- [2] R. G. Coyle, R. Crawshay, and L. Sutton. Futures Assessment by Field Anomaly Relaxation. *Futures*, **26**(1): 25–43, 1994.
- [3] R. G. Coyle and G. R. McGlone. Projecting Scenarios for South-east Asia and the South-west Pacific. *Futures*, **27**(1):65–79, 1995.
- [4] C. A. De Kluyver and H. Moskowitz. Assessing Scenario Probabilities via Interactive Goal Programming. *Management Science*, **30**(3):273–278, 1984.
- [5] C. Dilek. *Scenario Analysis Tool Suite - User Guide*. Submitted for publication in General Document Series, Defence Science and Technology Organisation, Australia, 2008.
- [6] Guy Duczynski. Effects-Based Operations between Australia and the United States: Achieving Interoperability at the Strategic Level through Shared End-States. *Security Challenges, Special Edition: Effects-Based Strategy*, **2**(1):75–89, 2006.
- [7] Michel Godet, François Bourse, Pierre Chapuy, and Isabelle Menant. *Futures studies: a tool-box for problem solving*. UNESCO, 1991.
- [8] Kurt Hornik. The R FAQ, 2007. <http://CRAN.R-project.org/doc/FAQ/R-FAQ.html>. Accessed on December, 2007.
- [9] Larry Jenkins. Selecting a Variety of Futures for Scenario Development. *Technological Forecasting and Social Change*, **55**:15–20, 1997.
- [10] Peter Johnston. *Alternate Futures Scenario Planning*. (Research Note RN 2003/10), Department of National Defence Canada, Operational Research Division, Directorate of Operational Research (Corporate), 2003.
- [11] Richard R. Laferriere and Stephen M. Robinson. Scenario Analysis in US Army Decision Making. *Military Operations Research Society, PHALANX*, (March Issue):10–14, 2000.
- [12] R. E. Linneman and H. E. Klein. The use of multiple scenarios by U.S. industrial companies. *Long Range Planning*, **12**: 83–90, 1979.
- [13] R. E. Linneman and H. E. Klein. The use of multiple scenarios by U.S. industrial companies: a comparison study, 1977–1981. *Long Range Planning*, **16**: 94–101, 1983.
- [14] H. Linstone and M. Turoff. *The Delphi Method: Techniques and Applications*. Addison-Wesley, 1975.
- [15] A. Makhorin. *GLPK (GNU Linear Programming Kit)*. <http://www.gnu.org/software/glpk/glpk.html>. Accessed on December, 2006.
- [16] J. P. Martino and K. Chen. Cluster analysis of cross impact model scenarios. *Technology Forecasting and Social Change*, **12**:61–71, 1978.
- [17] M.-T. Nguyen and M. Dunn. *Some Methods for Scenario Analysis in Defence Strategic Planning*. Will appear in Technical Report Series, Defence Science and Technology Organisation, Australia, 2008.

- [18] R. Rhyne. Technological forecasting within alternative whole futures projections. *Technological Forecasting and Social Change*, **6**:133–162, 1974.
- [19] R. Rhyne. Whole-Pattern Futures Projection, Using Field Anomaly Relaxation. *Technological Forecasting and Social Change*, **19**:331–360, 1981.
- [20] R. Rhyne. Field anomaly relaxation: the art of usage. *Futures*, **27**:657–674, 1995.
- [21] T Ritchey. Problem structuring using computer-aided morphological analysis. *Journal of the Operational Research Society*, **57**:792–801, 2006.
- [22] R. K. Sarin. A sequential approach to cross impact analysis. *Futures*, **10**:53–62, 1978.
- [23] R. K. Sarin. An approach for long-term forecasting with an application to solar electric energy. *Management Science*, **25**:543–554, 1979.
- [24] Ashley KW Stephens. *Future Urban States: a Field Anomaly Relaxation Study*. (DSTO-TR-1910), Defence Science and Technology Organisation, Australia, 2006.
- [25] Sun Microsystems and CollabNet. *NetBeans IDE 6.0*. <http://www.netbeans.org>. Accessed on January, 2008.
- [26] Larry W. Van Tassell, E. Tom Bartlett, and John E. Mitchell. *Projected use of grazed forages in the United States: 2000 to 2050: A technical document supporting the 2000 USDA Forest Service RPA Assessment*. Gen. Tech. Rep. RMRS-GTR-82. Fort Collins, CO: U.S. Department of Agriculture, Forest Service, Rocky Mountain Research Station, 2001.
- [27] Jung-Fa Tsai, Ming-Hua Lin, and Yi-Chung Hu. Finding multiple solutions to general integer linear programs. *European Journal of Operational Research*, **184**:802–809, 2008.
- [28] U. von Reibnitz. *Scenario Techniques*. McGraw Hill, New York, USA, 1985.
- [29] F. Zwicky. *Discovery, Invention, Research through the Morphological Approach*. Macmillan, New York, USA, 1967.

## A Note on Two-Stage Interval Time Minimization Transportation Problem

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### Abstract

This paper discusses two stage interval time minimization transportation problem, where minimum amount available at each source is shipped to the destinations in the first stage & enough quantity of the product is dispatched in second stage so as to meet the demand at destinations exactly. An iterative algorithm is proposed to find a solution that minimizes the sum of first and second stage shipment times.

### Introduction

Hammer [2] first discussed the time minimization transportation problem (TMTP) in 1969. The mathematical structure proposed by Hammer [2] for this problem is as follows:

$$\min_{X=\{x_{ij}\} \in S} [\max_{I \times J} [t_{ij}(x_{ij})]]$$

where the set S is given by

$$S: \left\{ X = (x_{ij}) \in R^{mn} \begin{cases} \sum_{j \in J} x_{ij} = a_i & \forall i \in I, \\ \sum_{i \in I} x_{ij} = b_j & \forall j \in J, \\ x_{ij} \geq 0 & \forall (i, j) \in I \times J. \end{cases} \right.$$

This problem attracted the interest of many scholars, who later on tried this problem and proposed different solution methodologies. In literature, some of the available algorithms to solve this problem are given by Szwarc [8], Garfinkel et al. [3], Bhatia et al. [1], Prakash [6] and Arora and Puri [4, 5]. Sonia et al. [7] in 2004 discussed an invariant of this problem in the form of two stage interval time minimization

transportation problem, where in first stage, the sources ship all of their on-hand material to the demand points and the second stage shipment covers the demand that is not fulfilled in first stage. In each stage, aim is to minimize the duration of transportation and the overall goal is to minimize the sum of two stage shipment times. Mathematical formulation of the problem considered by them is as follows:

Let  $a_i$  and  $a'_i, i \in I$  denote respectively the minimum and maximum availability of a homogeneous product at the source  $i$  and  $b_j, j \in J$  the demand of the same at destination  $j$ , where

$$\sum_{i \in I} a_i < \sum_{j \in J} b_j < \sum_{i \in I} a'_i .$$

In the first stage of the two stage Interval (TMTP), the quantity  $a_i (< a'_i)$  is shipped from each source  $i, i \in I$  and after the completion, enough quantity of the product is dispatched in second stage so as to exactly satisfy the demand  $b_j$  at the destination  $j, j \in J$ . The stage-I problem is thus formulated as:

$$\min_{Y=\{y_{ij}\} \in S'} [\max_{I \times J} (t_{ij}(y_{ij}))] = \min_{Y \in S'} [T_1(Y)]$$

where the set  $S'$  is given by

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$$S' : \begin{cases} \sum_{j \in J} y_{ij} = a_i & \forall i \in I, \\ \sum_{i \in I} y_{ij} \leq b_j & \forall j \in J, \\ y_{ij} \geq 0 & \forall (i, j) \in I \times J. \end{cases}$$

Corresponding to a feasible solution  $Y = \{y_{ij}\}$  of stage-1 problem, let  $S'(Y)$  be the set of feasible solutions of Stage-2 problem which is stated below :

$$\min_{Z = \{z_{ij}\} \in S'(Y)} [\max_{I \times J} (t_{ij}(z_{ij}))] = \min_{Z \in S'(Y)} [T_2(Z)]$$

where the set  $S'(Y)$  is given by

$$S'(Y) : \begin{cases} \sum_{j \in J} z_{ij} \leq a'_i - a_i & \forall i \in I, \\ \sum_{i \in I} z_{ij} = b_j - b'_j & \forall j \in J, \\ z_{ij} \geq 0 & \forall (i, j) \in I \times J, \end{cases}$$

$$\text{and } b'_j = \sum_{i \in I} y_{ij}, j \in J.$$

Thus a two stage time minimization transportation problem can be defined as:

$$(P) \min_{Y = \{y_{ij}\} \in S'} [(T_1(Y)) + \min_{Z \in S'(Y)} [(T_2(Z))]]$$

Closely related to the problem (P) is the interval time minimizing transportation problem ( $P_\alpha$ ) defined as:

$$(P_\alpha) \min_{X \in S} [T(X)] = \min_{X \in S} [\max_{I \times J} (t_{ij}(x_{ij}))]$$

where

$$S : \begin{cases} a_i \leq \sum_{j \in J} x_{ij} \leq a'_i & \forall i \in I, \\ \sum_{i \in I} x_{ij} = b_j & \forall j \in J, \\ x_{ij} \geq 0 & \forall (i, j) \in I \times J. \end{cases}$$

Clearly a feasible solution of (P) provides a

feasible solution to the problem ( $P_\alpha$ ) and conversely. Associated with the problem ( $P_\alpha$ ) a balanced transportation problem is defined as:

$$(P_\beta) \min_{X \in \hat{S}} [\hat{T}(X)] = \min_{X \in \hat{S}} [\max_{\hat{I} \times \hat{J}} (\hat{t}_{ij}(x_{ij}))]$$

where

$$\hat{S} : \begin{cases} \sum_{j \in \hat{J}} x_{ij} = \hat{a}_i & \forall i \in \hat{I}, \\ \sum_{i \in \hat{I}} x_{ij} = \hat{b}_j & \forall j \in \hat{J}, \\ x_{ij} \geq 0 & \forall (i, j) \in \hat{I} \times \hat{J}. \end{cases}$$

where,

$$\hat{I} = \{1, 2, \dots, m, m+1, \dots, 2m\},$$

$$\hat{J} = J \cup \{n+1\},$$

$$\hat{a}_i = a_i, i \in I,$$

$$\hat{a}_{m+i} = a'_i - a_i, i \in I,$$

$$\hat{b}_j = b_j \forall j \in J,$$

$$\hat{b}_{n+1} = \sum_{i \in I} a'_i - \sum_{j \in J} b_j,$$

$$\hat{t}_{ij} = t_{ij} \forall (i, j) \in I \times J,$$

$$\hat{t}_{m+i, j} = t_{ij} \forall (i, j) \in I \times J,$$

$$\hat{t}_{i, n+1} = M \forall i \in I,$$

where M is a very large positive number,

$$\hat{t}_{m+i, n+1} = 0 \forall i \in I.$$

It has been proved by Sonia et al. [7], that ( $P_\alpha$ ) and ( $P_\beta$ ) are equivalent. In their method two sequences of Stage I and Stage II time are generated. One of the sequences consists of generating pairs of the form ( $T_1(\cdot), T_2(\cdot) : T_1(\cdot) > T_2(\cdot)$ ) by solving time minimization transportation problem of the form  $P_{LB}(T_2(\cdot))$  and cost minimization transportation problem of the form  $CP_{LB}(T_1(\cdot), T_2(\cdot))$  where the problem  $P_{LB}(T_2(\cdot))$  reduces the on hand shipment time for Stage II, and the problem  $CP_{LB}(T_1(\cdot), T_2(\cdot))$  gives the minimum shipment time for Stage II corresponding to the Stage I shipment time obtained from

$P_{LB}(T_2(\cdot))$ . Similarly the sequence of two stage shipment time of the form  $(T_1(\cdot), T_2(\cdot) : T_1(\cdot) < T_2(\cdot))$  is obtained by solving the problems  $P_{UB}(T_1(\cdot))$  and  $CP_{UB}(T_2(\cdot), T_1(\cdot))$ , where these problems play a similar role as played by  $P_{LB}(T_2(\cdot))$  and  $CP_{LB}(T_1(\cdot), T_2(\cdot))$  with their role for Stage I and Stage II reversed. Further it has been established by them theoretically that the global minimum value of the problem  $(P)$  is obtained from these generated pairs.

The algorithm developed in the current paper generates only one sequence of Stage I and Stage II time, where at each iteration, Stage I time decreases strictly and Stage II time increases.

### Theoretical Development

As shipment time in Stage-I and Stage-II are concave functions, two stage interval time minimization transportation problem aims at minimizing a concave function over a polytope. Hence  $(P)$  is also a concave minimization problem. As the global minimum of a concave minimization problem is attained at an extreme point only, it is desirable to investigate only its extreme points. Let the set of transportation time on various routes is partitioned into a number of disjoint sets,  $B_h, h = 1, 2, \dots, s$ ,

where  $B_h = \{(i, j) \in I \times J : t_{ij} = t^h\}$  and  $t^j > t^{j+1} \forall j = 1, 2, \dots, s-1$ .

Positive weights say  $\lambda_{s-h+1}, h = 1, 2, \dots, s$  are attached to these sets where,  $\lambda_{j+1} \gg \lambda_j \forall j = 1, 2, \dots, s-1$ . This yields a standard (CMTP):

$$\min \sum_{h=1}^s \lambda_h \left( \sum_{(i,j) \in B_h} x_{ij} \right),$$

where  $X = \{x_{ij}\}$  belongs to the transportation polytope over which original (TMTP) is being studied. To find an (OFS) of the Stage II problem we define the following (CMTP):

$$(CP) \quad \min_{\hat{S}} \sum_{I \times J} c_{ij} x_{ij},$$

where

$$c_{i,n+1} = M \quad \forall i \in I,$$

$$c_{m+i,n+1} = 0 \quad \forall i \in I,$$

$$c_{ij} = 0 \quad \forall (i, j) \in I \times J,$$

$$c_{m+i,j} = \lambda_{s-h+1}; t_{m+i,j} = t^h, \forall (i, j) \in B_h$$

and  $h = 1, 2, \dots, s$ .

Let at any given time of Stage I and Stage II say,  $T_1^{k-1}, T_2^{k-1}$  respectively, where  $T_1^{k-1}, T_2^{k-1} \in \{t_1, t_2, \dots, t_s\}, k \in \{1, 2, \dots, s+1\}$ . The restricted version of the problem  $(CP)$ , denoted by  $(CP_k), k \geq 1$  is defined below:

$$(CP_k) \quad \min_{\hat{S}} \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$$

where

$$c_{ij} = M \text{ if } t_{ij} \geq T^{k-1}, (i, j) \in I \times J$$

$$= 0 \text{ if } t_{ij} < T^{k-1}, (i, j) \in I \times J$$

$$c_{i,n+1} = M \quad \forall i \in I,$$

$$c_{m+i,n+1} = 0 \quad \forall i \in I,$$

$$c_{m+i,j} = \lambda_{s-h+1}; t_{m+i,j} = t^h, (i, j) \in B_h$$

&  $h = 1, 2, \dots, s$ .

An (OFS) of the problem  $(CP)$  is denoted by  $Y^0$  with corresponding stage I time  $T_1^0$  and the stage II time by  $T_2^0$  and let  $Y^k$  be an (OFS) of  $(CP_k)$  yielding corresponding time of Stage I and Stage II as  $T_1^k$  and  $T_2^k$  respectively.

**Theorem 1.**  $T_2^k$  is the minimum time of stage II corresponding to any given time of Stage I in the problem  $(CP_k)$ .

**Proof:** Let if possible there exist a pair  $(T_1, T_2)$  yielded by some feasible solution  $Y = \{y_{ij}\}$  of  $(CP_k)$  such that  $T_2 < T_2^k$  and  $T_1 < T_1^{k-1}$  where  $T_2 = t_p$  and  $T_2^k = t_q$  for

some  $p, q \in \{1, 2, \dots, s\}$ . Since  $T_2 < T_2^k$ , therefore  $p > q$ , which implies  $s - p + 1 < s - q + 1$ .

Therefore

$$\begin{aligned} Z(Y) &= \sum_{i \times j} c_{ij} y_{ij} = \sum_{h=1}^s \lambda_{s-h+1} \left( \sum_{(i,j) \in B_h} y_{ij} \right) \\ &= \sum_{h=p}^s \lambda_{s-h+1} \left( \sum_{(i,j) \in B_h} y_{ij} \right), \end{aligned}$$

also

$$\begin{aligned} Z(Y^k) &= \sum_{i \times j} c_{ij} y_{ij}^k = \sum_{h=1}^s \lambda_{s-h+1} \left( \sum_{(i,j) \in B_h} y_{ij}^k \right) \\ &= \sum_{h=q}^s \lambda_{s-h+1} \left( \sum_{(i,j) \in B_h} y_{ij}^k \right). \end{aligned}$$

Since  $\lambda_{i+1} \gg \lambda_i, i = 1, 2, \dots, s-1$

$$\begin{aligned} \Rightarrow \sum_{h=p}^s \lambda_{s-h+1} \left( \sum_{(i,j) \in B_h} y_{ij} \right) &< \sum_{h=q}^s \lambda_{s-h+1} \left( \sum_{(i,j) \in B_h} y_{ij} \right) \\ \Rightarrow Z(Y) &< Z(Y^k). \end{aligned}$$

But this contradict the optimality of  $Y^k$ , therefore  $T_2^k \leq T_2$ .

**Theorem 2** (CP) gives optimal time of Stage II.

**Proof:** It follows on the same lines as proof of Theorem 1.

From Theorem 1 & 2 it is clear that  $T_2^0$  is the optimal time of Stage II, let the optimal time of Stage I is denoted by  $T_1^l$

**Remark 1.** By construction of  $(CP_k)$ , it is clear that  $T_1^0 > T_1^1 > \dots > T_1^l$  further it has also been observed that  $T_2^0 \leq T_2^1 \leq \dots \leq T_2^l$ , because let if possible  $T_2^{k+1} < T_2^k$ , for some  $k$ . Let  $Z_k = Z(Y^k), Z_{k+1} = Z(Y^{k+1})$ . Since  $T_2^{k+1} < T_2^k$ , we see that  $Z_{k+1} < Z_k$ . As  $T_2^{k+1} < T_2^k$ ,  $Y^{k+1}$  is a feasible solution of  $(CP_k)$  with  $Z_{k+1} < Z_k$ , a contradiction to

the fact that  $Y^k$  is an (OFS) of  $(CP_k)$ .

**Remark 2.** Since optimal time of Stage-1 problem is  $T_1^l$ , (OBFS) of  $(CP_{l+1})$  is not M-feasible.

**Remark 3.** Let  $T_1^0 = t^r$  for some  $r \in \{1, 2, \dots, s\}$  then the maximum number of iterations required to solve this problem is  $s - r + 1$ .

**Remark 4.** Let  $\hat{T} (= t^{\hat{r}}, \hat{r} \in \{1, 2, \dots, s\})$  be the overall time of transportation of the problem  $(P_\beta)$  defined by Sonia et al. [7], then the proposed method becomes better if  $4\hat{r} - r < 3s - 3$ .

**Theorem 3.** Let the generated pairs of Stage I and Stage II time be  $(T_1^k, T_2^k), k \geq 0$ . Then the optimal value of the problem (P) is given by  $\min_{\{h=0,1,\dots,l\}} [T_1^h + T_2^h]$ .

**Proof:** Let if possible there exists a pair  $(Y_1, Y_2)$  yielding Stage I time and Stage II shipment time  $(T_1, T_2)$  such that  $T_1 + T_2 < \min_{\{h=0,1,\dots,l\}} [T_1^h, T_2^h]$ . Since  $T_1^0 > T_1^1 \dots > T_1^l$  and  $T_2^0 \leq T_2^1 \dots \leq T_2^l$ , then the following cases arise:

**Case 1.**  $T_1 > T_1^0$ . (1)

By construction of  $(CP), (Y_1, Y_2)$  is a feasible solution of  $(CP)$ . Since  $T_2^0$  is the optimal time for  $(CP)$ , therefore

$$T_2^0 \leq T_2. \quad (2)$$

Combining (1) and (2), we get,

$$\begin{aligned} T_1 + T_2 &> T_1^0 + T_2^0, \\ \Rightarrow T_1 + T_2 &> \min_{\{h=0,1,\dots,l\}} [T_1^h + T_2^h]. \end{aligned}$$

**Case 2.**  $T_1 < T_1^l$ .

Since  $T_1 < T_1^l, (Y_1, Y_2)$  is an M-feasible solution of  $(CP_l)$ , which is a contradiction as this problem is not M-feasible.

**Case 3.**  $T_1 \in [T_1^0, T_1^l]$ .

In this case, either  $T_1 = T_1^k$  for some  $k = 0, 1, \dots, l$  or

$$T_1 \in (T_1^k, T_1^{k+1}) [\because T_1^{k+1} > T_1^k].$$

(i) If  $T_1 = T_1^0$ , then by construction of  $(CP)$ ,  $(Y_1, Y_2)$  is a feasible solution of  $(CP)$ .

$\Rightarrow T_2 \geq T_2^0$ , [ $\because T_2^0$  is the optimal time of stage II in  $(CP)$ ]

$$\Rightarrow T_1 + T_2 \geq T_1^0 + T_2^0,$$

$$\Rightarrow T_1 + T_2 \geq \min_{\{h=0,1,\dots,l\}} [T_1^h + T_2^h].$$

Similarly for the case when  $T_1 = T_1^k, k \in \{1, 2, \dots, l\}$  it can be shown that

$$T_1 + T_2 \geq T_1^k + T_2^k \geq \min_{\{h=0,1,\dots,l\}} [T_1^h + T_2^h].$$

(ii)  $T_1 \in (T_1^k, T_1^{k+1})$ .

Then  $(Y_1, Y_2)$  is a feasible solution of  $(CP_k)$  [ $\because T_1 < T_1^{k+1}$ ]. Also  $T_2 \geq T_2^k$  and  $T_1 > T_1^k$ ,

$$\Rightarrow T_1 + T_2 > T_1^k + T_2^k,$$

$$\Rightarrow T_1 + T_2 > \min_{\{h=0,1,\dots,l\}} [T_1^h + T_2^h].$$

Therefore there does not exist a feasible solution  $Y = (Y_1, Y_2)$  of  $(CP_k)$  yielding time less than  $\min_{\{h=0,1,\dots,l\}} [T_1^h + T_2^h]$ . Thus the optimal value of  $(P)$  is given by

$$\min_{\{h=0,1,\dots,l\}} [T_1^h + T_2^h].$$

### The Procedure

**Initial Step.** Find an (OBFS) of  $(CP)$  and thus obtain the corresponding times  $T_1^0$  and  $T_2^0$  of Stage I and Stage II respectively.

**General Step.** If  $k \geq 1$  at a given pair  $(T_1^{k-1}, T_2^{k-1})$  of Stage I and Stage II times, solve the problem  $(CP_k)$ . From the (OBFS)

of  $(CP_k)$  construct the pairs  $(T_1^{k+1}, T_2^{k+1})$ .

**Terminal Step.** If (OBFS) of problem  $(CP_k)$  is not M-feasible, then Stop. The optimal value of  $(P)$  is given by  $\min_{\{h=0,1,\dots,k\}} [T_1^h + T_2^h]$ .

### Numerical Illustration

Consider the two stage interval time minimization transportation problem given in Table 1. The problem considered here is same as discussed by Sonia et al [7].

The partition of various time routes is given by

$$t^1 (= 59) > t^2 (= 48) > t^3 (= 40) > t^4 (= 38) > t^5 (= 26) > t^6 (= 23) > t^7 (= 20) > t^8 (= 19) \text{ as } t^s = t^8 = 19, \text{ therefore } s = 8.$$

The corresponding problem  $(P_\beta)$  is shown in Table 2.

An (OBFS) of the problem  $(CP)$  yields Stage I time as  $T_1^0 = 40$  and Stage II time as  $T_2^0 = 19$ , where 19 is the optimal time of stage II. Next pair is obtained by solving the time minimization transportation problem  $(CP_1)$ , an (OBFS) of which yields Stage I time as 38 and Stage II time as 20, where 20 is the minimum time for Stage II corresponding to the stage I time 38. Similarly proceeding in the same way after solving further restricted problem  $(CP_2)$ , the pair obtained is (26,38) and (23,40) is obtained by solving  $(CP_3)$ . Algorithm terminates here as  $(CP_4)$  is no more M-feasible. Thus

$$\min \{40+19, 38+20, 26+38, 23+40\} = 58.$$

Hence the optimal value of problem  $(P)$  corresponds to the pair (38,20). The transportation schedule which gives this optimal value is shown in the Table 3.



Table 1

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	a <sub>i</sub>	a' <sub>i</sub>
S <sub>1</sub>	26	23	59	38	19	20	6	8
S <sub>2</sub>	40	48	20	19	23	59	15	29
S <sub>3</sub>	26	38	48	20	19	40	12	18
b <sub>j</sub>	6	9	3	14	10	5		

Table 2

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>	â <sub>i</sub>
S <sub>1</sub>	26	23	59	38	19	20	M	6
S <sub>2</sub>	40	48	20	19	23	59	M	15
S <sub>3</sub>	26	38	48	20	19	40	M	12
S <sub>1</sub>	26	23	59	38	19	20	0	2
S <sub>2</sub>	40	48	20	19	23	59	0	14
S <sub>3</sub>	26	38	48	20	19	40	0	6
$\hat{b}_j$	6	9	3	14	10	5	8	

Table 3

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>	â <sub>i</sub>
S <sub>1</sub>	0	0 <b>3</b>	M	0	0	0 <b>3</b>	M	6
S <sub>2</sub>	M	M	0 <b>3</b>	0 <b>2</b>	0 <b>10</b>	M	M	15
S <sub>3</sub>	0 <b>6</b>	0 <b>6</b>	M	0	0	M	M	12
S <sub>4</sub>	λ <sub>4</sub>	λ <sub>3</sub>	λ <sub>8</sub>	λ <sub>5</sub>	λ <sub>1</sub> <b>0</b>	λ <sub>2</sub> <b>2</b>	0	2
S <sub>5</sub>	λ <sub>6</sub>	λ <sub>7</sub>	λ <sub>2</sub>	λ <sub>1</sub> <b>12</b>	λ <sub>3</sub>	λ <sub>8</sub>	0 <b>2</b>	14
S <sub>6</sub>	λ <sub>4</sub>	λ <sub>5</sub>	λ <sub>7</sub>	λ <sub>2</sub>	λ <sub>1</sub>	λ <sub>6</sub>	0 <b>6</b>	6
$\hat{b}_j$	6	9	3	14	10	5	8	

An (OBFS) of the problem ( $CP_1$ ) is depicted in Table 3, where the entries in the lower left hand corner represents the associated cost and the highlighted entries show the values of the basic variables.

### Concluding Remarks

Present methodology tries to reduce the computational complexity as only one sequence of Stage I and Stage II pairs is adopted in contrast to the two way

procedure discussed by Sonia et al. [7] and problems such  $P_{LB}$  and  $CP_{LB}$  are avoided as there is no need to reduce Stage II time separately corresponding to the given time of Stage I.

As mentioned in Remark 4, for certain values of  $r$ , convergence rate of the proposed algorithm is better than the one discussed by Sonia et al. [7].

For the same problem as discussed by Sonia et al. [7], the current algorithm

suggests maximum number of 6 iterations as compared to the maximum 14 iterations suggested by Sonia et al. [7].

### Acknowledgement

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### References

- [1] H.L. Bhatia K. Swarup and M.C. Puri, "A procedure for time minimizing transportation problem." *Indian Journal of Pure and Applied Mathematics* vol. 8(8), pp. 920-929, 1977.
- [2] P.L. Hammer, "Time minimization transportation problem." *Naval Research Logistics Quarterly*, vol. 18, pp. 345-357, 1969.
- [3] R.S. Garfinkel and M.R. Rao, "The bottleneck transportation problem" *Naval Research Logistics Quarterly*, vol. 18, pp.

465-472, 1971.

- [4] S. Arora and M.C. Puri, "On lexicographic optimal solutions in transportation problem." *Optimization*, vol. 39, pp. 383-403, 1997.
- [5] S. Arora and M.C. Puri, "On a standard time transportation problem." *ASOR Bulletin*, vol. 20(4), pp. 2-14, 2001.
- [6] S. Prakash, "On minimizing the duration of transportation." *Proceedings of Indian Academy of Science*, vol. 91(1), pp. 53-57, 1982.
- [7] Sonia, R. Malhotra and M.C. Puri, "Two stage interval time minimization transportation problem." *ASOR Bulletin* vol. 23(1), pp. 2-14, 2004.
- [8] W. Szwarc, "Some remarks on time transportation problem." *Naval Research Logistics Quarterly* vol. 18, pp. 465-472, 1971.

## The 20th National Conference of the Australian Society for Operations Research 2009

28-30 September 2009,  
Gold Coast, Australia

For further details visit  
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## A Report on Operations Research Conference at Canberra

On the 7 & 8 July, 2008, the Defence and Security Applications Research Centre (DSARC) at UNSW@ADFA in conjunction with the Research Network for a Secure Australia (RNSA) and the Canberra Chapter of the Australian Operations Research Society (ASOR) held their inaugural OR conference.

The structure of the conference was to invite a range of OR professionals from different backgrounds to give their perspectives on what OR is, what it can do and where it should be heading. This conference was intended to be interactive and generous question and answer times were included in the program.

Prof John Baird, Rector of UNSW@ADFA welcomed the 40 strong attendees followed by an erudite opening speech from Dr Len Sciacca, Chief Operating Officer, DSTO. Both two days were full packed by exciting talks and discussions. Day 1 was basically dominated by hard-OR but fascinating topics. The Conference dinner rounded off a successful day one. The venue at Old Parliament House gave the attendees the opportunity to further discuss OR applications and chat informally with the practitioners and their counterparts. Day two of the Conference again saw some more thought provoking topics and attendees were particularly regaled by Prof Mosche Sniedovich with his presentation, "Responsible decision-making in the face of severe uncertainty". The Conference was deemed a success with much positive feedback from all.

Four renown OR academics flew in from different parts of Australia. They were Prof Natasha Boland from the University of Newcastle, Prof Lou Cacetta from Curtin University, Prof Mosche Sniedovich from Melbourne University and Prof Amrik Sohal from Monash university. Among other presenters, Dr Jeremy Manton, Prof. Neville Curtis, Dr. Bruce Fairlie and Dr. Paul Whitbread from DSTO; Dr Richard Davis, Office of National Security, Prime Minister & Cabinet Department; Mr Russell Hay and Mr Paul Trushell, Geoscience Australia and Attorney General Department; and Prof. Hussein Abbass and Dr Ruhul Sarker from UNSW@ADFA. Each of these hour long presentations commenced with an introduction to their specialist area, discussing advanced concepts, challenges and open problems in their area. A 10 minute interactive question time concluded the presentations. The detailed program is shown below.



## Program Day 1

- 8:30 Registration  
9:00 Prof. John Baird, Rector, UNSW@ADFA, Welcome  
9:05 Dr Len Sciacca, DSTO, Opening  
9:15 Dr. Jeremy Manton, DSTO, The need for OR in organisations  
10:00 Morning Coffee
- 10:30 Prof. Natasha Boland, University of Newcastle, Progress and challenges in linear programming, integer programming, and their applications  
11:30 Prof. Louis Caccetta, Curtin University, Effective computational models for constrained path problems  
12:30 Lunch [Officers Mess]
- 13:30 Prof. Amrik Sohal, Monash University, Applying Operations Research to Designing and Managing Supply Chains  
14:30 Dr. Ruhul Sarker, UNSW@ADFA, Why use evolutionary computation for solving optimisation problems?  
15:30 Afternoon Coffee
- 16:00 Prof. Neville Curtis, DSTO, Operations Research at the front end  
17:00 Close  
1900-2200 Conference Dinner, Old Parliament House, King George Terrace, Parkes ACT 2600]

## Program Day 2

- 9:00 Dr. Richard Davis, Office of National Security, Prime Minister & Cabinet Department, Challenges for OR in the national security domain  
10:00 Morning Coffee
- 10:30 A/Prof. Moshe Sniedovich, Melbourne University, Responsible decision-making in the face of severe uncertainty  
11:30 Prof. Hussein Abbass, UNSW@ADFA, Managing the known and preparing for the unknown: computational scenarios for analysis and planning  
12:30 Lunch [Officers Mess]
- 13:30 Dr. Bruce Fairlie, DSTO, Defence OR in the air domain  
14:30 Dr. Paul Whitbread, DSTO, OR for command and control analysis  
15:30 Afternoon Coffee
- 16:00 Russell Hay & Paul Trushell, Geoscience Australia and Attorney General Department, CIPMA: A computational tool to support government & business decision making  
17:00 Close

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We invite you to try the Beta Version at [www.palgrave-journals.com/iaor](http://www.palgrave-journals.com/iaor) and provide feedback which will help us improve the final product. To activate the site, you will be asked to provide your email address and set up a password, which you may use each time after that for as long as the Beta Version is available. Each person who does so prior to the end of September and completes the short online User Survey will be entered into a drawing with the opportunity to win an iPod Nano.

The Beta Version contains approximately 20,000 indexed abstracts from the years 2002-2007, which for trial purposes is sufficient for a realistic test of literature searching. When released to the public, the new IAOR Online will contain more than 55,000 indexed Operations Research and Management Science abstracts from 1989 to the present, and will be updated weekly from the current literature.

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The more comments we receive now, the better this publication will become, so we greatly appreciate your help.

Hugh Bradley  
IFORS Project Manager

## Forthcoming Conferences

### **The XIV Latin-Ibero American Congress on Operations Research (CLAIO 2008)**

9–12 September 2008, Cartagena de Indias, Colombia

[www.socio.org.co/CLAIO2008/index\\_eng.php](http://www.socio.org.co/CLAIO2008/index_eng.php)

### **Operational Research Practice for Africa**

October 10 -11, 2008, Marriott Wardman Park Hotel, Washington, D.C.

<http://www.orpagroup.net/ORPA2008/index.html>

### **2008 IEEE International Conference on Systems, Man, and Cybernetics**

October 12-15, 2008, Suntec Singapore

<http://www.smc2008.org/>

### **3<sup>rd</sup> Int. Conference on Bio-inspired Optimization Methods and their Applications**

(BIOMA2008), 13 - 14 October 2008, Ljubljana, Slovenia

<http://bioma.ijs.si/conference/2008>

### **43<sup>rd</sup> Annual Conference of the ORSNZ**

24-25<sup>th</sup> November, Wellington, New Zealand

Website: [www.orsnz.org.nz](http://www.orsnz.org.nz)

### **9<sup>th</sup> Asia-Pacific Industrial Eng. and Management Systems (APIEMS) Conference**

Bali, Indonesia, 3 - 5 December 2008

<http://www.apiems2008.org>

### **12<sup>th</sup> Asia Pacific Symposium on Intelligent and Evolutionary Systems (IES'08)**

7 - 8 December 2008, The University of Melbourne, Victoria, Australia

<http://www.complexity.org.au/ies2008/>

### **International conference on “Operations Research for a Growing Nation”**

15-17<sup>th</sup> December, 2008.

Sri Venkateswara University, Tirupati-517502, Andhra Pradesh, India

Website: [www.orsicon2008.com](http://www.orsicon2008.com)

### **The forth International Symposium on Scheduling (Int.S.S.09)**

4-6 July 2009, Nagoya, Japan

<http://www.fujimoto.mech.nitech.ac.jp/iss2009/>

### **EURO 2009 Conference**

July 5 – 8, 2009, Bonn

<http://www.euro-2009.de>

### **International Conference on Computers & Industrial Engineering (CIE39)**

July 6-8, 2009- Troyes, France

<http://www.utt.fr/cie39/>

### **18<sup>th</sup> World IMACS Congress and International Congress on Modelling and Simulation**

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We look forward to seeing you at the Conference.

Yours sincerely,  
Erhan Kozan  
Chair,  
ASOR Conference 2009



# BULLETIN

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It aims to provide news, world-wide abstracts, Australian problem descriptions and solution approaches, and a forum on topics of interests to Operations Research practitioners, researchers, academics and students.

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[1] Higgins, J.C. and Finn, R. Managerial Attitudes Towards Computer Models for Planning and Control. Long Range Planning, Vol. 4, pp 107-112. (Dec. 1976).

[2] Simon, H.A. The New Science of Management Decision. Rev. Ed. Prentice-Hall, N.J. (1977).

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