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## Editorial

In this issue, N. H. Shah and A. R. Patel have contributed a technical paper on *Optimal Pricing and Ordering Policy for Stock-Dependent Demand under Delay in Payments*. In addition, M.Valliathal and R. Uthayakumar have contributed a paper on *A Deterministic Two-Warehouse Inventory Model for Deteriorating Items with Stock-Level-ependent Demand Rate*. T. P. Hutchinson has prepared a technical note on *Interpretation of Data Showing Something has One Effect Sometimes and a Different Effect in other Circumstances: Theories of Interaction of Factors*. We are delighted to be publishing them here for the Bulletin readers.

I am pleased to inform you that the electronic version of ASOR Bulletin is now available at the ASOR national web site: http://www.asor.org.au/. Currently, the electronic version is prepared only as one PDF. We like to thank our web-master Dr Andy Wong for his hard work in redesigning and smoothly managing our national web site. Your comments on the new electronic version, as well as ASOR national web site, is welcome.

ASOR Bulletin is the only national publication of ASOR. I would like to request all ASOR members, ASOR Bulletin readers and OR organizations in the country to contribute to the ASOR Bulletin. The editorial policy is available either from the Bulletin web site or from the inside back cover of the Bulletin. The detailed instructions for preparing the manuscripts is available in the URL: http://www.asor.org.au/.

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# Optimal Pricing and Ordering Policy for Stock-Dependent Demand under Delay in Payments

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## Abstract:
In this paper, optimal pricing and ordering policy for inventory model for a retailer is developed when the demand is stock dependent and the supplier offers a trade credit. An algorithm for a retailer is suggested to maximize the total profit per unit time. The intuition that the cycle time and order quantity will increase under trade credit is contradicted. Numerical example is given to illustrate the proposed model. Sensitivity analysis for stock dependent parameter is carried out.

**Key – words:** Pricing, inventory, stock dependent demand, delay in payments.

## 1. Introduction:
In global market, supplier uses trade credit as a promotion tool to increase his sale and attract new retailers. Brigham (1995) gave financial management term "net credit". "net credit" means a supplier offers the retailer a time (say) 30 days, to settle the total amount against the purchases made. However, if the payment is not settled within the allowable trade credit period, the interest is charged on the unsold stock under the agreed terms and conditions. Therefore, a retailer can earn the interest on the generated revenue during permissible delay period and delay the payment up to the last day of the delay period offered by the supplier. The permissible trade credit reduces the retailer's holding cost because it reduces the amount of capital investment in stock for the duration of the offered trade credit. However, offering trade credit increases default risk to the supplier.

Goyal (1985) formulated an economic order quantity model under permissible delay in payment. He calculated interest earned on the purchase cost and concluded that the cycle time and ordering quantity increases marginally under the permissible delay in payments. Dave (1985) rectified Goyal's model by assuming that the selling price is necessarily higher than the purchase price. Shah (1993a, 1993b), Aggarwal and Jaggi (1995) then extended Goyal's model for deteriorating items. Jamal et al. (1997) generalized model to allow for shortages and deterioration. Hwang and Shinn (1997) derived optimal pricing and ordering policies for the retailer under the condition of trade credit. Liao et al. (2000) formulated an inventory model for stock dependent demand when delay in payments is permissible. Chang and Dye (2001) extended the model of Jamal et al. (1997) for time dependent deterioration. They assumed that the black logging rate is inversely proportional to the waiting time. Almost all above stated articles ignore the difference between the selling price and purchase cost. Jamal et al. (2000) and Sarker et al. (2000) computed interest earned on the selling price and concluded that the retailer should settle his account relatively sooner as the unit selling price increases relative to the unit purchase cost. Teng (2002) proved that it is beneficial for a well-established retailer to put order of smaller size and take the benefits of the permissible delay more frequently. Chang et al. (2003) determined an economic order quantity model for deteriorating items in which the supplier offers a trade credit to the retailer if the order quantity is greater than or equal to a pre-specified quantity. Teng et al. (2005) developed retailer's optimal ordering and pricing policy for a deteriorating inventory when demand is deterministic and constant.

In this paper, an attempt is made to develop a model from a retailer's point of view when supplier offers a permissible delay in payments. Demand is considered to be dependent on the stock displayed and selling price. Here, the retailer has to decide the unit selling price and the quantity to be replenished. In order to solve this problem, we use backward induction. Consequently, we first derive the optimal lot size for a given price and then determine the optimal sale price that maximizes retailer's total profit per unit time. The theoretical result suggests that the cycle time and order quantity decreases under the permissible trade credit. Computationally, it is established that a higher value of allowable trade credit lowers unit selling price and increases the profit.

## 2. Assumptions and Notations:

The proposed concept is formulated using following assumptions:

1. The inventory system under consideration deals with a single item.
2. Replenishment rate is infinite.
3. Shortages are not allowed.
4. The lead-time is zero or negligible.
5. The demand for the item is decreasing function of the selling price and increasing function of stock-dependent parameter. Consider $R\left(P, I\left(t\right)\right) = \alpha\left(1+\beta I\left(t\right)\right)P^{-\eta}$ where $\alpha$ constant demand, $\beta$ denotes rate of change of demand due to displayed stock is and $\eta > 1$ is constant price elasticity. $\alpha > 0$, $\alpha >> \beta$ and $0 \le \beta < 1$.
6. The supplier offers the retailer a credit period of (say) M days. During this time, the retailer deposits generated revenue in an interest bearing account. At the end of this period, the retailer pays off all units sold, keeping the rest for day-to-day expenses, and starts paying for the interest charges on the unsold stock.

In addition, the following notations are used throughout this paper:

h the unit inventory holding cost per year excluding the interest charges.

P the selling price per unit (a decision variable).

C the unit purchase cost with $C < P$.

A the ordering cost per order.

$I_c$ the interest charged per \$ in stock per year by the supplier.

$I_e$ the interest earned per \$ per annum by the retailer.

M the offered trade credit by the supplier to the retailer to settle the account against the purchases made.

Q the order quantity (a decision variable).

T the cycle time (a decision variable).

$I\left(t\right)$ the inventory level at any instant of time t, $0 \le t \le T$.

$R\left(P, I\left(t\right)\right)$ the demand given by $R\left(P, I\left(t\right)\right) = \alpha\left(1+\beta I\left(t\right)\right)P^{-\eta}$ where $\alpha > 0$ is constant demand $0 \le \beta < 1$ denotes rate of change of demand due to display of the stock. $\eta > 1$ price elasticity.

$Z\left(P, T\right)$ the total annual profit per unit time.

The total profit per unit time comprises of : (a) the sales revenue; (SR), minus (b) cost of placing orders; (OC), (c) cost of purchasing; (PC), (d) inventory holding cost excluding interest charges; (IHC), (e) cost of interest payable for unsold items after the permissible delay period, M (this occurs only when M < T ); (IC), plus (f) interest earned from the generated revenue during the permissible trade credit; (IE).

## 3. Mathematical Model:

The inventory level *I (t)* depletes due to stock-dependent demand and selling price of the unit. The rate of change of inventory is governed by the following differential equation:

$$\frac{dI(t)}{dt} = -R(P, I(t)), \qquad 0 \le t \le T \tag{1}$$

with the initial condition *I (0) = Q* and the boundary condition *I(T) = 0*. Consequently, the solution of (1) is given by

$$I(t) = \frac{1}{\beta}\left[e^{\alpha\beta P^{-\eta}(T-t)} - 1\right], \quad o \le t \le T \tag{2}$$

and the order quantity is

$$Q = \frac{1}{\beta}\left[e^{\alpha\beta P^{-\eta}T} - 1\right] \tag{3}$$

Next, we compute the different components of the total annual profit per unit time.

(a) Sales revenue; $SR = PR(P, I(T)) = \alpha P^{-\eta+1}$ (4)

(b) Cost of placing an order; $OC = \frac{A}{T}$ (5)

(c) Cost of purchasing; $PC = \frac{CQ}{T} = \frac{C}{\beta T}\left[e^{\alpha\beta P^{-\eta}T} - 1\right]$ (6)

(d) Inventory holding cost excluding interest charges;

$$IHC = \frac{h}{T}\int_0^T I(t)\, dt = \frac{h}{\alpha\beta P^{-\eta}T}\left[e^{\alpha\beta P^{-\eta}T} - \alpha\beta P^{-\eta}T - 1\right] \tag{7}$$

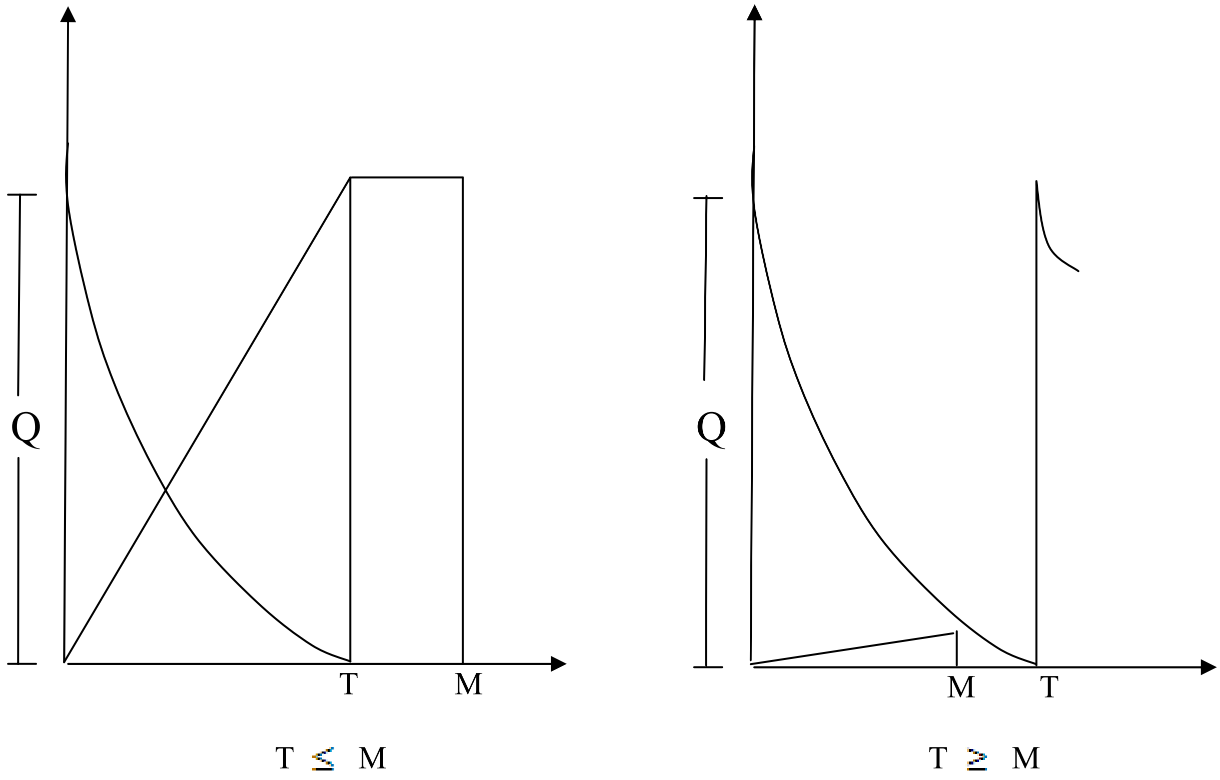


Fig. 1 The inventory - time graph

Regarding interest charged and earned (i.e. costs of (e) and (f)) we have following two cases depending on the lengths of T and M. These two cases are graphed in Fig. 1.

**Case: 1.** $T \le M$

In this case, the retailer sells $R\left(P, I\left(T\right)\right) T$ units by the end of the cycle time and has $CR\left(P, I\left(T\right)\right) T$ in his account to pay the supplier in full by the end of the credit period M. Hence, the interest earned per unit time is

$$IE_1 = \frac{PI_e}{T}\left[\int_0^T R\left(P, I\left(t\right)\right) t \, dt + R\left(P, I\left(T\right)\right) T\left(M - T\right)\right]$$

$$= \frac{PI_e}{\alpha\beta^2 P^{-\eta} T}\left[e^{\alpha\beta P^{-\eta} T} - \alpha\beta P^{-\eta} T - 1\right] + PI_e \alpha P^{-\eta}\left(M - T\right) \tag{8}$$

Therefore, the retailer's total profit per unit time; $Z_1\left(P, T\right)$ is

$$Z_1\left(P, T\right) = SR - PC - OC - IHC + IE_1 \tag{9}$$

**Case: 2.** $M \le T$

Using assumption (3), the buyer sells $R\left(P, I\left(M\right)\right) M$ - units by the end of the allowable trade credit M and has $CR\left(P, I\left(M\right)\right) M$ in his account to pay the supplier. The unsold items in stock are charged at interest rate $I_c$ by the supplier at the beginning of time T.

Hence, interest charged per unit time is

$$IC_2 = \frac{CI_c}{T}\int_M^T I\left(t\right) \, dt = \frac{CI_c}{\alpha\beta^2 P^{-\eta} T}\left[e^{\alpha\beta P^{-\eta}\left(T - M\right)} - \alpha\beta P^{-\eta}\left(T - M\right) - 1\right] \tag{10}$$

During the credit period, the retailer sells items and deposites the generated revenue into an account bearing account at the interest rate $I_e$ per dollar per year.

Therefore, the interest earned per unit time is

$$IE_2 = \frac{PI_e}{T}\int_0^M R\left(P, I\left(t\right)\right) t \, dt$$

$$= \frac{PI_e \, e^{\alpha\beta P^{-\eta} T}}{\alpha\beta P^{-\eta} T}\left[1 - e^{-\alpha\beta P^{-\eta} M} - \alpha\beta P^{-\eta} M \, e^{-\alpha\beta P^{-\eta} M}\right] \tag{11}$$

Hence, the total profit; $Z_2\left(P, T\right)$ per time unit is

$$Z_2\left(P, T\right) = SR - PC - OC - IHC - IC_2 + IE_2 \tag{12}$$

Hence, the total profit; $Z\left(P, T\right)$ per time unit is

$$Z\left(P, T\right) = \begin{cases} Z_1\left(P, T\right) \; ; & T \le M \\ Z_2\left(P, T\right) \; ; & M \le T \end{cases}$$

One can easily check that $Z_1\left(P, M\right) = Z_2\left(P, M\right)$. $Z_1\left(P, T\right)$ is continuous function of T either in $\left(0, M\right)$ or $\left(M, \infty\right)$ but not in both.

## 4. Determination of the optimal cycle time for any given price:

For low stock-dependent parameter, we have exponential series as

$$e^{\beta T} = 1+\beta T+\frac{(\beta T)^2}{2} \tag{13}$$

Hence, the total profit per time unit will be given by

$$Z_1(P,T) = AZ_1(P,T)$$

$$= (P-C)\alpha P^{-\eta} - \frac{C\alpha^2\beta P^{-2\eta}T}{2} - \frac{A}{T} - \frac{h\alpha P^{-\eta}T}{2} + PI_e\alpha P^{-\eta}\left(M-\frac{T}{2}\right) \tag{14}$$

and

$$Z_2(P,T) = AZ_2(P,T)$$

$$=(P-C)\alpha P^{-\eta} - \frac{C\alpha^2\beta P^{-2\eta}T}{2} - \frac{A}{T} - \frac{h\alpha P^{-\eta}T}{2} - \frac{(CI_c-PI_e)\alpha P^{-\eta}T}{2}$$

$$+CI_c\alpha P^{-\eta}M - \frac{(CI_c-PI_e)\alpha P^{-\eta}M^2}{2T} \tag{15}$$

Then, the approximation of total profit; $AZ(P,\ T)$ per time unit is

$$AZ(P,\ T)=\begin{cases} AZ_1(P,\ T) & ; \quad T\le M \\ AZ_2(P,\ T) & ; \quad M\le T \end{cases}$$

Note that the approximation is useful to obtain the closed – form solution for the optimal cycle time; T. By taking the first – order and second – order derivative of $AZ_k(P,\ T)$, for k = 1 and 2 with respect to T, we obtain

$$\frac{\partial AZ_1(P,T)}{\partial T} = \frac{A}{T^2} - \left[h + C\alpha\beta P^{-\eta} + PI_e\right]\alpha P^{-\eta}$$

(16)

$$\frac{\partial AZ_2(P,T)}{\partial T} = \frac{1}{T^2}\left[A+(CI_c-PI_e)\alpha P^{-\eta}M^2\right] - \frac{\alpha P^{-\eta}}{2}\left[C\alpha\beta P^{-\eta}+h+CI_c\right]$$

(17)

$$\frac{\partial^2 AZ_1(P,T)}{\partial T^2} = -\frac{2A}{T^3} < 0 \tag{18}$$

$$\frac{\partial^2 AZ_2(P,T)}{\partial T^2} = -\frac{2}{T^3}\left[A+(CI_c-PI_e)\alpha P^{-\eta}M^2\right] \tag{19}$$

The second order conditions given in eq. (18) suggests that for fixed P, $AZ_1(P,\ T)$ is concave function of T. Thus, there exists a unique value of $T=T_1$ which is given by

$$T_1=\sqrt{\frac{2A}{g_1\alpha P^{-\eta}}} \quad ; \tag{20}$$

where

$$g_1 = h + PI_e + C\alpha\beta P^{-\eta} \tag{21}$$

To ensure $T_1 \le M$ , we substitute (20) in the inequality $T_1 \le M$ , this holds if and only if

$$2A \le M^2 g_1\alpha P^{-\eta} \tag{22}$$

knowing $T_1$, the optimal purchase quantity $Q_1$ is given by

$$Q_1 = \sqrt{\frac{2A\alpha p^{-\eta}}{g_1}} + \frac{A\beta\alpha P^{-\eta}}{g_1} \tag{23}$$

Using (20), eq. (14) becomes function of *P* only.

Eq. (19) suggests that for fixed P, $AZ_2(P, T)$ is a concave function of T. Thus there exists a unique value of $T = T_2$, given by

$$T_2 = \sqrt{\frac{2L}{g_2 \alpha P^{-\eta}}} \tag{24}$$

where

$$L = A + \frac{1}{2}(CI_c - PI_e)\alpha P^{-\eta} M^2 \tag{25}$$

$$g_2 = h + CI_c + C\alpha\beta P^{-\eta} \tag{26}$$

Substituting (24) in inequality $T_2 \geq M$ gives $2A \geq \left(h + PI_e + C\alpha\beta P^{-\eta}\right)\alpha P^{-\eta} M^2$ (27)

and optimal procurement quantity $Q_2$ for case: 2 is given by

$$Q_2 = \sqrt{\frac{2L\alpha p^{-\eta}}{g_2}} + \frac{L\beta\alpha P^{-\eta}}{g_2} \tag{28}$$

Substituting (24) into eq. (15) reduces profit function to be only in terms of P.

When $\beta = 0$, the developed model reduces to that of Teng et al. (2005).

We have next theorem using (22) and (27).

**Theorem: 1.** For the low stock – dependent demand rate, we have

(1) If $2A < \left(h + PI_e + C\alpha\beta P^{-\eta}\right)\alpha P^{-\eta} M^2$ then $T^* = T_1$

(2) If $2A > \left(h + PI_e + C\alpha\beta P^{-\eta}\right)\alpha P^{-\eta} M^2$ then $T^* = T_2$

(3) If $2A = \left(h + PI_e + C\alpha\beta P^{-\eta}\right)\alpha P^{-\eta} M^2$ then $T^* = M$

**Proof:** It follows from (22) and (27)

Solving $2A = \left(h + P_0 I_e + C\alpha\beta P_0^{-\eta}\right)\alpha P_0^{-\eta} M^2$ (29)

We obtain value of $P_0$. Section 4 suggests that now approximated total profit per unit time is a function of *P* only.

i.e. $$AZ(P) = \begin{cases} AZ_1(P) \quad ; & P \leq P_0 \\ AZ_2(P) \quad ; & P \geq P_0 \end{cases} \tag{30}$$

To obtain the optimal price, differentiate (30) with respect to *P* and set it equal to zero.

## 5. An algorithm:

The steps to determine an optimal solution for the developed model are as follows:

Step 1: Determine $P_0$ by solving (29).

Step 2: Find $P_1$ by setting $\frac{\partial AZ_1(P)}{\partial P} = 0$. If $P_1 \le P_0$ then Case: 1 is optimal otherwise go to step 3.

Step 3: Find $P_2$ by setting $\frac{\partial AZ_2(P)}{\partial P} = 0$.

Step 4: Find corresponding cycle time and total profit per time unit.

## 6. Numerical Example:

Consider, the following parametric values $[\alpha, A, C, h, I_c, I_e, \eta]$ = $[10000, 10, 4.5, 0.5, 0.09, 0.06, 1.5]$ in proper units. Using the Algorithm, we obtain values of decision variables and objective function for different values of *M*. The results are exhibited in Table 1 and Table 2 for $\beta = 0.05$, $0.10$ respectively. The following observations are made based on Table 1 and Table 2.

1. The selling price increases with increase in the demand rate $\beta$ and decreases total profit per unit time of the retailer.
2. Increase in delay period *M* increases total profit per time unit but decreases values of optinal sale price and cycle time.
3. If $M \le T^*$, then a higher value *M* lowers optimum procurement quantity.
4. If $T^* \le M$, then increase in *M* increases optimum purchase units.

**Table 1:** Variations in delay period when $\beta = 0.05$

| M(days) | $P_0$ | P | T | Q(T) | AZ |
|---|---|---|---|---|---|
| 5 | 5.96537 | $P_2$=19.84566 | $T_2$=0.08188 | 11.40506 | 1491.8495 |
| 10 | 9.49560 | $P_2$=19.82114 | $T_2$=0.08162 | 11.38852 | 1492.4804 |
| 15 | 12.48589 | $P_2$=19.79135 | $T_2$=0.08127 | 11.36073 | 1493.1130 |
| 20 | 15.18748 | $P_2$=19.75629 | $T_2$=0.08082 | 11.32163 | 1493.7453 |
| 25 | 17.70588 | $P_2$=19.71594 | $T_2$=0.08027 | 11.27116 | 1494.3744 |
| 30 | 20.09879 | $P_1$=19.67598 | $T_1$=0.07973 | 11.22126 | 1498.0342 |
| 40 | 24.64258 | $P_1$=19.64419 | $T_1$=0.07954 | 11.22271 | 1501.7413 |
| 50 | 29.00183 | $P_1$=19.61250 | $T_1$=0.07936 | 11.22415 | 1505.4515 |
| 60 | 33.28582 | $P_1$=19.58092 | $T_1$=0.07918 | 11.22558 | 1509.1646 |
| 70 | 37.57309 | $P_1$=19.54944 | $T_1$=0.07899 | 11.22701 | 1512.8807 |

**Table 2:** Variations in delay period when $\beta = 0.10$

| M(days) | $P_0$ | P | T | Q(T) | AZ |
|---|---|---|---|---|---|
| 5 | 7.51289 | $P_2$=22.39073 | $T_2$=0.07028 | 8.76738 | 1402.6151 |
| 10 | 11.95294 | $P_2$=22.45154 | $T_2$=0.07006 | 8.75395 | 1403.1398 |
| 15 | 15.70798 | $P_2$=22.41528 | $T_2$=0.06974 | 8.73144 | 1430.6663 |
| 20 | 19.09428 | $P_2$=22.37112 | $T_2$=0.06934 | 8.69982 | 1404.1918 |
| 25 | 22.24448 | $P_2$=22.31904 | $T_2$=0.06883 | 8.65904 | 1404.7130 |
| 30 | 25.23090 | $P_1$=22.30005 | $T_1$=0.06872 | 8.65834 | 1408.7301 |
| 40 | 30.87999 | $P_1$=22.26398 | $T_1$=0.06858 | 8.65914 | 1412.2123 |
| 50 | 36.26805 | $P_1$=22.22802 | $T_1$=0.06841 | 8.65994 | 1415.6974 |
| 60 | 41.52765 | $P_1$=22.19218 | T1=0.06826 | 8.66073 | 1419.1853 |
| 70 | 46.75135 | $P_1$=22.15646 | T1=0.06810 | 8.661518 | 1422.6760 |

## 7. Conclusions:

In this study, optimal pricing and ordering model for a retailer is developed when demand is stock – dependent and the supplier offers a trade credit period using series expansion, closed – form optimal solution is established. The analytic results are obtained to decide retailer's optimal policy. Numerical example reveals that a higher value of the permissible delay period increases the total profit of the retailer but lowers selling price and cycle time.

The model can be generalized to allow for shortages, discounts, time dependent deterioration.

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# A Deterministic Two-Warehouse Inventory Model for Deteriorating Items with Stock-Level-Dependent Demand Rate

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## Abstract:
In this paper we discuss a deterministic two- warehouse inventory model for deteriorating items with stock-dependent demand rate. Shortages are not allowed. We study the effects of deterioration on the optimal cycle length, optimal order quantity,optimal quantity per shipment, the number of transporting items from RW to OW and the total profit of a deterministic two- warehouse inventory model. Necessary and sufficiency conditions for how much to order the optimum order quantity are given. Theoretical results are given to strengthen the model. Sensitivity analysis of the optimal solution with respect to the major parameter is carried out. Numerical examples are presented to demonstrate the developed model.

**Keywords:** Inventory, Deterministic model, Two-warehouse system, Deteriorating items.

## 1. Introduction
The warehouse storage capacity is defined as the amount of storage space needed to accommodate the materials to be stored to meet a desired service level which specifies the degree of storage space availability. Stock items to be delivered exactly when needed are impractical. Therefore, it is important to investigate the influence of warehouse capacity in various inventory policy problems. In recent years, various researchers have discussed a two-warehouse inventory system. This kind of system was first discussed by Hartley in Chap.12 [11]. Goswami and Chaudhuri [9] proposed an economic order quantity model for items with two levels of storage for a linear trend in demand. Sarma [21] studied a deterministic order level inventory model for deteriorating items with two storage facilities. Pakkala and Achary [18] studied a deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate. Zhou [25] developed an optimal EOQ model for deteriorating items with two warehouses and time varying demand and Kar et al. [14] established the deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon.

Most of the existing inventory models in the literature assume that items can be stored indefinitely to meet the future demands. However, certain types of commodities either deteriorate or become obsolete in the course of time and hence are unstable. Deterioration refers to the damage, spoilage, dryness, vaporization, etc. of the products. Inventory problems for deteriorating items have been studied extensively by many researchers from time to time. Research in this area started with the work of Whitin [22] who considered fashion goods deteriorating at the end of prescribed storage period. Ghare and Schrader [8] established a model for exponentially decaying inventories. Covert and Philip [4] studied an EOQ model for items with Weibull distribution deterioration. Dave [6] developed a discrete in time order level inventory model deteriorating items, Raafat et al.[19] provided an inventory model for deteriorating items and Heng et al. [12] studied an order level lot size inventory model for deteriorating items with finite replenishment rate.

The stock-dependency nature of demand rate is possible for most of the consumable goods. A retailer may display each of his items in large quantities to generate greater demand. Inventory models for a single deteriorating item with stock-dependent demand rate have been studied extensively in the last decade. Datta and Pal [5] developed a deterministic inventory system for deteriorating items with inventory level-dependant demand rate and shortages. Goyal and Giri [10] presented a review on inventory model for deteriorating items which gives useful information about two warehouse inventory problem. Yang [24] studied a two-warehouse partial backlogging inventory model for deteriorating items under inflation. Hsieh et al. [13] discussed a two- warehouse inventory system with deterioration and shortages using net present value. Banerjee and Agrawal [2] proposed a two-warehouse inventory model for items with three-parameter Weibull distribution deterioration, shortages and linear trend in demand. Chung et al. [3] developed a two-warehouse inventory model with imperfect quality production processes source. Gayen and Pal [7] provided a two ware house inventory model for deteriorating items with stock dependent demand rate and holding cost. Rong, et al. [20] presented a two warehouse inventory model for a deteriorating item with partially/fully backlogged shortage and fuzzy lead time. Recently, researches related to this area such as Niu and Xie [17], Kofjac et al. [15], Lee and Hsu [16], Yang [23], Zhou [26]and so on. However, a few two warehouse inventory models deal with inventory-level-dependent demand pattern, but they do not consider demand rate as a polynomial form of current inventory level except Zhou and Yang [27].

In this paper, we extend Zhou and Yang [27] model for deteriorating items. Here, we discuss a two-warehouse inventory model with deteriorating items under stock-level-dependent demand rate. The proposed model is suitable for all instantaneous deteriorating items stored in the warehouses. This paper is presented as follows. In section 2, the assumptions and notations are given. In section 3, we present the mathematical model. In section 4, numerical examples are given to illustrate the model. Finally, we conclude the paper.

## 2. Assumptions and notations:

To develop the mathematical model the following assumptions and notations are being made:

### *2.1 Assumptions*

- Replenishment is instantaneous with a known, constant lead time.
- The time horizon of the inventory system is infinite.
- The demand rate, R(t) is assumed to be dependent on the current inventory level and of polynomial form- that is to say R(t) = $\alpha I(t)^{\beta}$, $\alpha, \beta > 0$.The advantages of this type of demand –rate pattern can be found in Baker and Urban's paper [1].
- Shortages are not allowed to occur.
- The time of transporting items from RW to OW is ignored.
- There is no replenishment or repair of deteriorated units.
- The rented ware house has unlimited capacity. Each shipment from RW to OW will restore OW to W units, which means $q \le W$.

- Based on the practical observation, the transportation cost for q units per shipment is assumed as $T_c$(q) = A for 0< q≤y and $T_c$(q) = A + b (q-y) for y < q ≤W

  where y is the maximum number of units which can be shipped under a fixed transportation cost A and b is the variable charge to be paid for every additional unit after y.

### 2.2 Notations

| | |
|---|---|
| W | the storage capacity of OW |
| T | the length of replenishment cycle |
| Q | the replenishment quantity per replenishment |
| m | the number of transporting items from RW to OW |
| q | the quantity per shipment |
| $T_o$ | the fixed time interval between two successive shipments from RW to OW |
| $T_c(q)$ | the transportation cost for q units per shipment from RW to OW |
| P″ | the selling price per unit item |
| P | the purchasing cost per unit item |
| $h_{ow}$ | the holding cost per unit per unit time in OW |
| $h_{RW}$ | the holding cost per unit per unit time in RW and $h_{RW} \geq h_{OW}$ |
| $K_1$ | the fixed replenishment cost per replenishment for a two-warehouse system |
| K | the fixed ordering cost per order for a single warehouse system, generally, $K_1 \geq K$ (extra cost may be included for the two-warehouse system due to transportation) |
| I(t) | the inventory level at time t, $t\in [0,T]$ |
| R(t) | the demand rate at time t, $t\in [0,T]$ ( a function of current inventory level in OW) |
| $\theta$ | the constant deterioration rate where $0\leq \theta < 1$ |
| TP | the total profit per unit time |

## 3. Model formulation

Our problems to be discussed in this paper are:

- How the decision- maker knows whether or not to rent RW to hold more items under the situation defined above.
- What order lot-size shipment policy from RW to OW the decision- maker should make if he needs indeed to rent RW.

For answering the first question, we first simplify depict the single warehouse system.

### 3.1 Single warehouse model

The inventory system with a single warehouse can be stated as follows:

The inventory level of the system is Q (i.e. the order quantity) at the beginning of each replenishment cycle. An inventory level I(t) in the replenishment cycle [0,T] in OW satisfies the following differential equation:

$$\frac{dI(t)}{dt} = -\theta I(t) - \alpha I(t)^{\beta}; \qquad 0 \leq t \leq T \qquad (1)$$

The solution of the differential equation (1) is

$$I(t) = Q - t(\theta Q + \alpha Q^{\beta}) \qquad 0 \leq t \leq T \qquad (2)$$

and total profit is = $\frac{1}{T}$ {sales revenue- setup cost-holding cost-deterioration cost} (3)

Holding cost HC $= h_{OW} \int_0^T I(t)dt$

$$=h_{OW}\left[QT - \frac{T^2}{2}\left(\theta Q + \alpha Q^{\beta}\right)\right] \quad (4)$$

Setup cost = K (5)

$$\text{Deterioration cost} = P\,\theta\int_0^T I(t)dt$$

$$= P\theta\left[QT - \frac{T^2}{2}\left(\theta Q + \alpha Q^{\beta}\right)\right] \quad (6)$$

Therefore, total profit of single warehouse system is

$$TP_{OW} = \frac{1}{T}\left\{(P'-P)Q - K - (h_{OW} + \theta p)\left[QT - \frac{T^2}{2}\left(\theta Q + \alpha Q^{\beta}\right)\right]\right\} \quad (7)$$

$$I(T) = 0 \text{ implies } T = \frac{1}{\left(\theta + \alpha Q^{\beta-1}\right)} \quad (8)$$

Using (8) equation (7) becomes,

$$TP_{OW} = \left\{\left[(P'-P)Q - K\right]\left(\left(\theta + \alpha Q^{\beta-1}\right)\right) - (h_{OW} + \theta P)\left[Q - \frac{\left(\theta + \alpha Q^{\beta-1}\right)^{-1}}{2}\left(\theta Q + \alpha Q^{\beta}\right)\right]\right\} \quad (9)$$

The necessary conditions for $TP_{OW}$ (Q) to be maximum are:

- $\frac{dTP_{OW}}{dQ} = 0 \; and \; \frac{d^2TP_{OW}}{dQ^2} < 0$

$\frac{dTP_{OW}}{dQ} = 0$ implies:

$$\alpha(\beta-1)Q^{(\beta-2)}\left\{(P'-P)Q - K\right\} + \left(\theta + \alpha Q^{\beta-1}\right)(P'-P) - (h_{OW} + \theta P)$$

$$\left[1 - \frac{\left(\theta + \alpha Q^{\beta-1}\right)^{-1}}{2}\left(\theta + \alpha\beta Q^{(\beta-1)}\right) - \left[\frac{-\left(\alpha(\beta-1)Q^{(\beta-2)}\left(\theta + \alpha Q^{\beta-1}\right)^{-2}\right)}{2}\left(\theta Q + \alpha Q^{\beta}\right)\right]\right] \quad (10)$$

Solving (10) we get $Q^*$. When $Q^* < W$, we use single warehouse inventory system. Otherwise, we use two warehouse inventory systems.

In the next section 3.2, we will show the mathematical formulation of the two-warehouse system.

### 3.2 Two-warehouse inventory model formulation

At the beginning of each replenishment cycle, the system receives Q units out of which W units are kept in OW and the remaining parts are kept in RW. Items in OW are used to satisfy customers' demand until the inventory level in OW drops to (W-q) units. At this moment, q units from rented warehouse are shipped to OW to restore the stock to the original level W. Then the process is repeated until m shipments are completed. After the $m^{th}$ shipment, no units are left in RW. The remaining W units in OW are used up to the end of the replenishment cycle. The order quantity for each cycle is W+ m q =Q and the inventory level I (t) in the $i^{th}$ shipment cycle in OW satisfies the following differential equation:

$$\frac{dI(t)}{dt} = -\theta I(t) - R(t) \; ; \qquad (i-1)T_0 \le t \le iT_0, i = 1,2,...n \quad (11)$$

With the boundary condition $I(i-1)T_0$ =W,

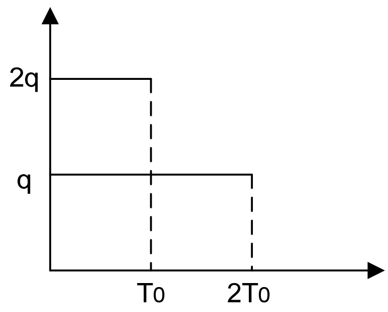


Fig.1 Inventory level for n=2 in RW

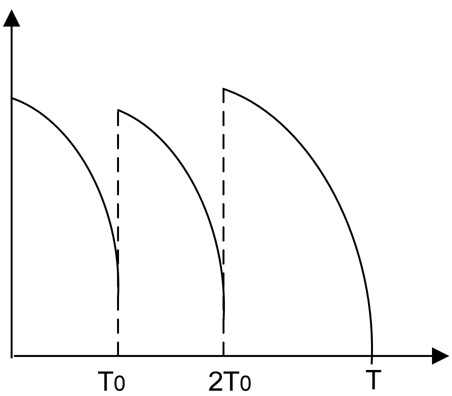


Fig.2 Inventory level for n=2 in OW

Solution of the differential equation (1) is

$$I(t) = W - (\theta W + \alpha W^{\beta})(t-(i-1)T_0); \quad (i-1)T_0 \le t \le iT_0, i = 1,2,...n \tag{12}$$

Thus the inventory level in OW at the end of the shipment of the i$^{th}$ shipment cycle becomes

$$I(iT_0) = W - (\theta W + \alpha W^{\beta})T_0; \tag{13}$$

And the amount of items transported from RW to OW in each shipment cycle is

$$q = W - I(iT_0) = (\theta W + \alpha W^{\beta})T_0 \tag{14}$$

$$T_0 = (\theta W + \alpha W^{\beta})^{-1} \tag{15}$$

This indicates that the shipment cycle length $T_0$ depends on the variable q.

The holding cost of items in OW in the shipment period is

$$HC = h_{OW} \int_{(i-1)T_0}^{iT_0} I(t)dt$$

$$= h_{OW} T_0 \left(W - (\theta W + \alpha W^{\beta})\frac{T_0}{2}\right) \tag{16}$$

The inventory level I(t) in OW in the interval $[mT_0, T]$ is given by

$$I(t) = W - (\theta W + \alpha W^{\beta})(t - mT_0); \quad mT_0 \le t \le T \tag{17}$$

$$I(T) = 0, \text{ (7) implies } T = m\,T_0 + \frac{1}{(\theta + \alpha W^{\beta-1})} \tag{18}$$

The holding cost of items in the interval [m $T_0$, T] in OW is

$$\int_{mT_0}^{T} h_{OW}\, I(t)\,dt = h_{0W} \int_{mT_0}^{T} \left[W - \left(\theta W + \alpha W^{\beta}\right)\right]\left[t - mT_0\right]dt$$

$$= h_{0W}\left[ W(T - mT_0) - (\theta W + \alpha W^{\beta})\left(\frac{(T - mT_0)^2}{2}\right)\right] \tag{19}$$

So the holding cost of the item in OW is given by

$$HC_{OW} = \left( \begin{array}{l} h_{ow} T_0 \,(W - (\theta W + \alpha W^{\beta})\frac{T_0}{2}) + \\ h_{ow}\left[ W(T - mT_0) - (\theta W + \alpha W^{\beta})\left(\frac{(T - mT_0)^2}{2}\right)\right] \end{array} \right) \tag{20}$$

The holding cost of the items in RW is

$$HC_{RW} = h_{RW}\left[mqT_0 + (m-1)qT_0 + \ldots + 2qT_0 + qT_0\right]$$

$$= h_{RW}\; q\, m\, (m+1)\frac{T_0}{2} \tag{21}$$

Therefore, the average total profit of the system can be expressed as

$$TP = \frac{1}{T}\,(P' - P)Q - \frac{1}{T}K_1 - \frac{1}{T}mT_c\,(q) - \frac{1}{T}h_{OW}\, T_0 (W - (\theta W + \alpha W^{\beta})\frac{T_0}{2}) -$$

$$\frac{1}{T}h_{0W}\left[ W(T - mT_0) - (\theta W + \alpha W^{\beta})\left(\frac{(T - mT_0)^2}{2}\right)\right] - \frac{1}{T}h_{RW}\, q\, m(m+1)\frac{T_0}{2} \tag{22}$$

The average total profit function depends only on the discrete variable m and a continuous variable q (denoted by TP (m, q)). Now our objective is to find the optimal values of m and q in order to keep TP (m, q) maximum. The average total profit is =(1/T){sales revenue- setup cost-holding cost in own warehouse- holding cost in the rented warehouse}

Therefore, the average total profit function can be written as follows:

$$TP_1\,(m, q) = \frac{1}{T}\,(P' - P)Q - \frac{1}{T}K_1 - \frac{1}{T}mA - \frac{1}{T}h_{OW}\, T_0 (W - (\theta W + \alpha W^{\beta})\frac{T_0}{2}) -$$

$$\frac{1}{T}h_{0W}\left[ W(T - mT_0) - (\theta W + \alpha W^{\beta})\left(\frac{(T - mT_0)^2}{2}\right)\right]$$

$$- \frac{1}{T}h_{RW}\, q\, m(m+1)\frac{T_0}{2} \quad \text{for} \quad 0 < q \le y \tag{23}$$

$$TP_2(m, q) = \frac{1}{T}\left(P'-P\right)Q - \frac{1}{T}K_1 - \frac{1}{T}m\text{A} - \frac{1}{T}\text{m b (q-y)} - \frac{1}{T}h_{OW}T_0(\text{W}-(\theta W + \alpha W^{\beta})\frac{\text{T}_0}{2}) - \frac{1}{T}h_{0W}\left[\text{W(T - mT}_0) - (\theta\text{W} + \alpha\text{W}^{\beta})\left(\frac{(\text{T - mT}_0)^2}{2}\right)\right] - \frac{1}{T}h_{RW}\,q\,m(m+1)\frac{T_0}{2}$$

$$\text{for } \; y< q \le \text{W} \tag{24}$$

In order to find the optimum solution we use the following necessary and sufficiency conditions.

The necessary conditions for $TP_{RW}$ (Q) to be maximum are:

- $\frac{d\text{TP}_{1\ \text{RW}}}{\text{dQ}} = 0 \; and \; \frac{d^2\text{TP}_{1\ \text{RW}}}{\text{dQ}^2} < 0$ and
- $\frac{d\text{TP}_{2\ \text{RW}}}{\text{dQ}} = 0 \; and \; \frac{d^2\text{TP}_{2\ \text{RW}}}{\text{dQ}^2} < 0$

$\frac{d\text{TP}_{1\ \text{RW}}}{\text{dQ}} = 0$ and using the equations W+ m q = Q ,(5) we have

$$\left\{\begin{matrix} \left(\begin{matrix} \left(P'-P\right)(\text{W}+\text{mq}) - K_1 - mA \\ -\text{h}_{\text{OW}}\begin{bmatrix}\text{Wq}(\theta\text{W}+\alpha\text{W}^{\beta})^{-1} - \\ (\theta\text{W}+\alpha\text{W}^{\beta})^{-1}\frac{\text{q}^2}{2}\end{bmatrix} \\ -\text{h}_{\text{OW}}\begin{bmatrix}W(\theta+\alpha\text{W}^{\beta-1})^{-1} - \\ (\theta+\alpha\text{W}^{\beta-1})^{2}\frac{(\theta W+\alpha\text{W}^{\beta})}{2}\end{bmatrix} \\ -\text{h}_{\text{RW}}\left[\frac{\text{q}^2}{2}(\theta\text{W}+\alpha\text{W}^{\beta})^{-1}m(m+1)\right] \end{matrix}\right) \\ \left[mq\left((\theta\text{W}+\alpha\text{W}^{\beta})^{-1} + (\theta+\alpha\text{W}^{\beta-1})^{-1}\right)\right]^{-2} \end{matrix}\right\} * \left\{\begin{bmatrix} \left(\begin{matrix} \left(P'-P\right)m - \\ \text{h}_{\text{OW}}\begin{bmatrix}\text{W}(\theta\text{W}+\alpha\text{W}^{\beta})^{-1} - \\ (\theta\text{W}+\alpha\text{W}^{\beta})^{-1}q\end{bmatrix} \\ -\text{h}_{\text{RW}}\begin{bmatrix}q(\theta\text{W} \\ +\alpha\text{W}^{\beta})^{-1}m(m+1)\end{bmatrix}\end{matrix}\right) \\ *\begin{bmatrix} m\left[-(\theta\text{W}+\alpha\text{W}^{\beta})^{-1}\right] + \\ \left[mq\begin{pmatrix}(\theta\text{W}+\alpha\text{W}^{\beta})^{-1} + \\ (\theta+\alpha\text{W}^{\beta-1})^{-1}\end{pmatrix}\right]^{-1}\end{bmatrix} \end{bmatrix}\right\} = 0$$

$$for \; 0 < q \le y \tag{25}$$

$\frac{d\text{TP}_{2\ \text{RW}}}{\text{dQ}} = 0$ and using the equations W + m q = Q , (5) we have:

$$
\left\{ \begin{pmatrix} (P'-P)(\mathrm{W}+\mathrm{mq})-K_1-mb(q-y) \\ -\mathrm{h_{OW}}\begin{bmatrix} \mathrm{Wq}(\theta\mathrm{W}+\alpha\mathrm{W}^{\beta})^{-1}- \\ (\theta\mathrm{W}+\alpha\mathrm{W}^{\beta})^{-1}\dfrac{\mathrm{q}^2}{2} \end{bmatrix} \\ -\mathrm{h_{OW}}\begin{bmatrix} W(\theta+\alpha\mathrm{W}^{\beta-1})^{-1}- \\ (\theta+\alpha\mathrm{W}^{\beta-1})^{2}\dfrac{(\theta W+\alpha\mathrm{W}^{\beta})}{2} \end{bmatrix} \\ -\mathrm{h_{RW}}\left[\dfrac{\mathrm{q}^2}{2}(\theta\mathrm{W}+\alpha\mathrm{W}^{\beta})^{-1}m(m+1)\right] \end{pmatrix} \left(\left[mq\left((\theta\mathrm{W}+\alpha\mathrm{W}^{\beta})^{-1}+(\theta+\alpha\mathrm{W}^{\beta-1})^{-1}\right)\right]^{-2}\right) \right\} * \left\{ \begin{pmatrix} (P'-P)m-mb \\ -\mathrm{h_{OW}}\begin{bmatrix} \mathrm{W}(\theta\mathrm{W}+\alpha\mathrm{W}^{\beta})^{-1}- \\ (\theta\mathrm{W}+\alpha\mathrm{W}^{\beta})^{-1}q \end{bmatrix} \\ -\mathrm{h_{RW}}\begin{bmatrix} q(\theta\mathrm{W} \\ +\alpha\mathrm{W}^{\beta})^{-1}m(m+1) \end{bmatrix} \end{pmatrix} * \begin{pmatrix} m\left[-(\theta\mathrm{W}+\alpha\mathrm{W}^{\beta})^{-1}\right] \\ +\left[mq\begin{pmatrix} (\theta\mathrm{W}+\alpha\mathrm{W}^{\beta})^{-1} \\ +(\theta+\alpha\mathrm{W}^{\beta-1})^{-1} \end{pmatrix}\right]^{-1} \end{pmatrix} \right\} = 0
$$

$$
for \;\; y < q \le W \qquad (26)
$$

Our goal is to maximize the total profit given in equations (23) and (24), for that we study the following results. Summarizing the above results, we can now establish the following solution procedure to obtain the optimal solution of our problem.

**3.3 Solution Procedure**

The following solution procedure can be shown for the decision maker to determine the optimum quantity per shipment and the optimal profit when the number of transporting items from RW to OW is fixed.

**I. To find q*, TP*:**

***Step 1*:** Fix m,

***Step 2*:** Using (15), find q (Use Matlab).

***Step 3*:** If $0 \le q \le y$, find $TP_1(q)$, otherwise find $TP_1(y)$

***Step 4*:** Using (16), find $q_1$.

***Step 5:*** If $y < q \le w$, find $TP_2(q)$, otherwise find $TP_2$ (y)

***Step 6*:** $\mathrm{TP}^{*}_{\mathrm{RW}} = \text{maximum}\left\{\mathrm{TP}_{1\ \mathrm{RW}}, \mathrm{TP}_{2\ \mathrm{RW}}\right\}$

The following solution procedure can be shown for the decision maker to determine the optimal values of the number of transporting items from RW to OW, the optimum quantity per shipment when m is a variable, optimal order quantity and the optimal replenishment cycle for a two- warehouse system.

**II. To find m***

For doing this,

***Step 7*:** Let m = 1, 2, 3, …. respectively, determine the corresponding shipment quantity, q*(m), and the average total profit $\mathrm{TP}_{\mathrm{RW}}$(m), and compare the total profits of the system $\mathrm{TP}_{\mathrm{RW}}$(1), $\mathrm{TP}_{\mathrm{RW}}$(2), $\mathrm{TP}_{\mathrm{RW}}$(3),…… The values of m and q*(m) yielding the biggest value of average total profit are taken to be the optimal values of m and q.

***Step 8*:** Substituting the values of m and q into Q = W+ m Q and T = m $T_0$+ $(\theta + \alpha W^{\beta-1})$ we can obtain the optimal replenishment quantity and the optimal cycle of the two-warehouse system.

***Step 9*:** Solve equation (10) for finding Q* and compute TP*(Q) from equation (9) and T* from equation (8) for a single warehouse system.

We can easily find the optimal replenishment policies using the above solution procedure. In order to strengthen the proposed model to maximize the profit we derive some theoretical results.

### 3.4. Theoretical results

The necessary and sufficiency condition for the decision- maker to decide whether or not to rent RW to hold more items under the situation defined above is given in Theorem 1. Theorem 2 shows the relevant total profit in the rented warehouse. Theorem 3 identifies the maximum total profit of the model.

**Theorem 1:** The optimal order quantity Q* of the single-warehouse system is no less than W if and only if $(P''-P)\theta Q^{(2-\beta)}+(P''-P)\alpha\beta+A\alpha(1-\beta)\geq\frac{h_o}{2}Q^{3-\beta}$.

***Proof:*** For notational convenience, let $f(Q)=(P''-P)\theta Q^{(2-\beta)}+(P''-P)\alpha\beta+A\alpha(1-\beta)$ and $G(Q)=\frac{h_o}{2}Q^{3-\beta}$ (say), then the optimal replenishment quantity Q* for the single warehouse system satisfies the relation f(Q*) = G(Q*).If $W\leq Q^*$, $f(W)\geq G(W)$, it can be easily seen from fig. 3. i.e. $(P''-P)\theta Q^{(2-\beta)}+(P''-P)\alpha\beta+A\alpha(1-\beta)\geq\frac{h_o}{2}Q^{3-\beta}$.Conversely , if $f(W)\geq G(W)$,then $W\leq Q^*$,it can be shown in fig.3.

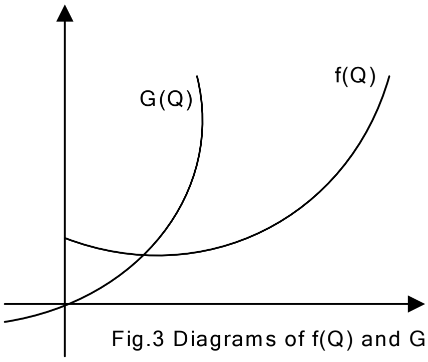

Fig.3 Diagrams of f(Q) and G(Q)

The following theorem is useful to find what order lot-size shipment policy from RW to OW the decision- maker should make if he needs indeed to rent RW.

**Theorem 2:** When $(P''-P)\theta Q^{(2-\beta)}+(P''-P)\alpha\beta+A\alpha(1-\beta)\geq\frac{h_o}{2}Q^{3-\beta}$, for any given y, the maximum of $TP_1$ with q- range constraint is arrived at q = q* if q* < y; otherwise at q = y, and the maximum of $TP_2$ with q- range constraint is arrived at q = q* if q* > y; otherwise at q = y.

***Proof:*** Using (25), we observe that $\frac{dTP_{1\ RW}(0)}{dQ}>0$ and $\frac{dTP_{1\ RW}}{dQ}=-\infty$.Hence there must be a unique root, q* to equation (25) in the interval (0, W) and $\frac{d^2TP_{1\ RW}}{dQ^2}<0$, q* is the global maximum point of $TP_1$ in the interval if

$(P''-P)\theta Q^{(2-\beta)}+(P''-P)\alpha\beta+A\alpha(1-\beta)\geq\frac{h_o}{2}Q^{3-\beta}$. When considering the q-range constraints the maximum of $TP_1$ will still remain at q = $q_1$* if $q_1$*< y. If $q_1^*\geq y$, the maximum of $TP_1$ will be at q = y. Using (26), we observe that $\frac{d TP_{2\ RW}(0)}{dQ} > 0$ and $\frac{d TP_{2\ RW}}{dQ} = -\infty$ .Hence there must be a unique root, q* to equation (26) in the interval (0, W) and $\frac{d^2 TP_{2\ RW}}{dQ^2} < 0$, q* is the global maximum point of $TP_2$ in the interval if (P″-P)$\theta Q^{(2-\beta)}$+ (P″-P)$\alpha\beta$ + A$\alpha(1-\beta) \geq \frac{h_o}{2}$ $Q^{3-\beta}$. When considering the q - range constraints the maximum of $TP_2$will still remain at q = $q_2$* if $q_2$* > y. Otherwise, maximum of $TP_2$ will be at q = y.

The following theorem shows when the decision- maker can obtain much more profit from renting RW to order items of more than W units.

**Theorem 3:** If (P'-P)$\theta Q^{(2-\beta)}$+ (P'-P)$\alpha\beta$ + A$\alpha(1-\beta) \geq \frac{h_o}{2}$ $Q^{3-\beta}$, the two-warehouse model's maximal profit is larger than one of the single- warehouse model.

***Proof:*** For a specified m-value, the maximum of the average total profit function, is the larger of $TP_1$* and $TP_2$*. The average total profit reaches its maximum at the inner point, q = q*, in the interval (0, W) for an arbitrarily given m if
(P'-P)$\theta Q^{(2-\beta)}$+ (P'-P)$\alpha\beta$ +A$\alpha(1-\beta) \geq \frac{h_o}{2}$ $Q^{3-\beta}$. This indicates that when (P'-P)$\theta Q^{(2-\beta)}$+ (P'-P)$\alpha\beta$ +A$\alpha(1-\beta) \geq \frac{h_o}{2}$ $Q^{3-\beta}$, the maximal average total profit $TP_{RW}$(m*,q*) for the inventory system with two warehouses should be greater than $TP_{RW}$(0,0).From (7) and (22), we obtain $TP_{RW}(0,0) > TP_{ow}(W)$. Thus,
TP(m*,q*) > $TP_{ow}$(W) if (P'-P)$\theta Q^{(2-\beta)}$+ (P'-P)$\alpha\beta$ +A$\alpha(1-\beta) \geq \frac{h_o}{2}$ $Q^{3-\beta}$.

**Note:** For calculation point of view
let S = (P'-P)$\theta Q^{(2-\beta)}$+(P'-P)$\alpha\beta$ +A$\alpha(1-\beta)$-$\frac{h_o}{2}$ $Q^{3-\beta}$ (say).

The purpose of this section is to illustrate the results of our models and demonstrate the performance of the solution procedures presented in section 3.3. Numerical examples are presented in section 4. The optimal replenishment policies for a two-warehouse model are shown in the Table 1 and for a single-warehouse models are shown in Table 2. Sensitivity analyses of various parameters are shown in Table 3.

## 4. Numerical Examples and Sensitivity Analyses

To illustrate the model let us consider the following examples:
Let us consider $K_1$= 13, K = 10, P″ = 20, P = 10, W = 400, β = 0.1, b = 0.2, y = 20, $h_{RW}$ = .8, θ β = 0.2, α= 20, A = 4 in appropriate units.

**Table 1:** Optimal values of the model for two- warehouse system

| m | q | T | Q | TP | S |
|---|---|---|---|---|---|
| 40 | 29.8637 | 7.8276 | 1794.5474 | 1182.9223 | + |

**Table 2:** Optimal values of the model for single- warehouse system

| Q | T | TP | S |
|---|---|---|---|
| 444.4446 | 3.5362 | 595.2341 | + |

The numerical values displayed in Table 1 and Table 2 indicates that for two- warehouse system, total profit is greater than that of single- warehouse system. Duration of the cycle time and the optimum order quantity of the two- warehouse system is greater than that of single- warehouse system. So, two-warehouse system under deterioration maximizes the profit.

In order to study how various parameters affect the optimal solution of the proposed inventory model, sensitivity analysis is performed. Keeping all the other parameters fixed and varying a single parameter at a time, for the same set of values we study the results. The results of the various parameters against the profit of our model are shown in the following Table.

**Table 3:** Sensitivity analysis of parameters

$\alpha$= 20, $\beta$= 0.2,

| $\theta$ α | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| .1 | 30 | 29.94 | 7.03 | 1498.34 | 1018.64 | + |
| .2 | 40 | 29.6 | 6.49 | 1783.97 | 1152.83 | + |
| .3 | 47 | 29.99 | 6.02 | 2009.63 | 2040.62 | + |
| .4 | 53 | 29.86 | 5.59 | 2182.46 | 2573.87 | + |
| .5 | 57 | 29.85 | 5.19 | 2301.26 | 3125.13 | + |

$\alpha$= 20, $\beta$= 0.3,

| α$\theta$ | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| .1 | | | 3.68 | 892.69 | 1932.26 | – |
| .2 | 43 | 29.86 | 5.44 | 1884.14 | 2127.14 | + |
| .3 | 48 | 29.59 | 5.02 | 2020.08 | 2674.36 | + |

$\theta$ = 0.2, $\beta$= 0.4,

| α | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| 6 | 40 | 29.8 | 6.36 | 1790.72 | 1576.3 | + |
| 9 | 42 | 29.81 | 5.69 | 1852.18 | 1939.72 | + |
| 12 | | | 2.41 | 813.58 | 2761.57 | − |
| 16 | | | 2.41 | 1313.59 | 4460.62 | − |

$\alpha$ = 20, $\beta$= 0.1,

| $\theta$ | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| .1 | 28 | 29.04 | 8.47 | 1413.09 | 707.00 | + |
| .2 | 40 | 29.86 | 7.83 | 1794.55 | 1182.92 | + |
| .3 | 51 | 29.53 | 7.27 | 2106.10 | 1670.63 | + |
| .4 | 59 | 29.83 | 6.8 | 2360.22 | 2174.03 | + |

$\alpha$=16, $\beta$=0.4,

| $\theta$ | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| .2 | | | 2.41 | 1313.59 | 4460.62 | – |
| .5 | 43 | 29.47 | 3.35 | 1876.22 | 4612.16 | + |
| .6 | 34 | 29.53 | 2.72 | 1603.85 | 5294.32 | + |

α =12, β =0.4,

| θθ | θ | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|---|
| .09 | | | | 4.46 | 1780.67 | 2938.53 | – |
| .2 | | | | 2.41 | 813.58 | 2761.57 | – |
| .5 | | 49 | 29.48 | 3.95 | 2054.22 | 4035.8 | + |
| .6 | | 45 | 29.79 | 3.46 | 1940.34 | 4669.71 | + |

α =9, β=0.4,

| θ | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| .09 | | | 4.45 | 1102.78 | 1819.16 | – |
| .2 | 42 | 29.81 | 5.69 | 1852.18 | 1939.72 | + |
| .4 | 51 | 29.75 | 4.85 | 2117.47 | 3036.14 | + |
| .5 | 52 | 29.75 | 4.41 | 2159.72 | 3615.23 | + |

α= 6, θ = 0.2

| β | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| .1 | 45 | 29.97 | 3.81 | 1948.54 | 3908.26 | + |
| .2 | 45 | 29.61 | 9.65 | 1932.59 | 974.76 | + |
| .3 | 40 | 29.67 | 7.64 | 1786.28 | 1215.054 | + |
| .4 | 40 | 29.8 | 6.36 | 1790.72 | 1576.3 | + |

α = 20, β= .4, θ = 0.2

| P' | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| 12 | 11 | 29.4857 | 2.1886 | 924.3430 | 173.6955 | + |
| 15 | 22 | 29.0168 | 4.0328 | 1238.3707 | 620.3606 | + |
| 30 | | | 5.2246 | 1766.0095 | 5519.1613 | + |

α= 20, β= 0.4, θ = 0.2

| P | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| 5 | | | 4.9052 | 1455.780 | 3498.2554 | – |
| 8 | | | 4.6568 | 1250.4467 | 2445.5854 | – |
| 15 | 22 | 29.0168 | 4.0328 | 1238.3707 | 620.3606 | + |

Θ= 20, β= 0.4, θ =0.2

| y | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| 15 | | | 2.4078 | 1904.9934 | 6470.1727 | – |
| 50 | | | 2.4078 | 1904.9934 | 6470.1727 | – |
| 60 | | | 2.4078 | 1904.9934 | 6470.1727 | – |

α= 20, β = 0.4, θ = 0.2

| b | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| .1 | | | 2.4078 | 1904.9934 | 6470.1727 | – |
| .6 | | | 2.4078 | 1904.9934 | 6470.1727 | – |
| .9 | | | 2.4078 | 1904.9934 | 6470.1727 | – |

α= 20, β= 0.4, θ = 0.2

| $h_{oW}$ | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| .7 | | | 2.2417 | 1524.9305 | 5304.140 | – |
| .9 | | | 2.1176 | 1288.6612 | 4542.1401 | – |
| 3.0 | | | 2.656 | 2653.24 | 8628.3118 | – |

α= 20, β= 0.4, $\theta$ = 0.2

| W | m | q | T | Q | TP | S |
|---|---|---|---|---|---|---|
| 300 | | | 4.45 | 1102.78 | 1819.16 | – |
| 500 | | | 2.41 | 1904.99 | 6470.17 | – |

From the above numerical examples we have the following results:

- Total profit and optimal replenishment quantity are more sensitive to alpha than that of beta.
- Total profit and optimal replenishment quantity are more sensitive to $h_{oW}$ than that of $h_{RW}$.
- Theta and the optimal replenishment quantity are inversely proportional.
- Theta and the total profit are directly proportional.
- Theta and the optimal replenishment duration are inversely proportional.
- Selling price and the optimal replenishment duration, total profit, optimal replenishment quantities are directly proportional.
- Purchasing cost, the optimal replenishment duration and total profit are inversely proportional.
- Total profit and optimal replenishment quantity are more sensitive to selling price than that of purchasing cost.
- Compared to other parameters y, b, $h_{RW}$ on the optimal policies are less effective.
- Compared to the parameters alpha, beta and theta the number of transporting items from RW to OW are more sensitive than that of the quantity per shipment.
- Purchasing cost and the optimal replenishment quantities are directly proportional.

The purpose of this section is to give some practical application of our model.

### 4.1 Managerial Implications

- In order to maximize the total profit the retailer should minimize the holding cost of the own warehouse.
- If the retailer wants to maximize the profit he could control the deterioration rate of the stored items.
- When the retailers control the deterioration rate, it will minimize the number of replenishment which gives them maximum profit.
- Without controlling the deterioration rate the retailers' durations of the items are less, that will increase the transportation cost and number of shipments from rented warehouse to own warehouse leads to reduce the profit.

The above stated managerial implications are also suitable for manufacturers.

## 5. Conclusions

In this paper, a two warehouses inventory replenishment model for a single item under inventory-dependent demand rate with deterioration is discussed. Deterioration is the main problem in inventory models. So this assumption is more realistic. Zhou and Yang [27] is a special case of our model. Moreover, they did not consider deterioration in their model. The necessary and sufficiency condition for the decision- maker to decide whether or not to rent RW to hold more items under the situation defined above is given in Theorem 1. Theorem 3 identifies the maximum total profit of the model. Theorem 2 shows the relevant total profit in the rented warehouse. Furthermore, sensitivity analysis not only justifies our theoretical results but also provides many reasonable managerial results.

This model can be extended in many ways. We could extend the deterministic model into a stochastic model. Finally, we could generalize the model to allow quantity discounts, shortages, inflation, etc.

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# Interpretation of Data Showing Something has One Effect Sometimes and a Different Effect in other Circumstances: Theories of Interaction of Factors

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## Abstract
A possible explanation of interaction is that quantities derived from the independent variables separately add together, but then a curvilinear relationship intervenes between their total and the dependent variable observed. It is shown that two different theories of this type are always available to explain crossover interaction in a 2x2 table. For example, one theory may say that a good outcome occurs when there is an approximate match between values associated with the independent variables, and the other theory that a good outcome occurs when the total of values associated with the independent variables is either decisively small or large, with poorer outcome resulting from intermediate values.

## Introduction
Factorial experimentation involves manipulating two (or more) x's, and observing y at all combinations of the values of $x_1$ and $x_2$. If the effect on y of $x_1$ depends upon what $x_2$ is, there is said to be an interaction between the x's.

A possible explanation of interaction is that (a) quantities derived from the independent variables separately add together, but (b) a curvilinear relationship intervenes between their total and the dependent variable observed (see Hutchinson, 2004). This idea is not new. Indeed, once it comes to mind, it is a very obvious one. The reason it is worth publicising is that it seems not to be widely taught and not to come to researchers' minds spontaneously.

Interaction was found, for example, in computer simulation of logistics operations by McGee et al. (2005). Consider a logistics system that supports operations, and the effects of characteristics of the system on the operational availability of equipment. Capability to use express service for shipments of parts and capability to repair parts locally will both be good, but to some extent they are substitutes for each other, and having both will not be much of an improvement on having one of them.

A common pattern in reports of research is for interaction to be found to be statistically significant, for the researchers to make a song and dance about the novelty and importance of this, but then for no substantive explanation or interpretation to be given.

The next Section discusses the results of McGee et al. (2005). The explanation that will be proposed is one of positive but decreasing returns for effort (i.e., the slope of the curvilinear relationship is positive but decreasing). Attention then turns to curvilinearity that actually reverses in direction, and leads to crossover interaction. Not merely one, but two simple and attractive explanations for interaction can always be found, if there are only two factors and only two categories of each. They may be relevant to, for example, the idea that that the tone of a message needs to match the personality of the audience receiving it, if attitude or behaviour change is to occur. Finally, there is a short discussion Section.

## A Logistics Simulation Example

McGee et al. (2005) had several factors in their study rather than two. It turns out that the specific pattern of the interactions gives some evidence for a theory in which things add together but then there is curvilinear dependence on the total. The results in McGee et al. indicate that all of the following four factors are good for the measure of performance, operational availability:

(A) Waiting until there is a truckload of parts is not necessary,
(D) Shipments may be made by express service,
(G) Equipment is reliable,
(J) There is capability to repair parts locally.

(The capital letters are identifiers as in McGee et al., but in the case of statements (A) and (D), positive coding in McGee et al. corresponded to the negation of the statement.)

All of those results of factors considered singly are in accordance with common sense. What is of interest is that there are interactions of (D) with each of (A), (G), and (J), and that these interactions are such that (D) being true is less important if (A) is true, if (G) is true, and if (J) is true. This rather suggests that positive quantities derived from (A), (D), (G), and (J) being true combine additively, with operational availability improving less than linearly with the sum. (This would imply that the other two-way interactions between these variables do exist, even though they were not large enough to be reported in McGee et al.)

The present paper is concerned with understanding a phenomenon, interaction, that may appear even in 2x2 tables. For simulation experiments, an enormous design space may be of interest --- there is a case study in Kleijnen et al., 2005, for which the starting point was 40 factors each at 40 levels --- and it should be conceded that the priorities for data processing may be utterly different. Note also that when interaction is only quantitative, rather than crossover, there is a choice over how seriously to take it, as it may be possible to find some reasonably simple and meaningful transformation of the dependent variable such that there is no longer any interaction.

McGee et al. were concerned with logistics in a military context. Other examples of interaction may be found in actual military operations. From a defender's point of view, it is bad to make mistakes, and it is bad if the ground attacker has good technology --- but the effect of a technology may be much greater when the defender makes a certain type of mistake, leading to utterly one-sided combat. Examples of this include the combination of air superiority and failure to detect a ground attack (being surprised is a disaster when the defenders are sheltering rather than manning their vehicles), and the combination of advanced sights and inadequate concealment of targets (advanced sights that penetrate darkness and sandstorm are only useful if the defenders fail to hide their vehicles behind a hill). Interactions between different errors by the defenders and between defenders' errors and attackers' technology were examined by Biddle (1996).

## Argument for 3 x 3 Experiments, Rather than 2 x 2

Let the categories of $x_1$ be A and B, the categories of $x_2$ be C and D, and the observations of the dependent variable y be as follows.

| | C | D |
|---|---|---|
| A | 1 | 4 |
| B | 3 | 2 |

There is "crossover" interaction: moving from C to D increases the response in condition A, but decreases it in condition B. Further, moving from A to B increases the response in condition C, but decreases it in condition D.

Suppose we are lucky enough to have a theory that specifies what it is about $x_1$ and $x_2$ that is adding together and determining the level of response, and roughly how much of it is associated with A, B, C, and D. Without loss of generality, these amounts can be taken as 0, 1, 0, and 2, respectively.

- First, let us add these together, and let the result be a total t.

| | C (0) | D (2) |
|---|---|---|
| A (0) | 0 | 2 |
| B (1) | 1 | 3 |

The totals t, in order from 0 to 3, are shown below along with the corresponding y's.

| | | | | |
|---|---|---|---|---|
| t: | 0 | 1 | 2 | 3 |
| y: | 1 | 3 | 4 | 2 |

Thus a theory in which the quantities add together, and then y is an inverted-U shaped function of the result (i.e., it first increases, then decreases), will explain the dataset.

- Second, let us subtract the quantities, and call the result a difference d.

| | C (0) | D (2) |
|---|---|---|
| A (0) | 0 | 2 |
| B (1) | -1 | 1 |

The differences d, in order from -1 to 2, are shown below along with the corresponding y's.

| | | | | |
|---|---|---|---|---|
| d: | -1 | 0 | 1 | 2 |
| y: | 3 | 1 | 2 | 4 |

Thus a theory in which the quantities are subtracted, and then y is a U-shaped function of the result (i.e., it first decreases, then increases), will explain the dataset.

Thus there will always be two different theories available to explain a 2x2 table. (Even more theories will be available if there are no preconceptions about the quantities associated with $x_1$ and $x_2$.) In the case where small values of y are better than high values, the first theory says that a good outcome occurs when the total t is either decisively small or decisively large, with poorer outcome resulting from intermediate values; and the second theory says that a good outcome occurs when there is an approximate match between the values associated with $x_1$ and $x_2$. Hutchinson (2008) discussed this in the context of the dependence of house prices on characteristics of the house and characteristics of its location. The two competing theories there were that for a house to be highly valued, (a) it and its location should in total be either highly urban or highly suburban, not in between, or (b) there should be a match between the characteristics (in an urban vs. suburban sense) of the house and its location.

Are these really different theories, or is one somehow a disguised version of the other? How can a decision be made between them? Yes, they are different, as can be seen by considering the result when a category intermediate between A and B is paired with a category intermediate between C and D.

- The sum of two intermediate quantities is intermediate, neither decisively small nor decisively large, so the first theory predicts the outcome will be poor.

- There is an approximate match between the category of $x_1$ and the category of $x_2$, so the second theory predicts the outcome will be good.

Consequently, a 3x3 table of results will enable us to decide between the two theories.

## Example Concerning Compatibility Between a Message and Its Audience

Attempts to change attitudes and behaviour have often had disappointing results. Yet great changes in public sentiment have occurred in regard to some issues in recent decades (e.g., smoking is less tolerated). An idea that has been proposed to explain this variation is that the tone of a message needs to match the personality of the audience receiving it. If the necessity of matching message to audience is a reality, it refers to crossover interaction: one thing is superior to another in condition 1, but is inferior in condition 2.

Goldstein (1959) found that a strong fear appeal receives greater acceptance among those he referred to as copers than among those he referred to as avoiders, while a minimal fear appeal receives greater acceptance among avoiders than among copers. He was able to refer to other literature supporting the idea of individual differences in reactions. There has been much subsequent research. According to a review by Atkin (2001, p. 23), "Effectiveness can be increased if message content, form, and style are tailored to the predispositions and abilities of the distinct subgroups". Later in that review (pp. 31-32), there is discussion of mechanisms causing health campaigns to fail. These mechanisms will apply to some audiences and in some circumstances, while for other audiences and in other circumstances, the campaign would have its intended effect. Evidently, then, the hypothesis is that what matters is the difference between some aspect of presentational style (e.g., how graphic and threatening it is) and some aspect of the people receiving the message (e.g., the extent to which they are sensation seekers), with effectiveness declining either side of some optimum. Jones and Owen (2006) draw attention to the variety of different findings concerning the effect of level of threat on likelihood of behavioural change, including the possibility of an inverted-U relationship.

The implication of the present paper is that if it is credible that maximum change in attitude or behaviour occurs when the difference between excitement (for example) of the message and of the audience is small, it will also be possible to invent a theory saying that excitement as a characteristic of the message and excitement as a characteristic of the audience add together, and maximum change occurs when the total is either small or big and is smaller in between. But perhaps it is unappealling that maximum change occurs when total excitement is either small or big and is smaller in between? The rely to this is that it is difficult to move from an abstract model to an appropriate name, and the problem may lie in the name. Suppose that "excitement" is really a distinction between excitement and rationality. Theorise that maximum change occurs when either total excitement is big or total rationality is big (and is smaller in between) --- the idea that was unappealling now has a certain plausibility.

## Discussion

To interpret the results from the multi-factor study of McGee et al., there were several steps.

- Propose a theory, in the sense of identifying that several things are expected to have a positive effect.
- Code the factors in such a way that all their individual effects will be positive.
- After fitting a model with individual effects and interactions, examine whether the interactions are positive or negative.
- When the interactions are found to be negative, conclude that it appears there are decreasing returns for effort.

Having a theory also appears important when there is crossover interaction.

- Faced with the puzzle of opposite effects in different circumstances, even the general idea of a U-shaped or inverted-U shaped dependence is a step forward.

- Two specific proposals are potentially of wide application: small sum or large sum both being good, or small difference being good.
- Then one needs some reason to think that category E (for example) lies between A and B on factor $x_1$ and that category F (for example) lies between categories C and D on factor $x_2$: the result for the combination EF will decide between the two proposals.

When an interaction is found, there is naturally a demand for theory to explain the complicated result. It seems fair to conclude that quite simple ideas may help, and may even suggest future lines of research.

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[2] Simon, H.A. The New Science of Management Decision. Rev. Ed. Prentice-Hall, N.J. (1977).

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